NANYANG TECHNOLOGICAL UNIVERSITY

School of Mechanical & Aerospace Engineering

Design, Machine, Control, Intelligence

MA4825

Robotics

Xie Ming, PhD (France)

mmxie@ntu.edu.sg

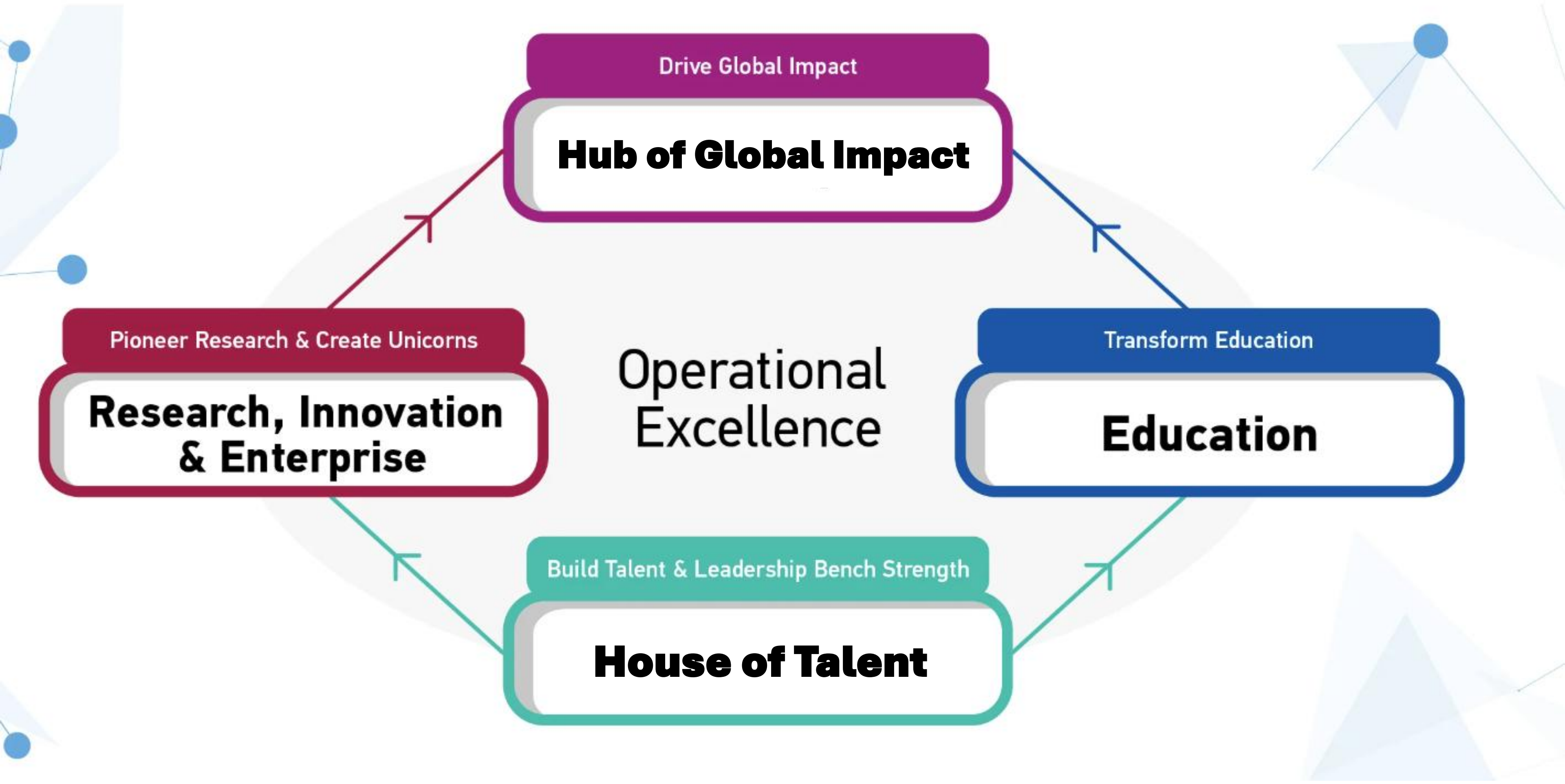
http://personal.ntu.edu.sg/mmxie

Outline

- Module 1: Robot’s Advanced Body
- Module 2: Robot’s Advanced Perception
- Module 3: Robot’s Advanced Planning
- Module 4: Robot’s Advanced Control

About NTU

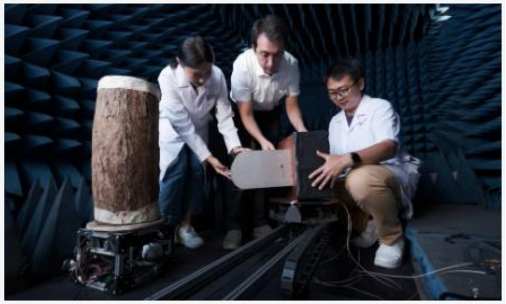
Remember NTU's Vision …



Remember NTU's Mission …

Education

We deliver transformative educational experiences that make our students both future- and AI-ready, so they are sought after by employers.



House of Talent

We attract, develop, and retain the very best people to drive excellence across the University.

Research, Innovation and Enterprise

We pursue breakthrough discoveries. We integrate technology and the humanities to address global challenges. We accelerate cutting edge innovation and create promising new enterprises.

Hub of Global Impact

We drive global impact in all that we do. We pursue long-lasting global partnerships with like-minded institutions across the world.

Education is to help citizens to fulfill their missions on Earth, which include: to understand the world and to improve the world …

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7 新加坡

李总理：排名不应是办大学终极考量

黄伟曼 报道
ngwaimun@sph.com.sg

本地大专学府的主要服务对象是新加坡学生，大学除了追求学术表现之外，也扮演重要的国家与社会角色，李显龙总理提醒本地大学不应为追求名次而忽略所肩负的重任，必须持续改善本地教育，为新加坡人提供精益求精的教育。

李总理昨晚主持新加坡国立大学大学城（University Town，简称UTown）开幕仪式时指出，本地大学的世界排名一直受到关注，但在衡量大学的表现时，不应只看名次先后，排名也不应是办大学的终极考量。

他向现场近1000名国大学生、教师与嘉宾解释说，许多在世界大专教育排行榜上名列前茅的大国顶尖大学收生广泛，例如美国哈佛大学、斯坦福大学、英国牛津大学、剑桥大学、中国北京大学和清华大学等每年录取的学生中，只有一小部分是当地学生，而本地的情况却不同，大专学府主要收本地生，必须履行更广泛的国家任务。

他说：“本地大学的责任在于发展学生的品格与社会良知、鼓励他们在求学时期建立长久的友谊与默契。大学要以灌输新加坡价值观和精神特质为出发点，教导学生取之社会用之社会的精神，让他们知道自己有责任推动新加坡前进。”

“要达到这个宗旨，每一所大学都应该建立自己独特的教育模式，不要一味复制其他大学。”

占地19公顷的大学城是国大搬到肯特岗校园现址后，最大型的一次扩展计划。设于大学城的四所寄宿型学院和为研究生所设的大学城宿舍，将为4000多名学生提供住宿，将教学融入住宿环境里，让来自不同背景和科系的学生一起生活、一起学习。

在大学城学院课程中，学生需在所住的学院里修读五个单元，包括两个写作单元和三个研讨课单元。他们也有机会聆听外国学者发表专题演讲，并参加各种不同的课外活动。

李总理昨天肯定国大为提升学生学习体验所做的努力，他认为大学城为学生营造了一个有利于相互学习的空间。他指出，大学城是国大的“新聚集点”，它将国大社群缩小，进一步拉近学生与同学和教授的关系。

他也以陈爱丽丝与彼德学院（College of Alice & Peter Tan)学生之前在“整合式课程”（Capstone Programme）下为乐龄人士制定健康饮食计划为例，表示希望看到大学城继续推行类似社会计划，鼓励学生积极参与这些关怀社会的活动。

国大校长陈祝全教授致辞时指出，除了设施与空间设计吸引人之外，大学城为国大社群以及国大的运作和成长带来了深刻且积极的影响。“大学城可说已成为国大校园的核心，在一些方面补足了之前肯特岗校园所缺少的生气。”

相关新闻刊第6页

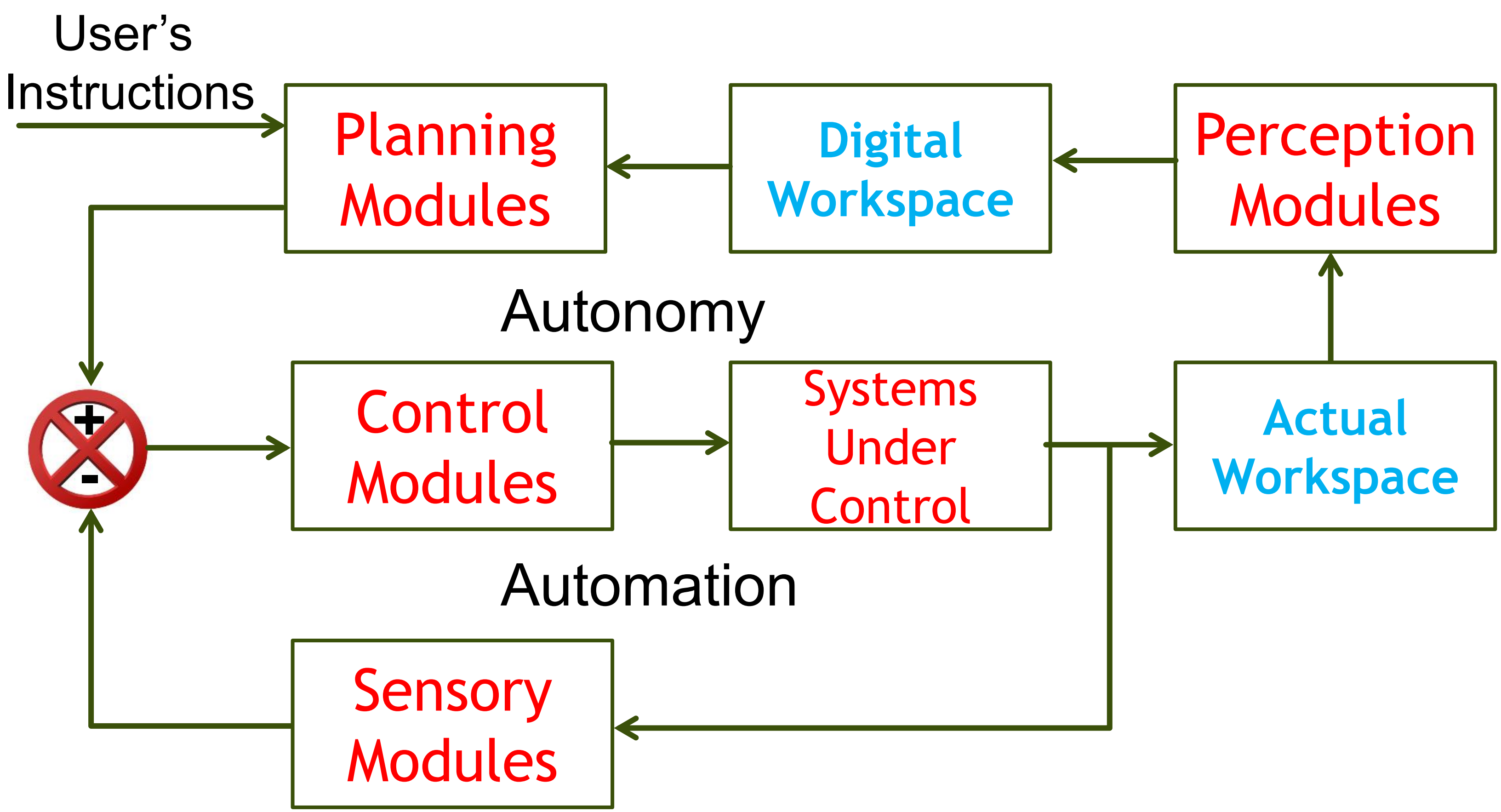
About You

Remember your mission as MAE undergraduates …

- You are here to grow your knowledge and skills so as to be able to design machines with controllable behaviors and hopefully in some intelligent ways.

How to fulfill your mission?

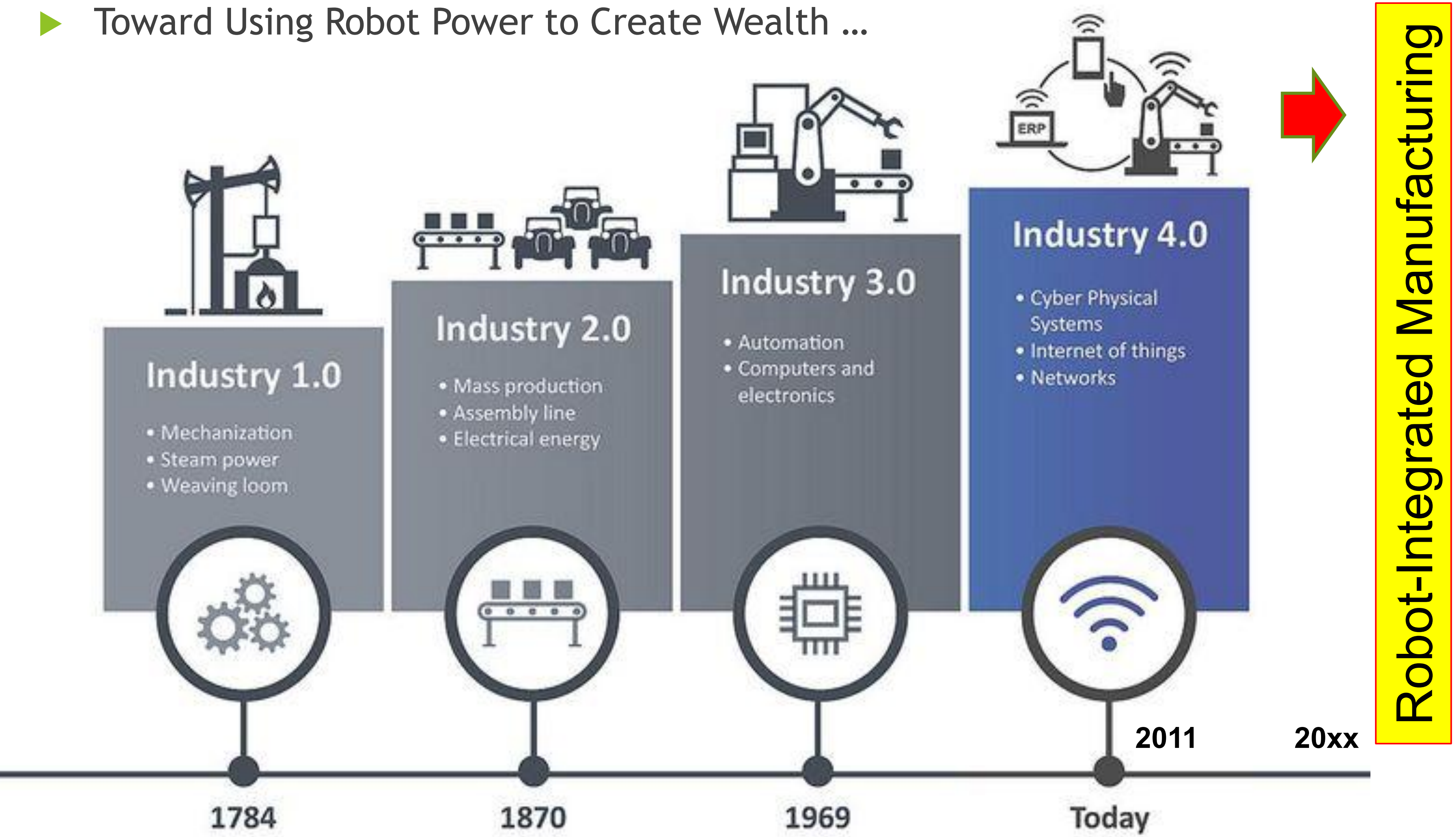
- To apply learnt knowledge and skills into the implementation of the following universal blueprint underlying all the intelligent machines or systems.



About Course

Why to study this course?

- Toward Using Robot Power to Create Wealth …



How to study this course?

- To put yourselves into the mindset of designers of robots as products:
  - Who are the users?
  - What are the needs of users?
  - What are your robots which could meet the needs of your users or buyers?
  - What are the solutions behind the design of your robots?

Market Demands or Needs
↓
Product Specifications
↓
Design Specifications
↓
Conceptual Design
↓
Selection of Materials/Components/Devices
↓
Embodiment Design
↓
Prototyping
↓
Optimizing
↓
Production
↓
Marketing

What to Learn?

Q1: What is the energy flow?

Q2: What is the signal flow?

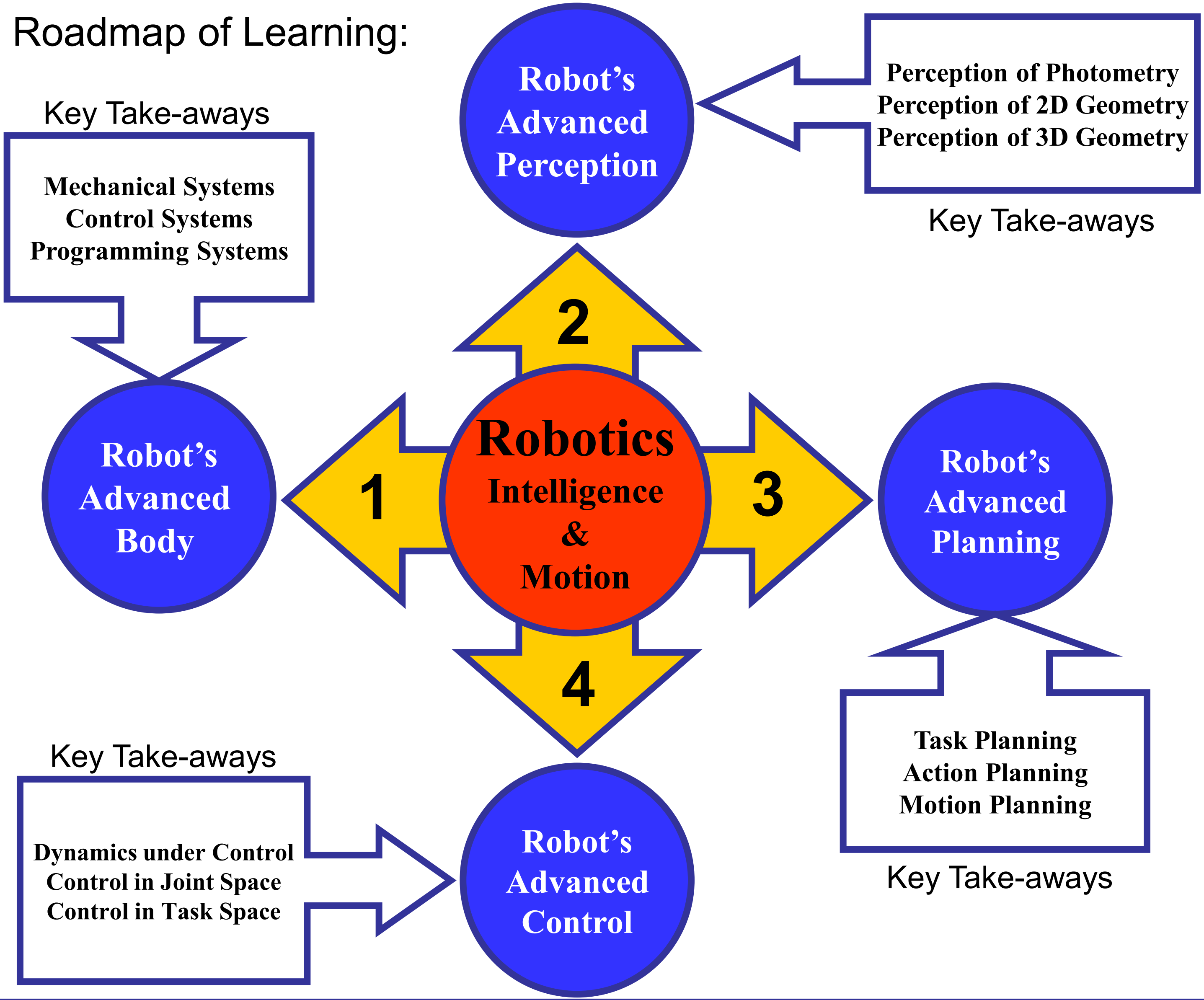
Q3: What is the knowledge flow?

Q4: What is the relationship between energy flow and signal flow?

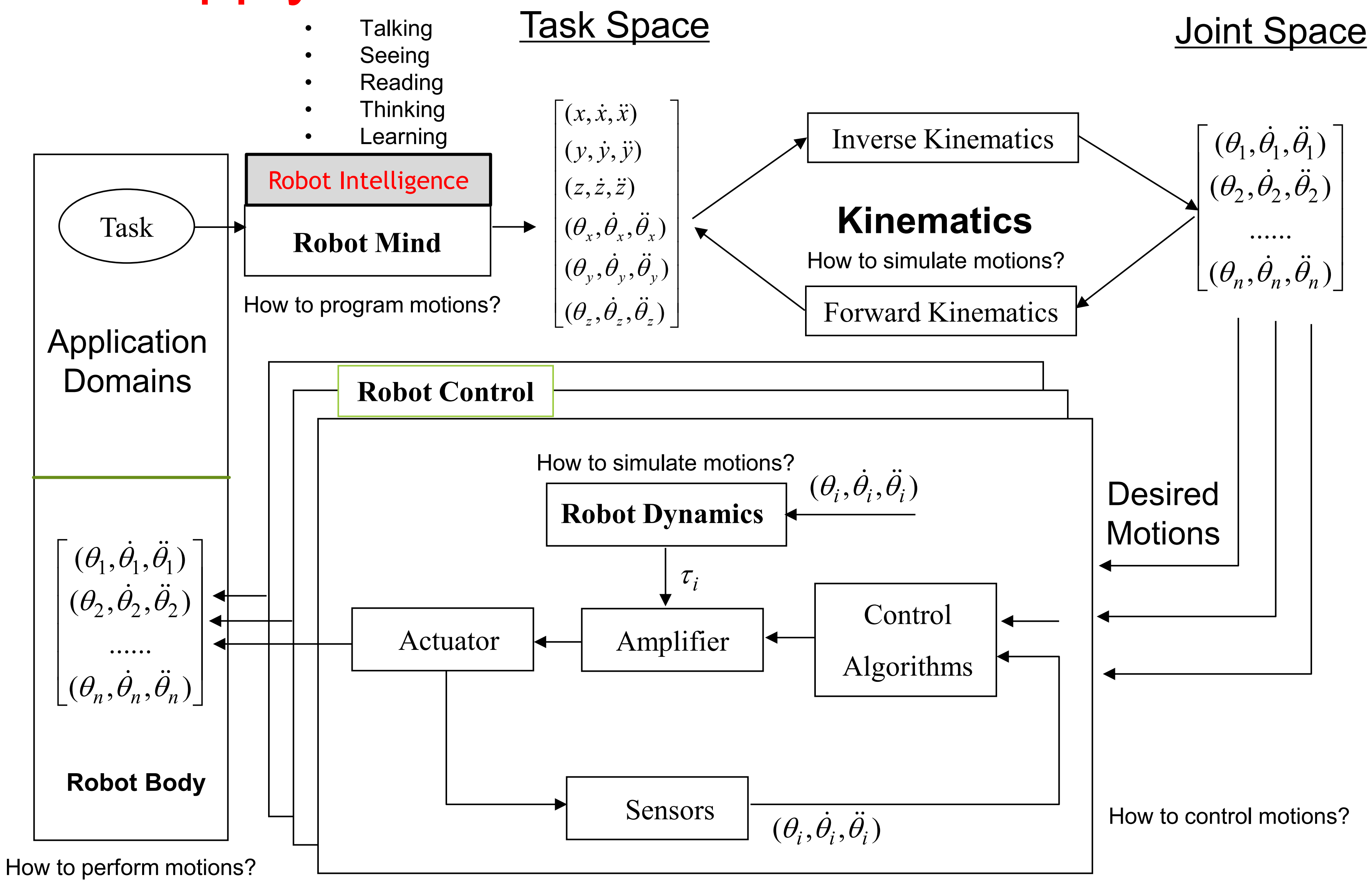
Q5: What is the relationship between signal flow and knowledge flow?

1. One Machine
2. Two Capabilities
3. Three Benefits
4. Four Pillars

Roadmap of Learning:



How to Apply?



Talking
Seeing
Reading
Thinking
Learning
Task Space
Joint Space
Robot Intelligence
Task
Robot Mind
How to program motions?
$(x,\dot{x},\ddot{x})$
$(y,\dot{y},\ddot{y})$
$(z,\dot{z},\ddot{z})$
$(\theta_x,\dot{\theta}_x,\ddot{\theta}_x)$
$(\theta_y,\dot{\theta}_y,\ddot{\theta}_y)$
$(\theta_z,\dot{\theta}_z,\ddot{\theta}_z)$
Inverse Kinematics
Kinematics
How to simulate motions?
Forward Kinematics
$(\theta_1,\dot{\theta}_1,\ddot{\theta}_1)$
$(\theta_2,\dot{\theta}_2,\ddot{\theta}_2)$
......
$(\theta_n,\dot{\theta}_n,\ddot{\theta}_n)$
Application Domains
Robot Control
How to simulate motions?
$(\theta_i,\dot{\theta}_i,\ddot{\theta}_i)$
Robot Dynamics
τ_i
Desired Motions
Control Algorithms
Actuator
Amplifier
$(\theta_1,\dot{\theta}_1,\ddot{\theta}_1)$
$(\theta_2,\dot{\theta}_2,\ddot{\theta}_2)$
......
$(\theta_n,\dot{\theta}_n,\ddot{\theta}_n)$
Robot Body
Sensors
$(\theta_i,\dot{\theta}_i,\ddot{\theta}_i)$
How to control motions?
How to perform motions?

Terminology Alert

- Advanced Robotics is about the study of advanced robots which could perform tasks in some intelligent ways.

- Advanced Robot is a machine which has
  - two capabilities (automatic control and autonomous control),
  - three benefits and
  - four pillars.

Today's Lectures …

- Module 1: Robot's Advanced Body
- Module 2: Robot's Advanced Perception
- Module 3: Robot's Advanced Planning
- Module 4: Robot's Advanced Control

School of Mechanical & Aerospace Engineering

Design, Machine, Control, Intelligence

Module 4

MA4825

Robot's Advanced Control

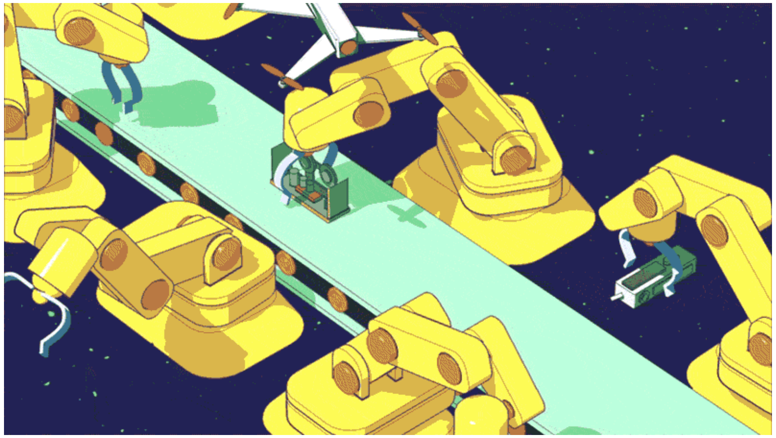
Xie Ming, PhD (France)

http://personal.ntu.edu.sg/mmxie

(Learning, Teaching) <o> (Research, Innovation) <o> (Leadership, Service)

Outline of Module 4

- Dynamics under Control
- Signal Flow Diagram
- Design of Control Systems
- Control in Joint-Space
- Control in Task-Space



Outline of Module 4

- Dynamics under Control
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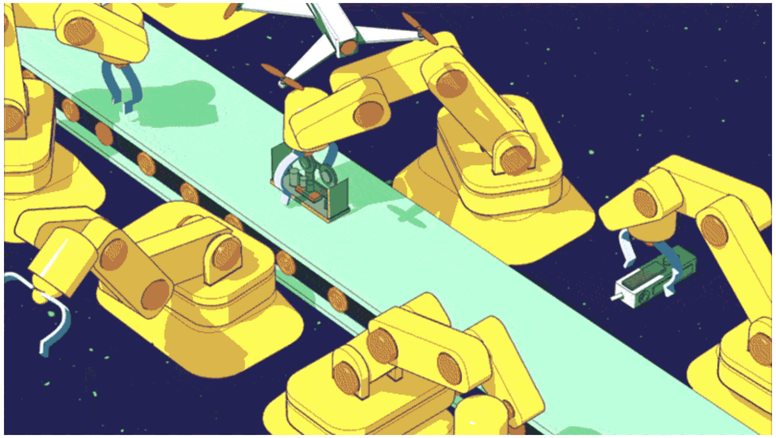


School of Mechanical & Aerospace Engineering

Design, Machine, Control, Intelligence

Module 4

MA4825 Robotics

Lecture 1

Dynamics under Control

Xie Ming, PhD (France)

http://personal.ntu.edu.sg/mmxie

Outline of Lecture 1

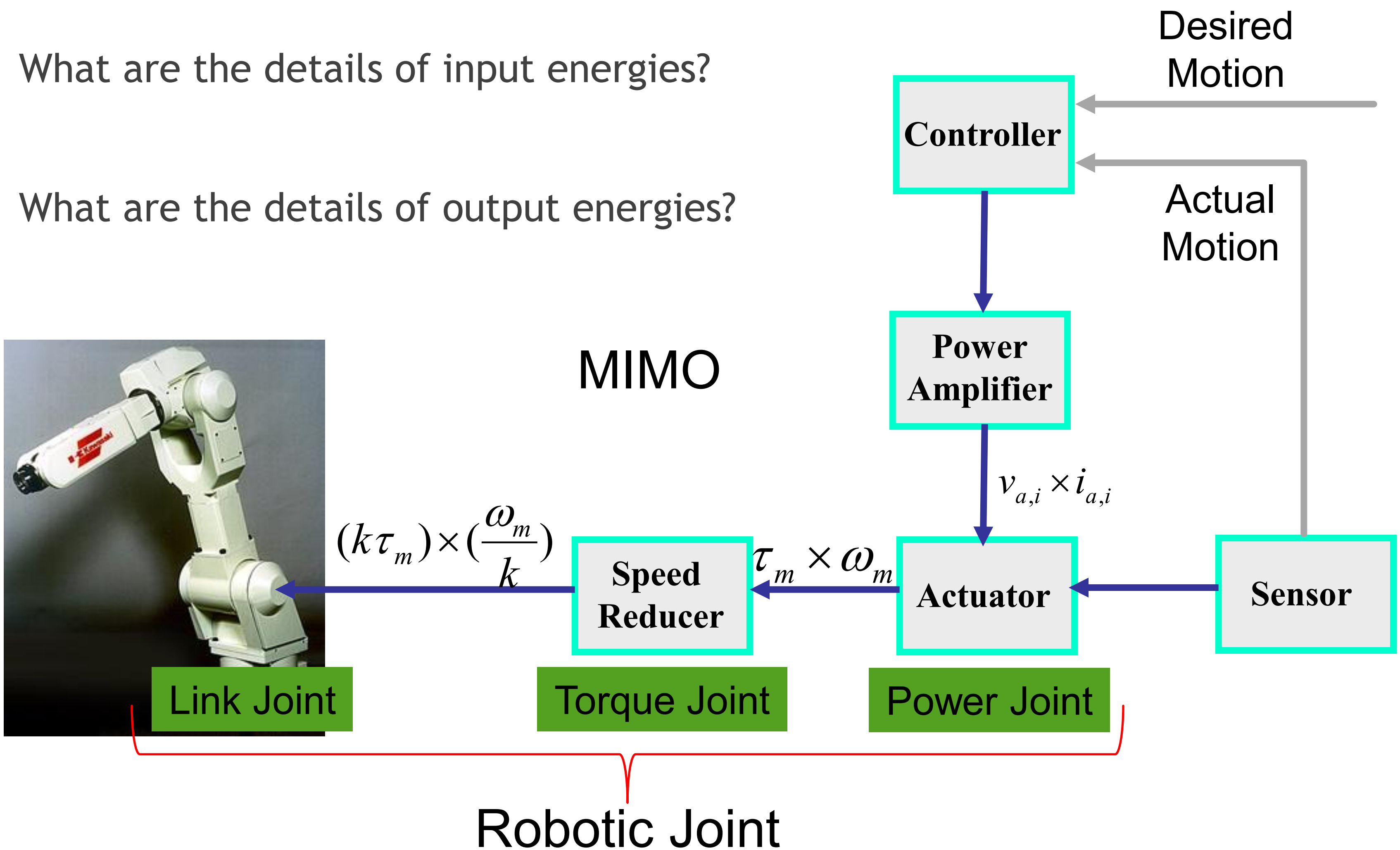
- Three Relationship Between Motion and Energy
- Dynamic Equations of Motion at Link Joints
- Dynamic Equations of Motion at Torque Joints
- Dynamic Equations of Motion at Power Joints
- Dynamic Equations of Motion at Controllers

Outline of Lecture 1

- Three Relationship Between Motion and Energy
- Dynamic Equations of Motion at Link Joints
- Dynamic Equations of Motion at Torque Joints
- Dynamic Equations of Motion at Power Joints
- Dynamic Equations of Motion at Controllers

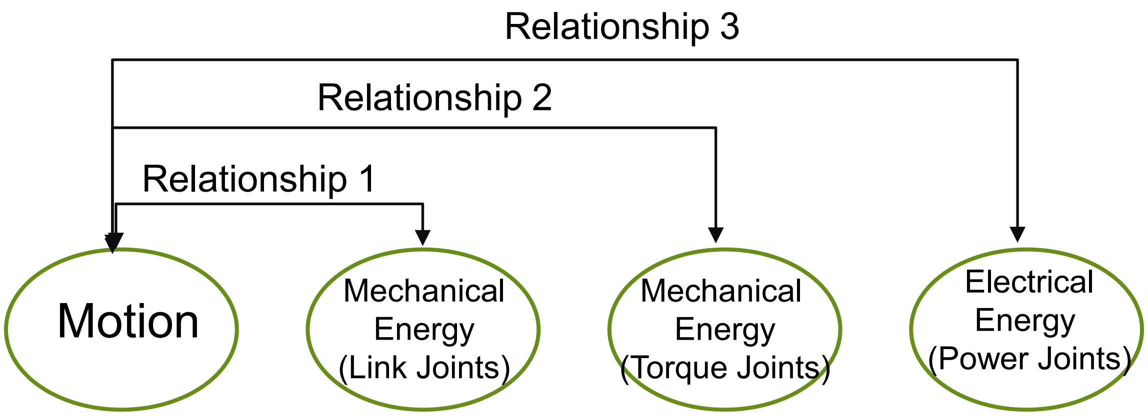
Dynamics refers to relationship between motion and energy. Then, the questions will be:

- What are the details of input energies?
- What are the details of output energies?



Robot's Dynamics Includes Three Relationships Between Motion and Energy …

- Motions are related to mechanical energy at link joints
- Motions are related to mechanical energy at torque joints
- Motions are related to electrical energy at power joints

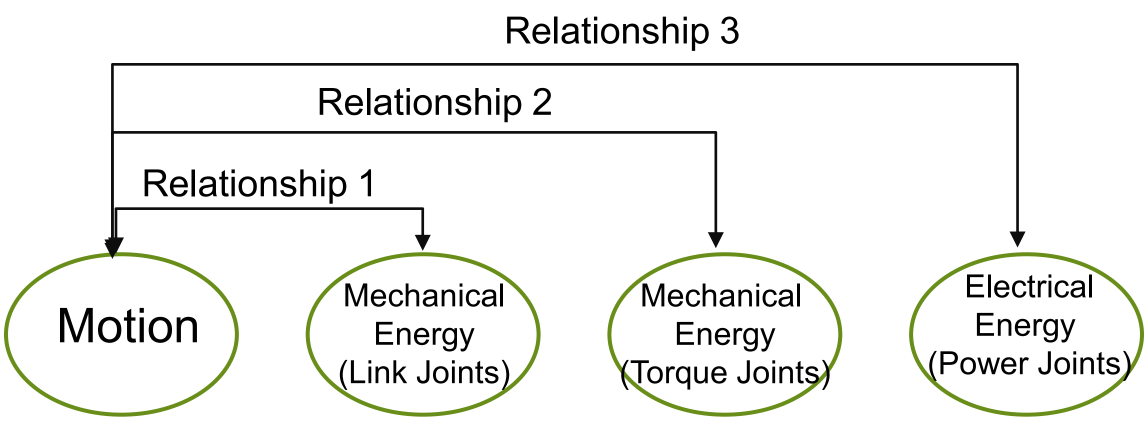


Outline of Lecture 1

- Three Relationship Between Motion and Energy
- Dynamic Equations of Motion at Link Joints
- Dynamic Equations of Motion at Torque Joints
- Dynamic Equations of Motion at Power Joints
- Dynamic Equations of Motion at Controllers

What is the relationship between motion and mechanical energy at link joints?

- Motions are related to mechanical energy at link joints
- Motions are related to mechanical energy at torque joints
- Motions are related to electrical energy at power joints



Background Knowledge: Newton's Second Law

Net Force = Acting Force – External Forces - Gravity

Net Force = Mass of Rigid Body x Acceleration

Acceleration = First Order Derivative of Velocity

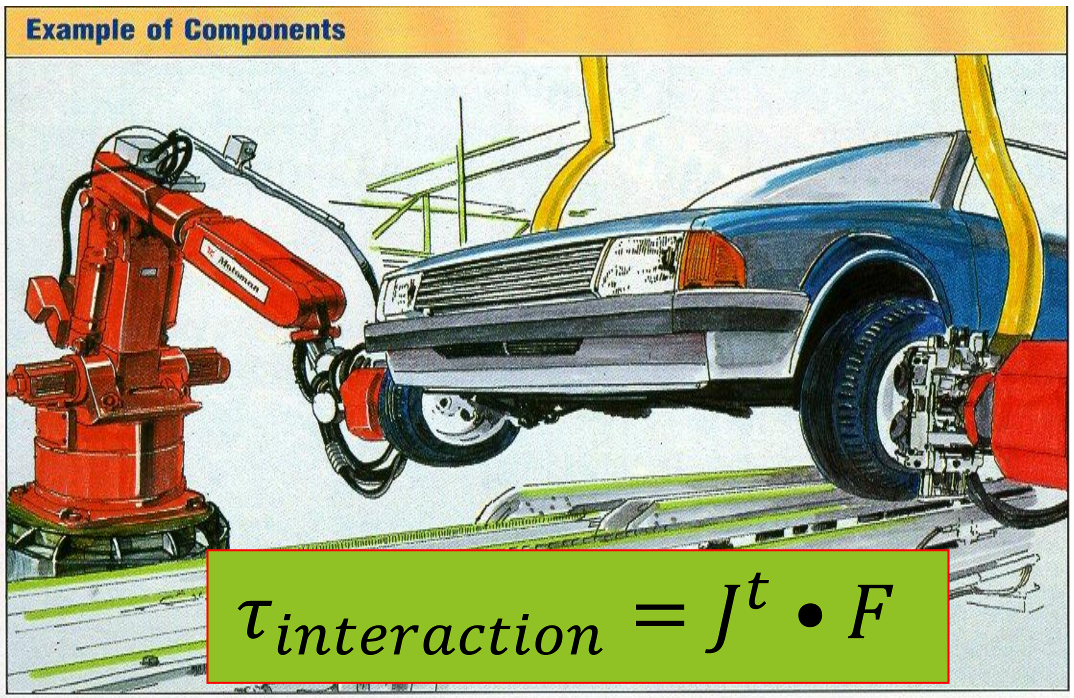
Velocity = First Order Derivative of Position

Typical External Forces

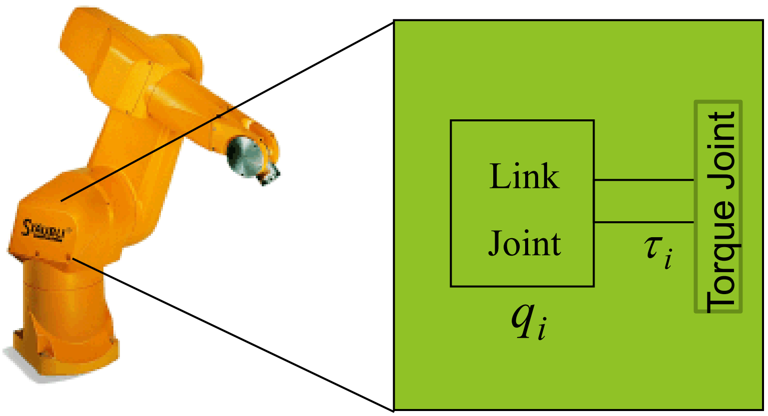
- External force due to gravity (i.e. mg)
- External force due to interaction
- External force due to viscous friction
- External force due to angular motion
- External force due to radial motion

External Force due to Constrained Motions

- When a robot's tool-tip has an interaction force with a work-piece or an environment, there should be the additional torques applied to the robot's joints in order to produce this interaction force.

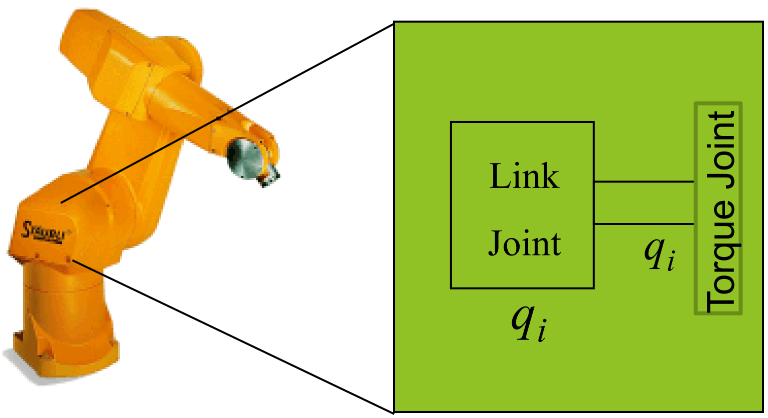


$$\tau_{interaction} = J^t \bullet F$$



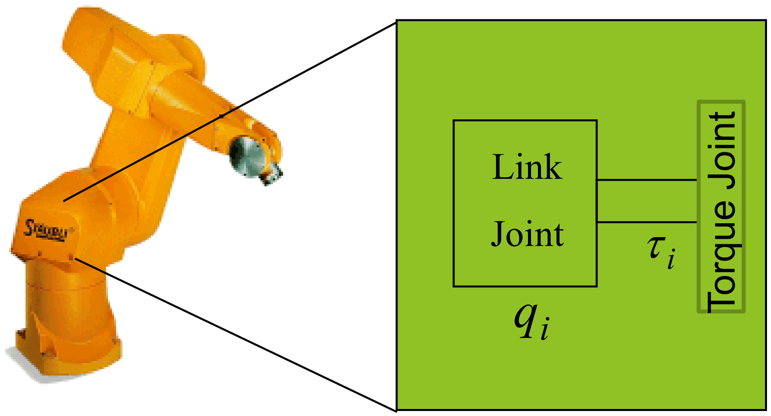
The vector of torques at robot's link joints:

$$\tau = \begin{pmatrix} \tau_1 \\ \tau_2 \\ \dots \\ \tau_n \end{pmatrix}$$



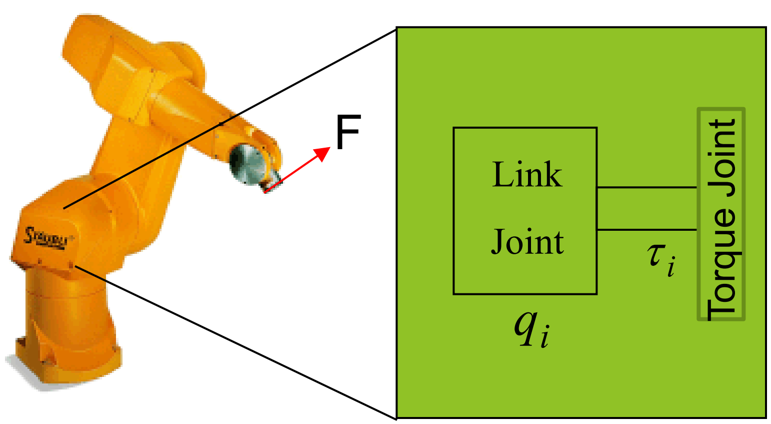
The vector of robot's link joint angles:

$$q = \begin{pmatrix} q_1 \\ q_2 \\ \dots \\ q_n \end{pmatrix}$$



The Jacobian matrix of a robot :

$$\begin{pmatrix} \text{Linear Velocity} \\ \text{Angular Velocity} \end{pmatrix} = \begin{pmatrix} \dot{O}_e \\ \omega_e \end{pmatrix} = J \bullet \frac{dq}{dt}$$



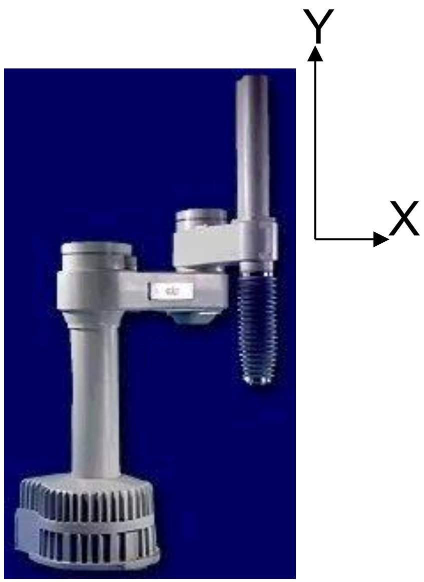
Additional torque to produce interaction force :

$$\tau_{\text{interaction}} = J^t \bullet F$$

External Force due to Viscous Friction

- When two objects in contact undergo relative motion, there is a frictional force which is proportional to the relative velocity. Such a force is called a viscous frictional force.

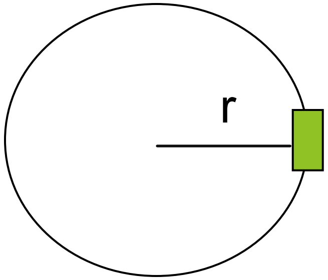
$$F_f = k\frac{dq}{dt}$$



External Force due to Angular Motion

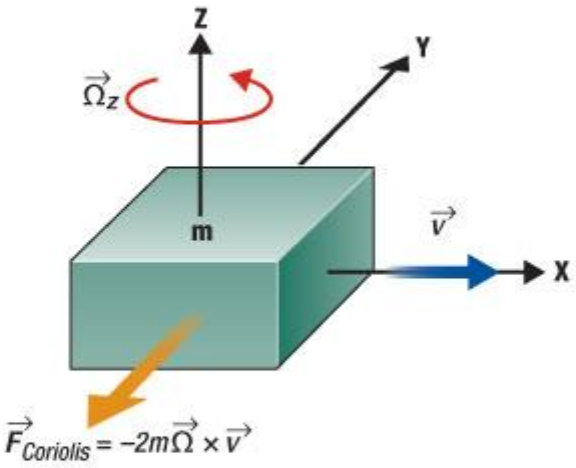
- When an object undergoes angular motion about an axis, there is a centrifugal force which is proportional to the square of circular velocity at the object's center of mass.

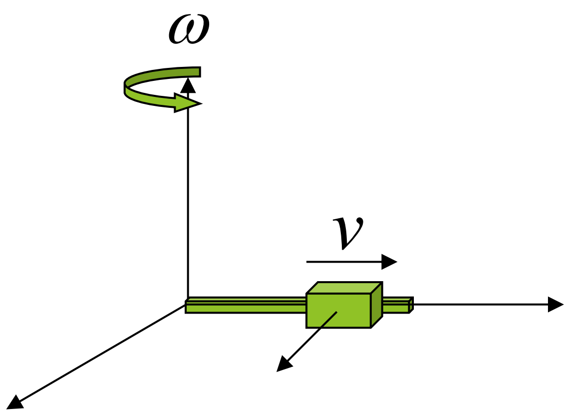
$$F_a = m\frac{v^2}{r} = m\frac{(r \bullet \omega)^2}{r} = m \bullet r \bullet (\frac{dq}{dt})^2$$



External Force due to Radial Motion

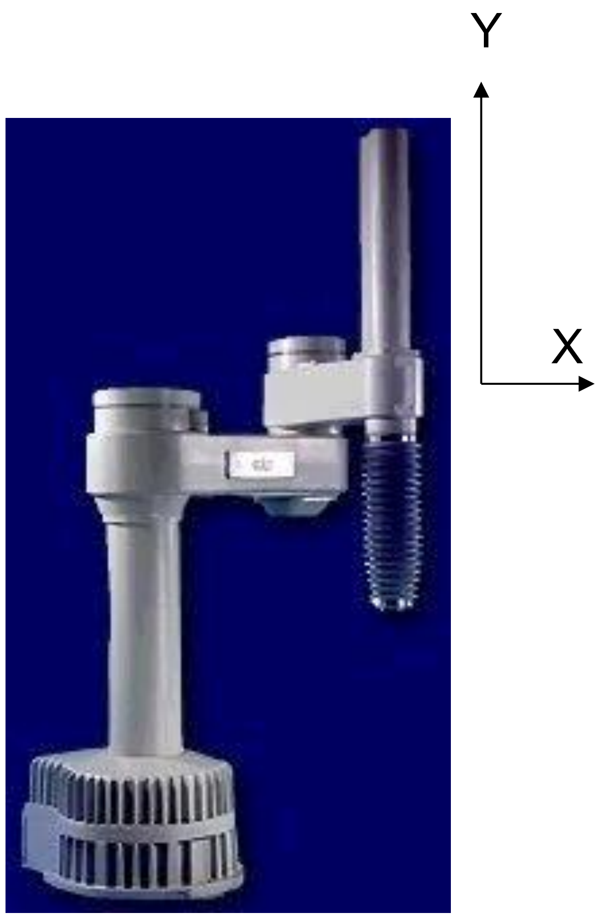
- When an object undergoes a linear motion in a radial direction on a rotation base, there will be a Coriolis force acting on this object.





Example of Robot Manipulator

- A SCARA robot has a vertical link which undergoes linear motion in vertical direction. Assume that the payload of the vertical link is M and that the input force from the torque joint coupled with a motor acting on vertical link is F. What is the relationship between the input force and the vertical position of vertical link?



Solution

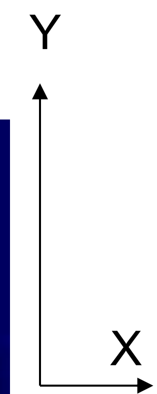
- Analysis:

The mass of the last link and its payload: M

The output force acting on the last link: F

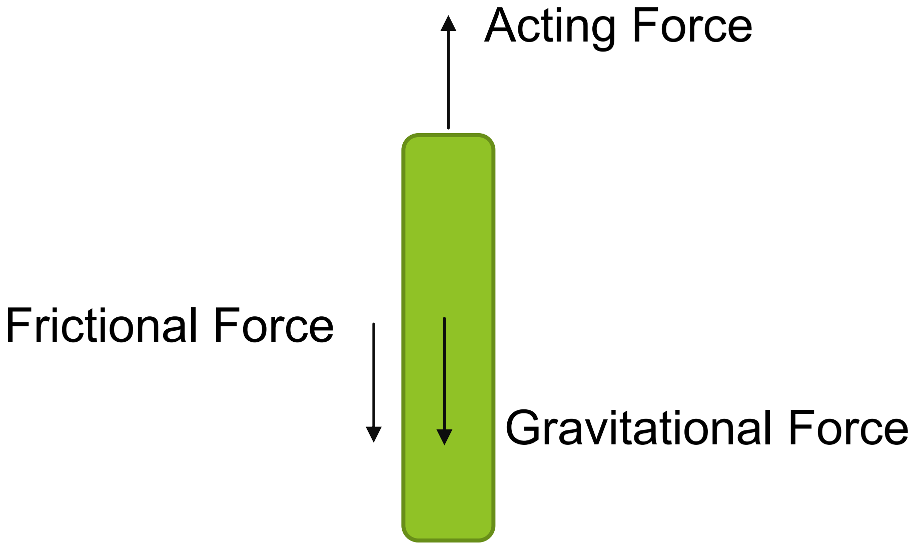
The vertical position of the last link: q

The viscous friction coefficient between the joint and the link: k



Solution (continued)

- Free body diagram:



Solution (continued)

- Equation:

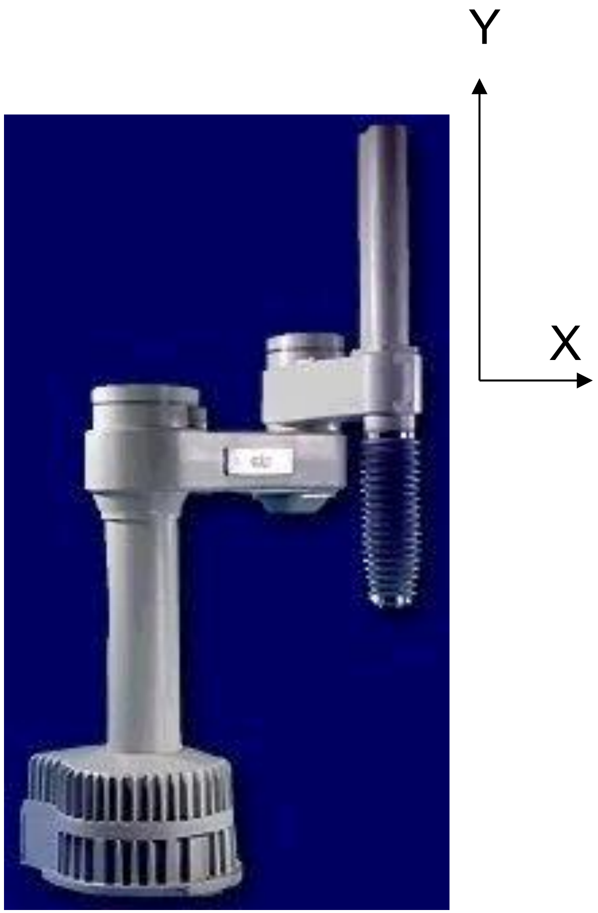
Net force acting on the link :

$$F_{net} = F - k\frac{dq}{dt} - M \bullet g$$

Equation of motion (Newton's Second Law) :

$$F_{net} = F - k\frac{dq}{dt} - M \bullet g = M\frac{d^2q}{dt^2}$$

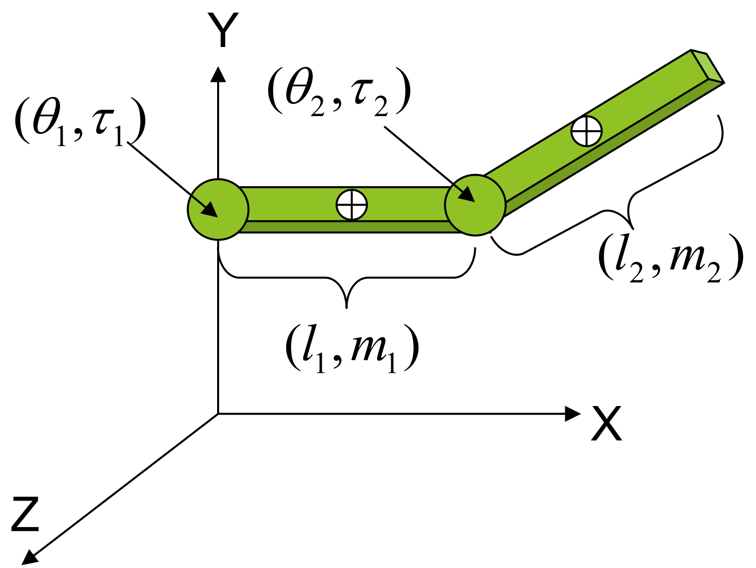
$$M\frac{d^2q}{dt^2} + k\frac{dq}{dt} + M \bullet g = F$$



Example of Robot Arm at Rest (Statics)

$$\tau_{interaction} = J^t \bullet F$$

- A robot arm has two links which can move within a vertical plan. Now, assume that we want to keep the arm at rest as shown in the figure. What should be the torques at joints 1 and 2?



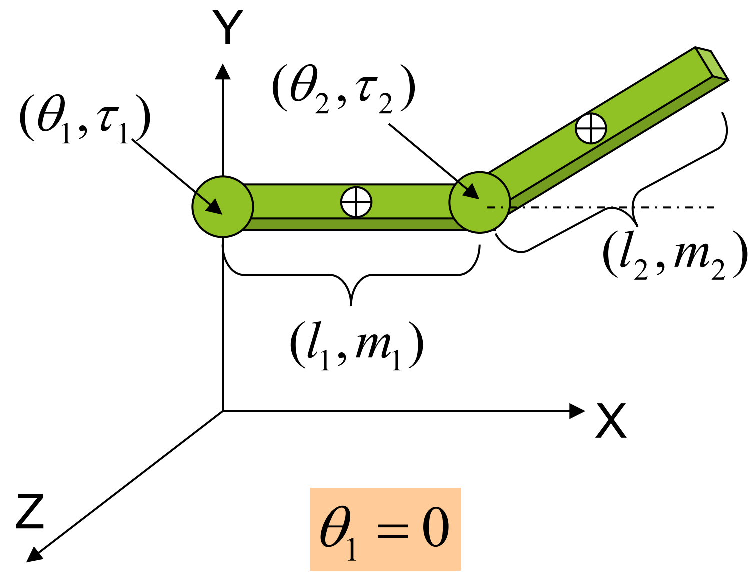
Solution

$$\tau_{interaction} = J^t \bullet F$$

- Torque at joint 2 when link 1 is horizontal:

If $\theta_1 = 0$, we have

$$\tau_2 = m_2 g \times \frac{l_2}{2} \cos(\theta_2)$$



Solution (continued)

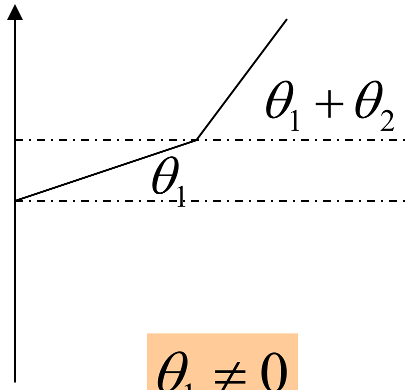
$$\tau_{interaction} = J^t \bullet F$$

$$F = \begin{pmatrix} m_1 g \\ m_2 g \end{pmatrix}$$

- Torque at joint 2 when link 1 is not horizontal:

If $\theta_1 \neq 0$, the torque at joint 2 should be:

$$\tau_2 = m_1 g \times 0 + m_2\, g \times \frac{l_2}{2} \cos(\theta_2 + \theta_1)$$



$\theta_1 \neq 0$

Solution (continued)

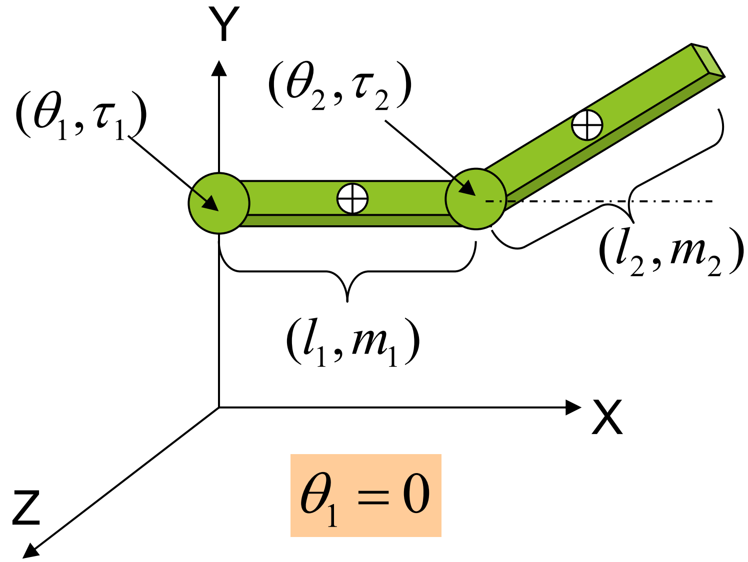
$$\tau_{interaction} = J^t \bullet F$$

$$F = \begin{pmatrix} m_1 g \\ m_2 g \end{pmatrix}$$

- Torque at joint 1 when link 1 is horizontal:

If $\theta_1 = 0$, we have

$$\tau_1 = m_1 g \times \frac{l_1}{2} + m_2 g \times \{l_1 + \frac{l_2}{2}\cos(\theta_2)\}$$



Solution (continued)

$$\tau_{interaction} = J^t \bullet F$$

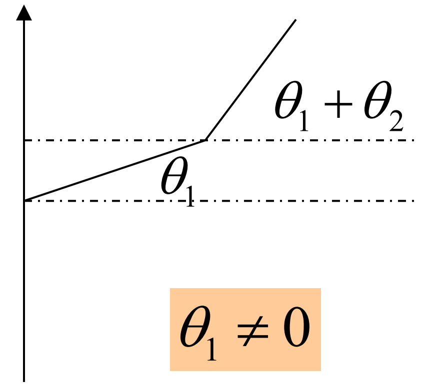
$$F = \begin{pmatrix} m_1 g \\ m_2 g \end{pmatrix}$$

- Torque at joint 1 when link 1 is not horizontal:

If $\theta_1 \neq 0$, the torque at joint 1 should be:

$$\tau_1 = m_1 g \times \frac{l_1}{2}\cos(\theta_1) + m_2 g \times \{l_1 \cos(\theta_1) + \frac{l_2}{2}\cos(\theta_2 + \theta_1)\}$$

$$F = \begin{pmatrix} m_1 g \\ m_2 g \end{pmatrix}$$



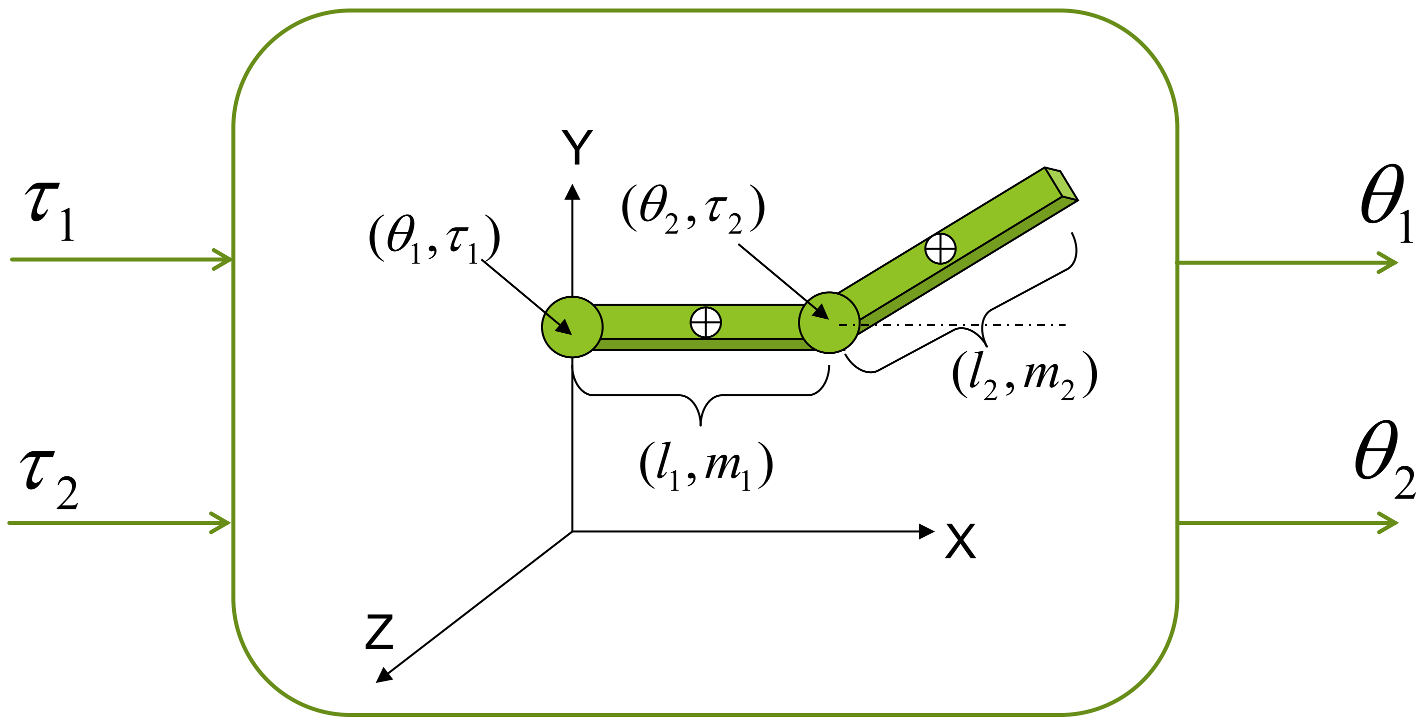
$\theta_1 \neq 0$

Solution (continued)

$$\tau_{interaction} = J^t \bullet F$$

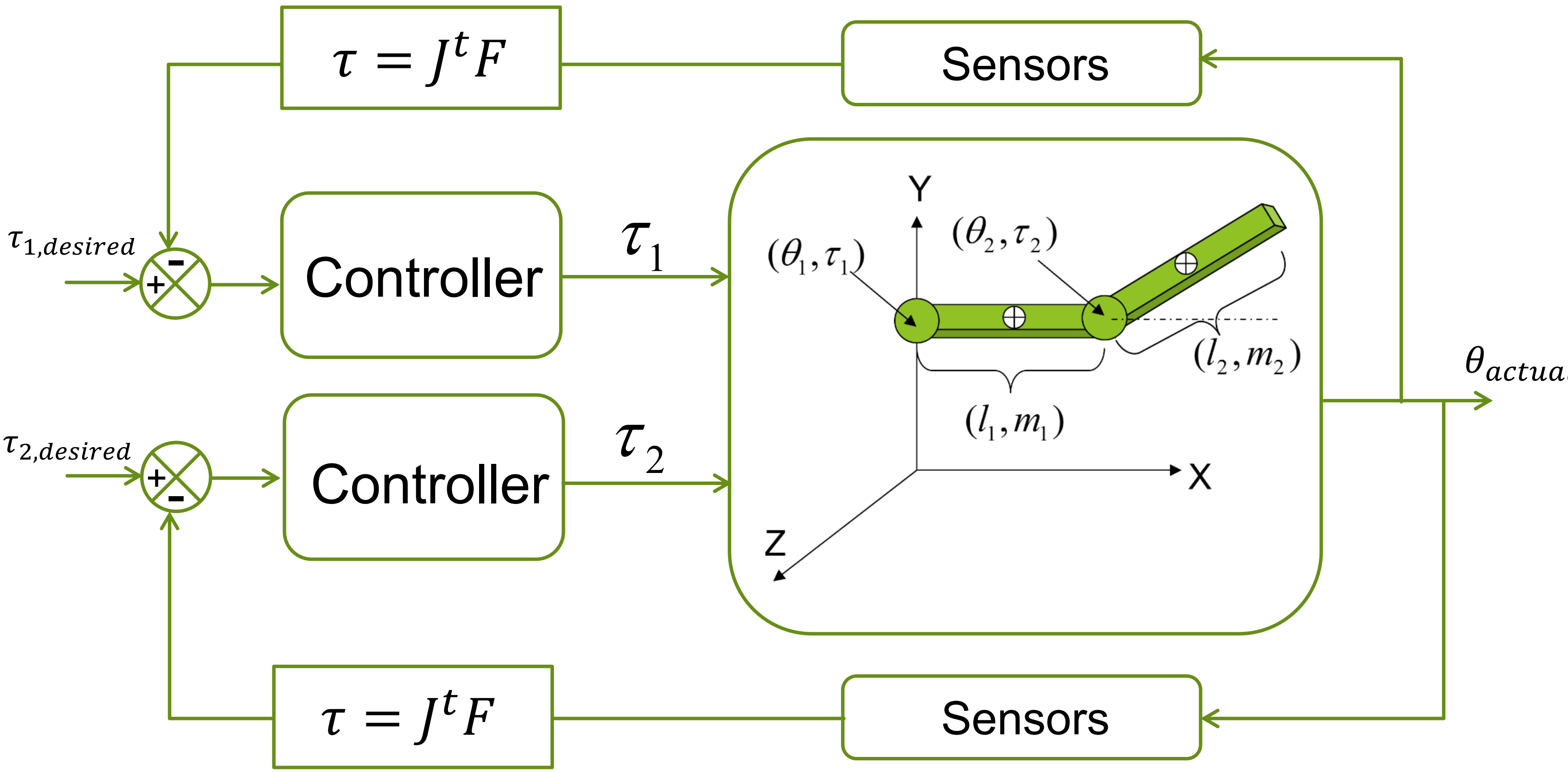
$$F = \begin{pmatrix} m_1 g \\ m_2 g \end{pmatrix}$$

- Relationship between Input and Output without Error Control Loops:



Solution (continued)

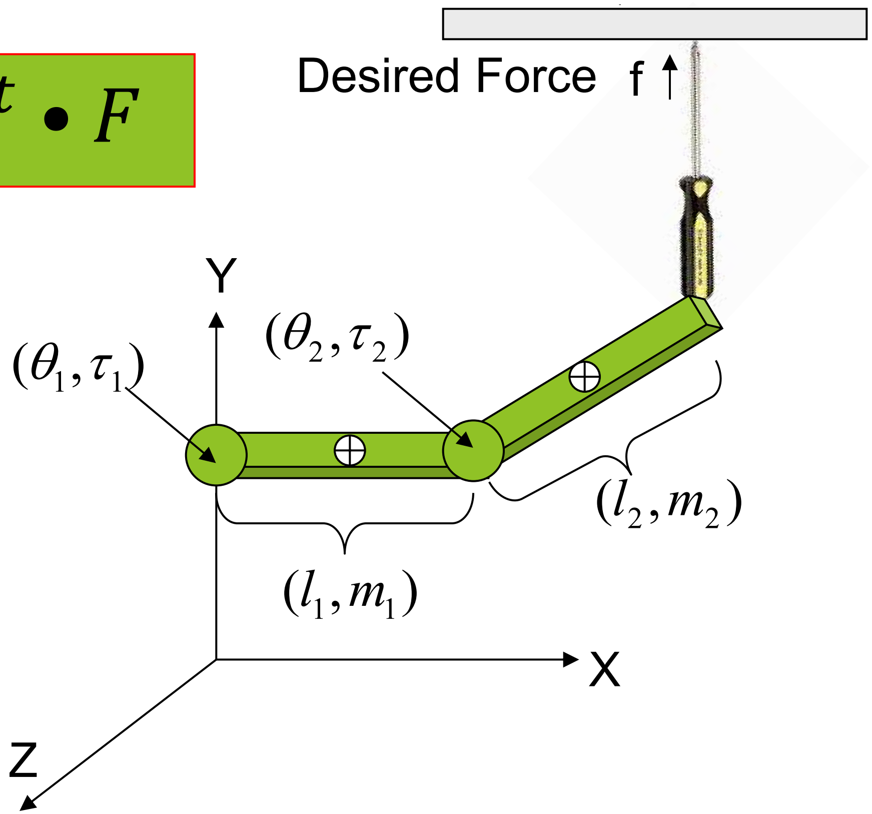
- Relationship between Input and Output with Error Control Loops:



Example of Example of Robot Arm at Rest (Statics)

- A robot arm has two links which can move in a vertical plan. Now, we want the robot to exert a desired force F onto a screw driver. Assume that the mass of the screw driver is negligible. How to produce the desired force F acting on the screw driver?

$$\tau_{interaction} = J^t \bullet F$$



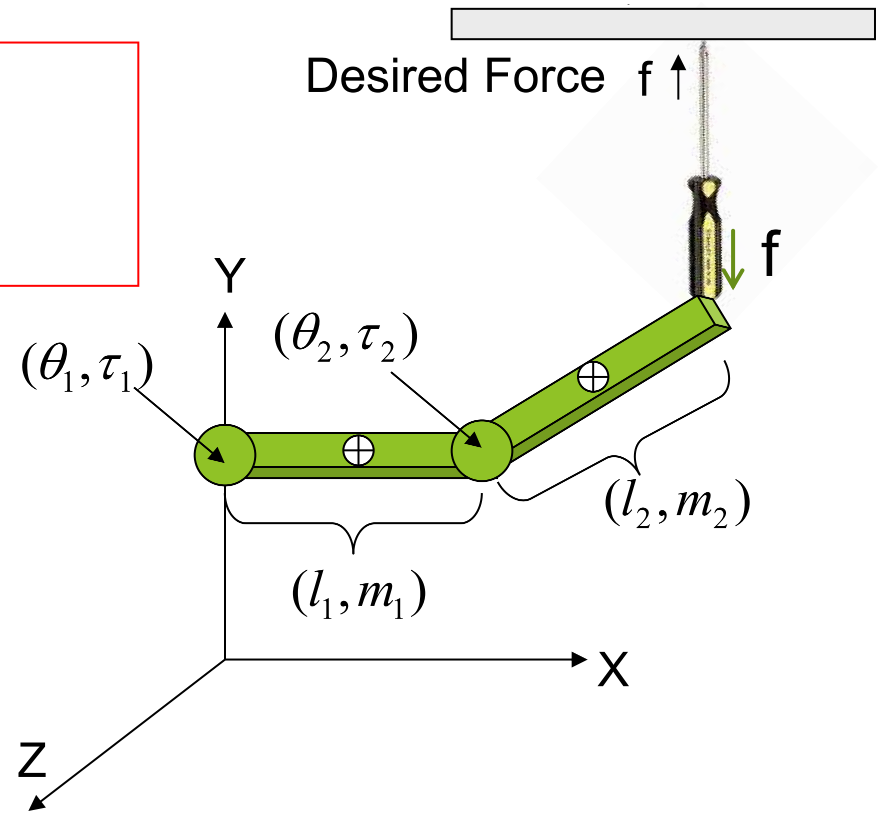
Solution

$$\tau_{interaction} = J^t \bullet F$$

- Desired torque at joint 2 when link 1 is horizontal

If $\theta_1 = 0$, we have

$$\tau_2 = m_2 g \times \frac{l_2}{2} \cos(\theta_2) + f \times l_2 \cos(\theta_2)$$



Solution (continued)

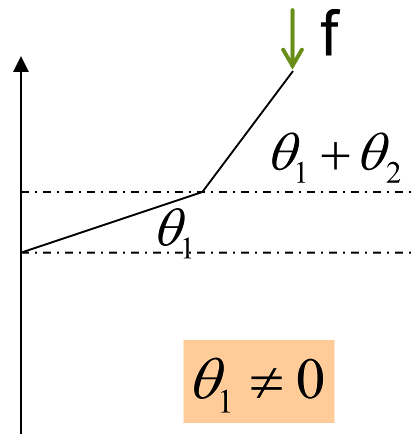
$$\tau_{interaction} = J^t \bullet F$$

$$F = \begin{pmatrix} m_1 g \\ m_2 g \\ f \end{pmatrix}$$

- Desired torque at joint 2 when link 1 is not horizontal

If $\theta_1 \neq 0$, the torque at joint 2 should be:

$$\tau_2 = m_1 g \times 0 + m_2 g \times \frac{l_2}{2} \cos(\theta_2 + \theta_1) + f \times l_2 \cos(\theta_2 + \theta_1)$$



Solution (continued)

$$\tau_{interaction} = J^t \bullet F$$

$$F = \begin{pmatrix} m_1 g \\ m_2 g \\ f \end{pmatrix}$$

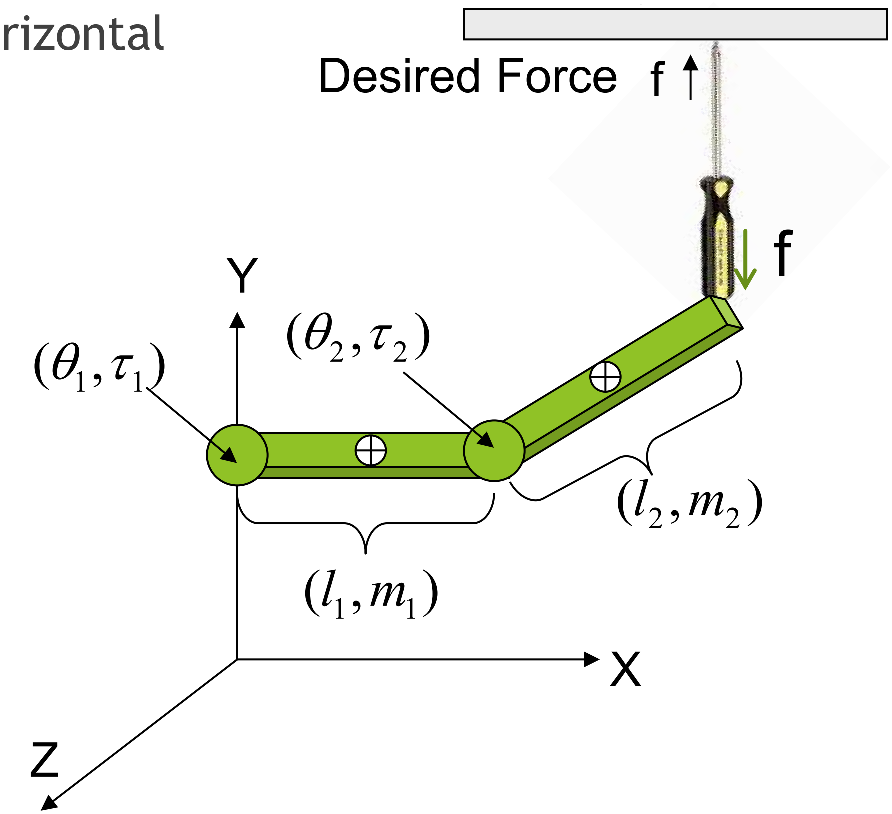
- Desired torque at joint 1 when link 1 is horizontal

If $\theta_1 = 0$, we have

$$\tau_1 = m_1 g \times \frac{l_1}{2} +$$

$$m_2 g \times \{l_1 + \frac{l_2}{2} \cos(\theta_2)\} +$$

$$f \times \{l_1 + l_2 \cos(\theta_2)\}$$



Solution (continued)

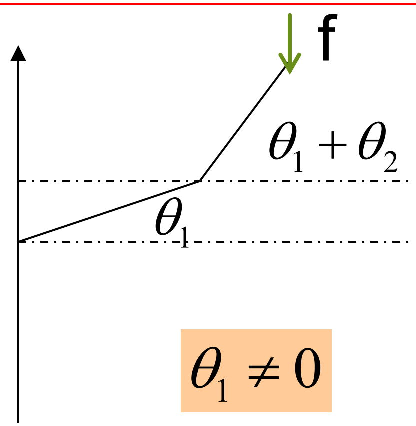
$$\tau_{interaction} = J^t \bullet F$$

$$F = \begin{pmatrix} m_1 g \\ m_2 g \\ f \end{pmatrix}$$

- Desired torque at joint 1 when link 1 is not horizontal

If $\theta_1 \neq 0$, the torque at joint 1 should be:

$$\tau_1 = m_1 g \times \frac{l_1}{2}\cos(\theta_1) + m_2 g \times \{l_1 \cos(\theta_1) + \frac{l_2}{2}\cos(\theta_2 + \theta_1)\} + f \times \{l_1 \cos(\theta_1) + l_2 \cos(\theta_2 + \theta_1)\}$$

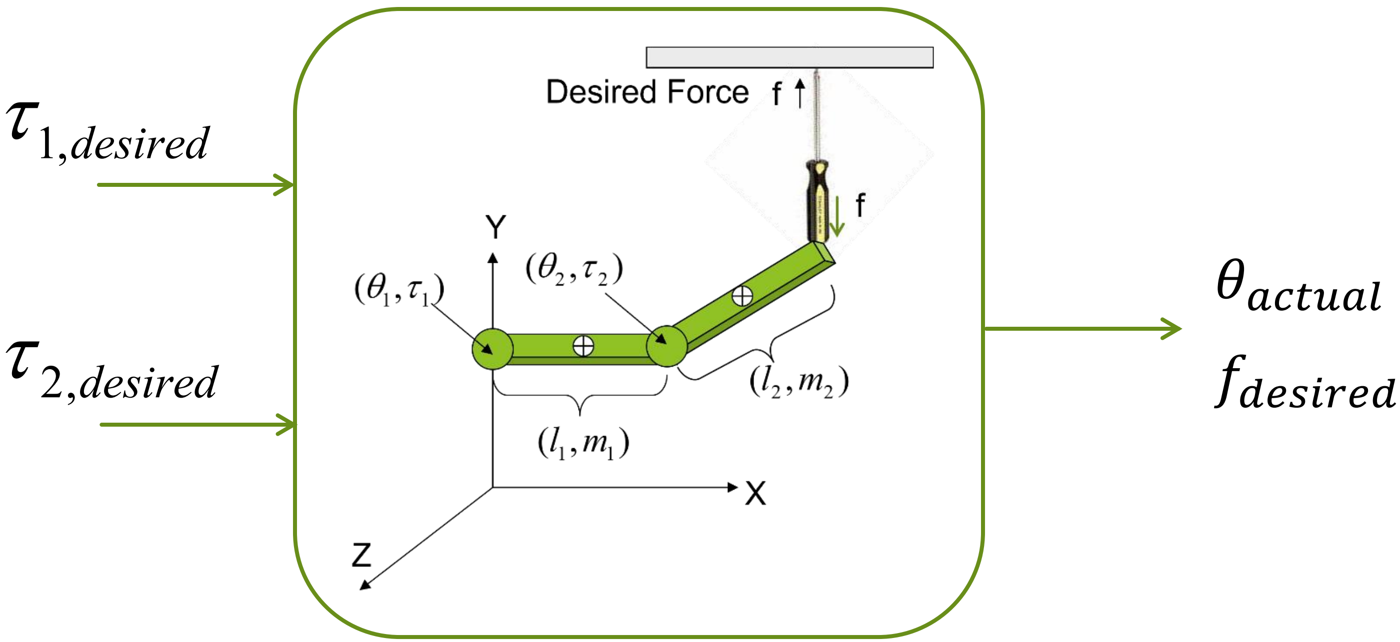


$\theta_1 \neq 0$

Solution (continued)

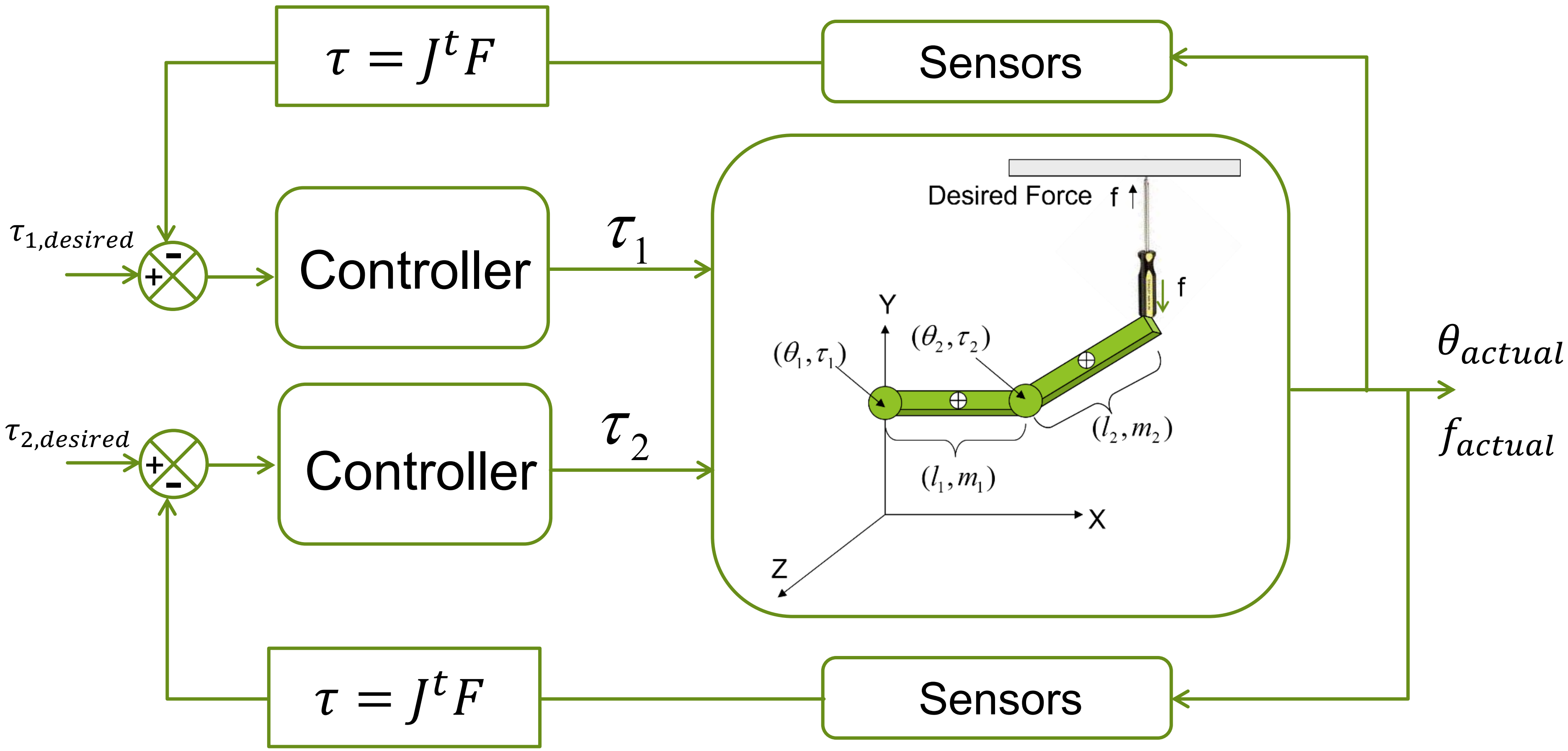
$$\tau_{interaction} = J^t \bullet F \qquad F = \begin{pmatrix} m_1 g \\ m_2 g \\ f \end{pmatrix}$$

- Desired force within the framework of systems without error control loops:



Solution (continued)

- Desired force within the framework of systems with Error Control Loops:



Example of Articulated Robot Manipulator

- When the torques acting on the link joints of a robot with six revolute joints can overcome all the external forces, what will be the equation of motion at the robot's link joints?

$$\tau_{applied} > J^t \bullet F$$

Answer

$$B(q)\ddot{q} = \tau - J^t F - C(q,\dot{q})\dot{q} - g(q)$$

q : Vector of generalized coordinates

$B(q)$: Matrix of Inertia

Kinetic Resistance

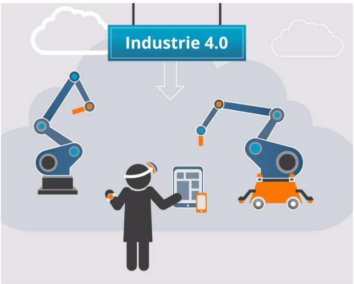
$C(q,\dot{q})$: Matrix of Centrifugal and Coriolis forces.

$g(q)$: Vector of gravitational effect

τ : Vector of generalized forces acting at the link joints

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau - J^t F$$

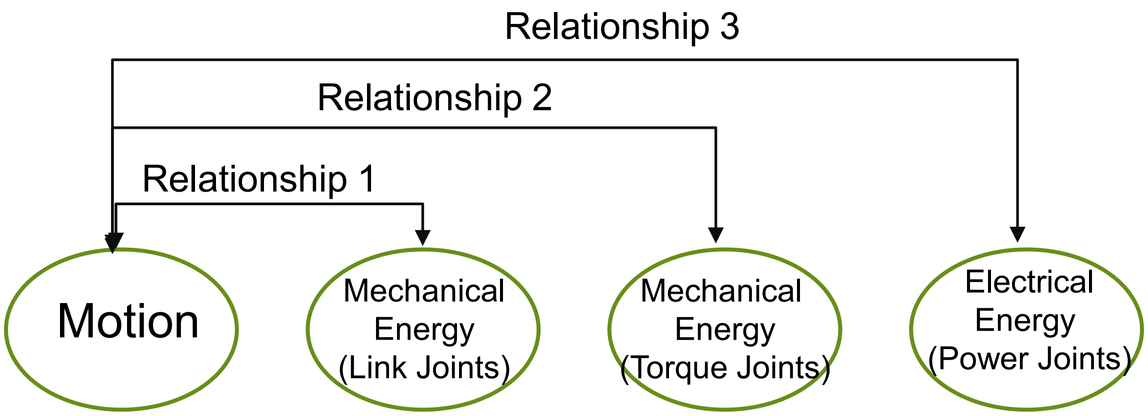
Outline of Lecture 1



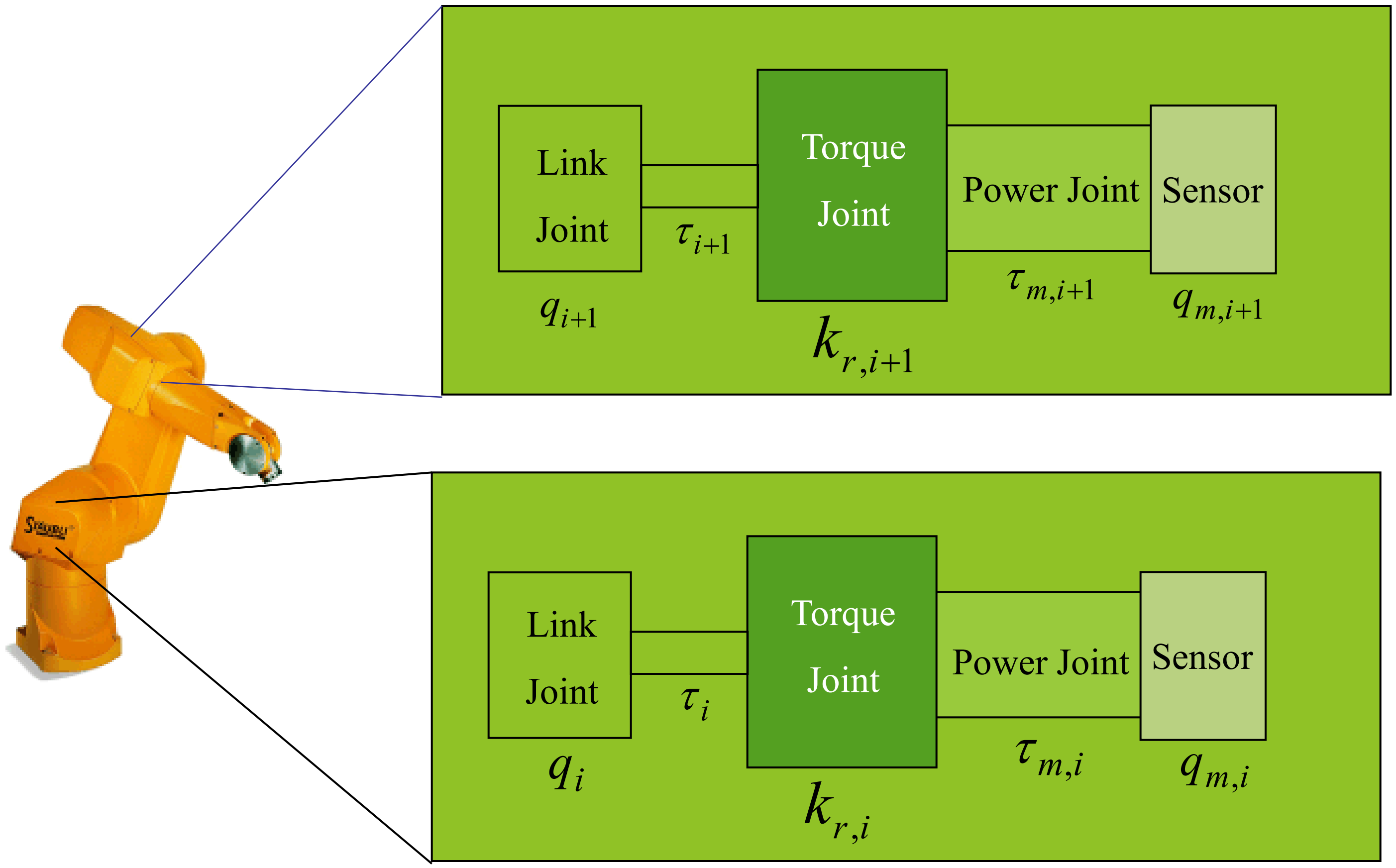
- Three Relationship Between Motion and Energy
- Dynamic Equations of Motion at Link Joints
- Dynamic Equations of Motion at Torque Joints
- Dynamic Equations of Motion at Power Joints
- Dynamic Equations of Motion at Controllers

What is the relationship between motion and mechanical energy at torque joints?

- Motions are related to mechanical energy at link joints
- Motions are related to mechanical energy at torque joints
- Motions are related to electrical energy at power joints

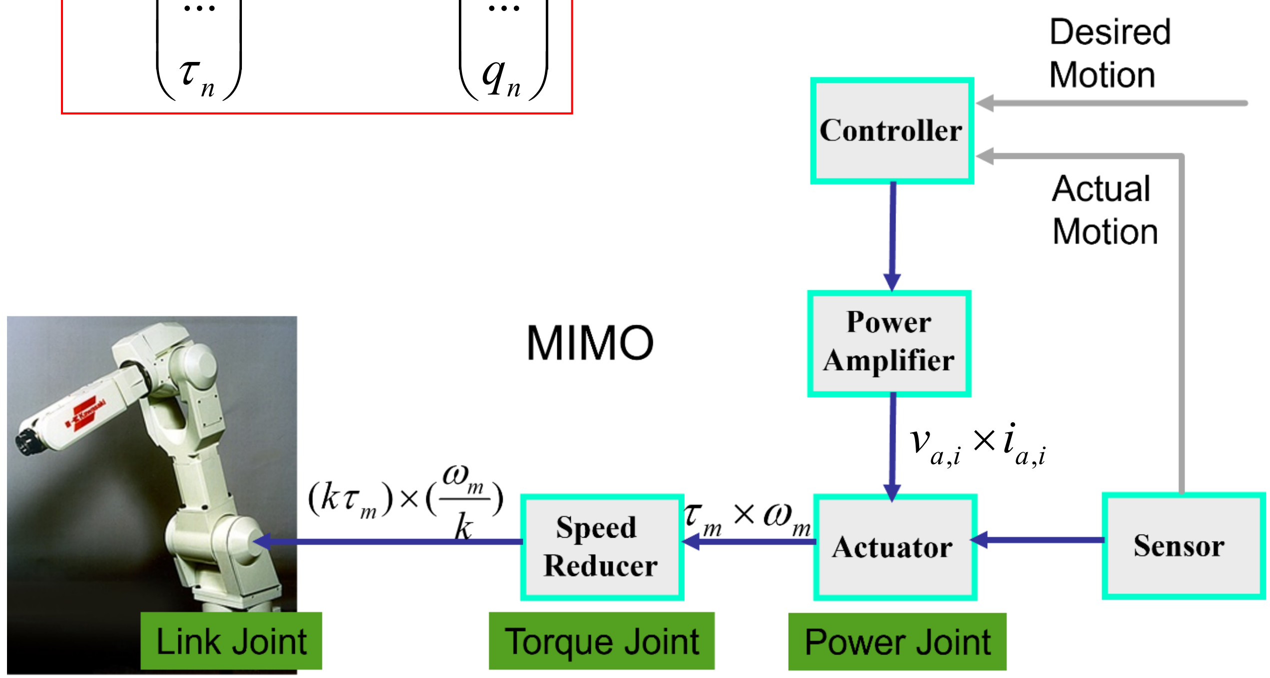


Link joints receive energies from torque joints (and power joints)



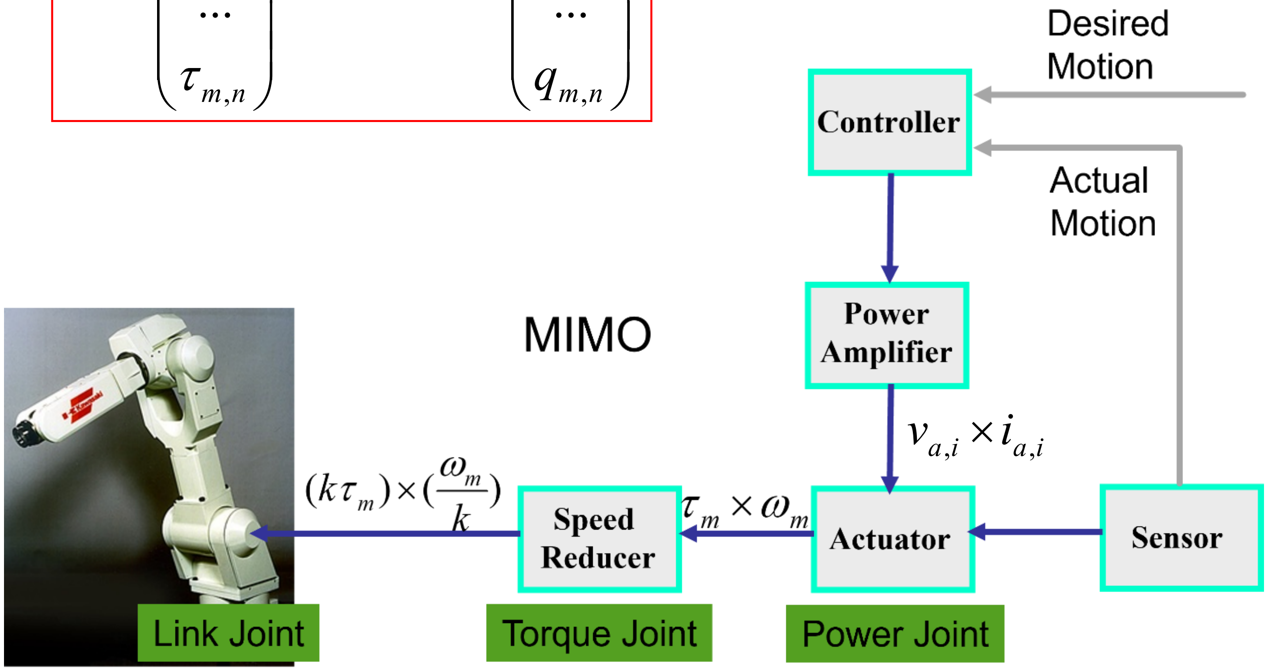
Torque vector and motion vector at input of link joints

$$\tau = \begin{pmatrix} \tau_1 \\ \tau_2 \\ \cdots \\ \tau_n \end{pmatrix} \quad \text{and} \quad q = \begin{pmatrix} q_1 \\ q_2 \\ \cdots \\ q_n \end{pmatrix}$$



Torque vector and motion vector at input of torque joints

$$\tau_m = \begin{pmatrix} \tau_{m,1} \\ \tau_{m,2} \\ \cdots \\ \tau_{m,n} \end{pmatrix} \quad \text{and} \quad q_m = \begin{pmatrix} q_{m,1} \\ q_{m,2} \\ \cdots \\ q_{m,n} \end{pmatrix}$$

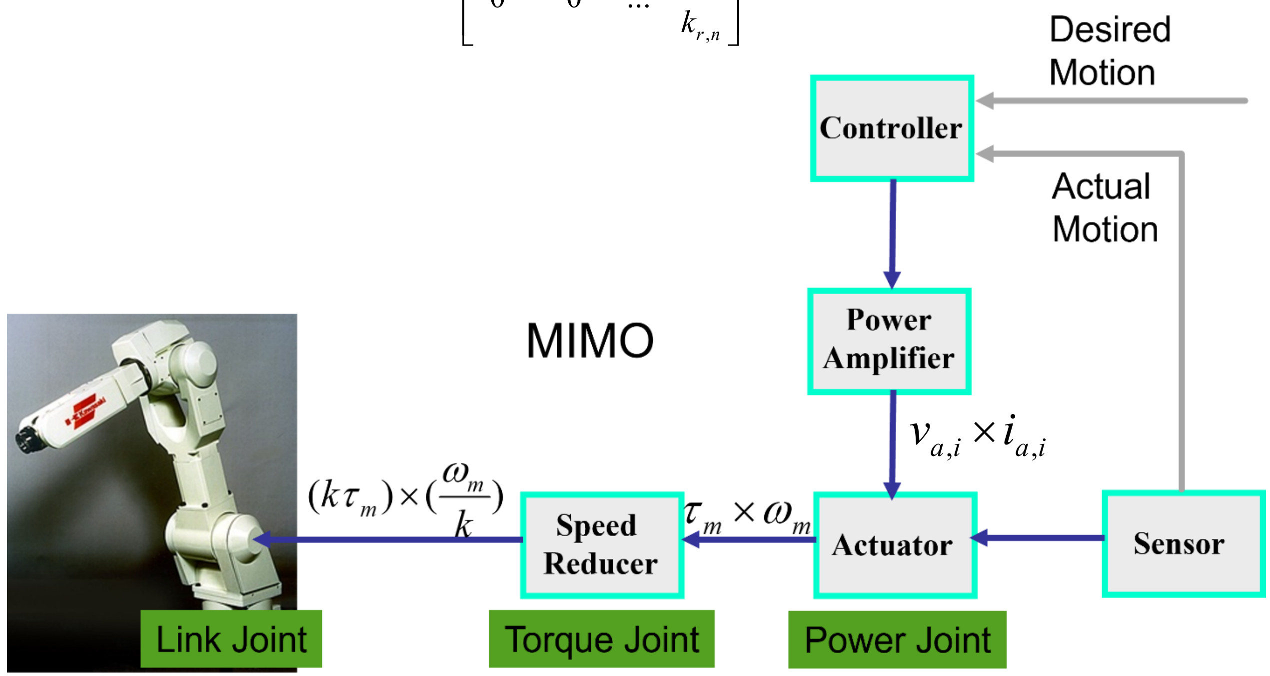


Property 1 of Torque Joints

$$\tau_i = k_{r,i} \bullet \tau_{m,i}, \; i = 1,2,\ldots,n$$

$$K_r = \begin{bmatrix} k_{r,1} & 0 & \ldots & 0 \\ 0 & k_{r,2} & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & k_{r,n} \end{bmatrix} \rightarrow \tau = K_r \bullet \tau_m$$

$$K_r^{-1} = \begin{bmatrix} \frac{1}{k_{r,1}} & 0 & \ldots & 0 \\ 0 & \frac{1}{k_{r,2}} & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & \frac{1}{k_{r,n}} \end{bmatrix} \rightarrow \tau_m = K_r^{-1} \bullet \tau$$

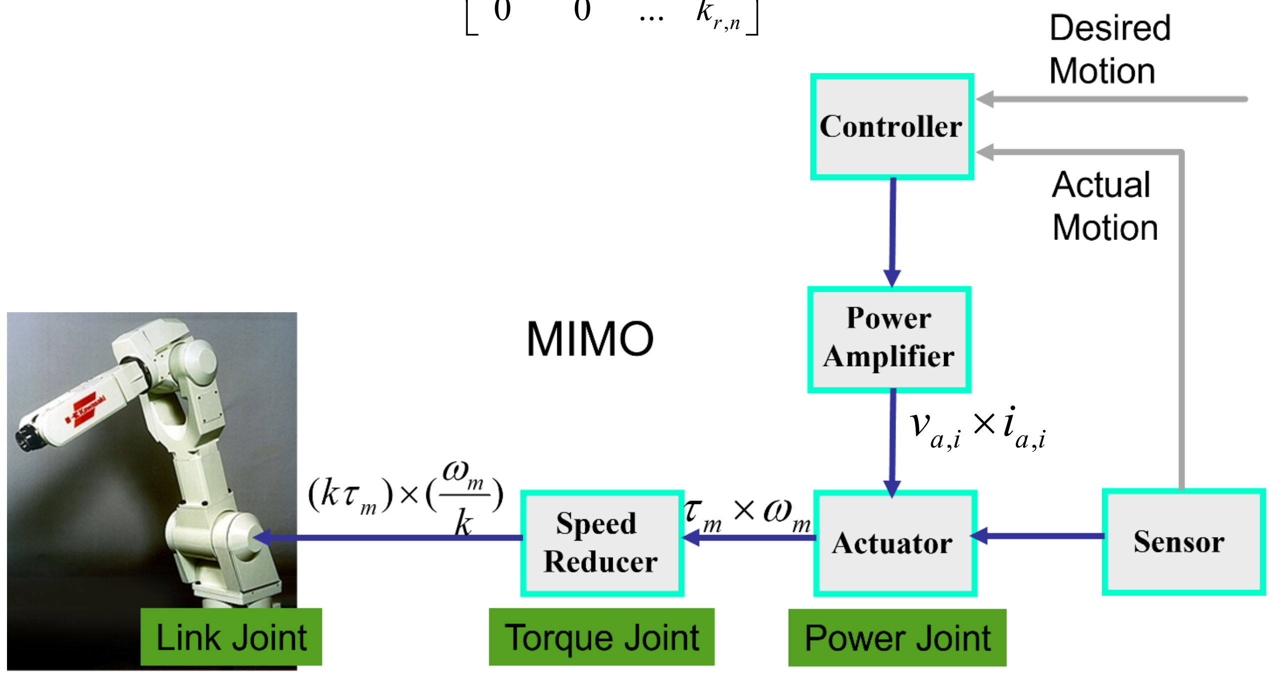


Property 2 of Torque Joints

$$q_i = \frac{1}{k_{r,i}} \bullet q_{m,i}, \quad i = 1,2,\ldots,n$$

$$K_r^{-1} = \begin{bmatrix} \frac{1}{k_{r,1}} & 0 & \cdots & 0 \\ 0 & \frac{1}{k_{r,2}} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \frac{1}{k_{r,n}} \end{bmatrix} \rightarrow q = K_r^{-1} \bullet q_m$$

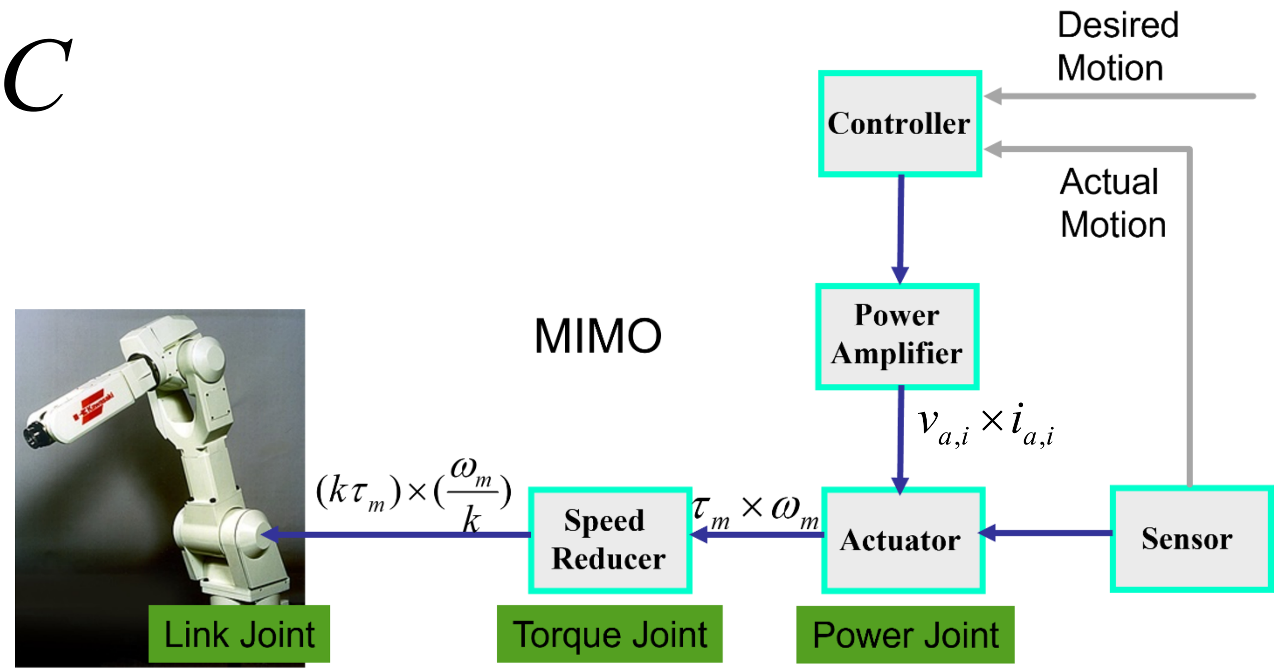
$$K_r = \begin{bmatrix} k_{r,1} & 0 & \cdots & 0 \\ 0 & k_{r,2} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & k_{r,n} \end{bmatrix} \rightarrow q_m = K_r \bullet q$$



If we define

$$B(q) = B_{diag} + \Delta B \quad \Rightarrow \quad \begin{bmatrix} 3 & 1 & 2 \\ 1 & 4 & 3 \\ 2 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 2 & 3 & 0 \end{bmatrix}$$

$$C(q, \dot{q}) = C$$



Then we have

$$B(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$$

$$q = K_r^{-1} q_m \qquad B(q) = B_{diag} + \Delta B$$

$$\tau = K_r \tau_m \qquad C(q,\dot{q}) = C$$

$$[B_{diag} + \Delta B] \bullet K_r^{-1} \bullet \ddot{q}_m + C K_r^{-1} \dot{q}_m + g(K_r^{-1} q_m) = K_r \tau_m$$

This is the dynamic equation of Robot at Torque Joints.

Example

- Prove the following equation and explain its physical meanings:

$$\tau_m = K_r^{-1} B_{diag} K_r^{-1} \ddot{q}_m + d$$

with :

$$d = K_r^{-1} \Delta B K_r^{-1} \ddot{q}_m + K_r^{-1} C K_r^{-1} \dot{q}_m + K_r^{-1} g(K_r^{-1} q_m)$$

Solution

$$[B_{diag} + \Delta B] \bullet K_r^{-1} \bullet \ddot{q}_m + CK_r^{-1}\dot{q}_m + g(K_r^{-1}q_m) = K_r\tau_m$$

$$\Downarrow$$

$$B_{diag}K_r^{-1}\ddot{q}_m + \Delta BK_r^{-1}\ddot{q}_m + CK_r^{-1}\dot{q}_m + g(K_r^{-1}q_m) = K_r\tau_m$$

Solution (continued)

$$B_{diag} K_r^{-1} \ddot{q}_m + \Delta B K_r^{-1} \ddot{q}_m + C K_r^{-1} \dot{q}_m + g(K_r^{-1} q_m) = K_r \tau_m$$

$$\Downarrow$$

$$K_r \tau_m = B_{diag} K_r^{-1} \ddot{q}_m + \Delta B K_r^{-1} \ddot{q}_m + C K_r^{-1} \dot{q}_m + g(K_r^{-1} q_m)$$

Solution (continued)

$$K_r \tau_m = B_{diag} K_r^{-1} \ddot{q}_m + \Delta B K_r^{-1} \ddot{q}_m + C K_r^{-1} \dot{q}_m + g(K_r^{-1} q_m)$$

⇓

$$\tau_m = K_r^{-1} B_{diag} K_r^{-1} \ddot{q}_m + K_r^{-1} \Delta B K_r^{-1} \ddot{q}_m + K_r^{-1} C K_r^{-1} \dot{q}_m + K_r^{-1} g(K_r^{-1} q_m)$$

Solution (continued)

$$\tau_m = K_r^{-1} B_{diag} K_r^{-1} \ddot{q}_m + \underbrace{K_r^{-1} \Delta B K_r^{-1} \ddot{q}_m + K_r^{-1} C K_r^{-1} \dot{q}_m + K_r^{-1} g(K_r^{-1} q_m)}_{d}$$

$$\Downarrow$$

$$\tau_m = K_r^{-1} B_{diag} K_r^{-1} \ddot{q}_m + d$$

with :

$$d = K_r^{-1} \Delta B K_r^{-1} \ddot{q}_m + K_r^{-1} C K_r^{-1} \dot{q}_m + K_r^{-1} g(K_r^{-1} q_m)$$

Example

- Prove the following equation and explain its physical meanings:

$$\ddot{q}_m = K_r B_{diag}^{-1} K_r (\tau_m - d)$$

with :

$$d = K_r^{-1} \Delta B K_r^{-1} \ddot{q}_m + K_r^{-1} C K_r^{-1} \dot{q}_m + K_r^{-1} g(K_r^{-1} q_m)$$

Solution

$$\tau_m = K_r^{-1} B_{diag} K_r^{-1} \ddot{q}_m + d$$

with :

$$d = K_r^{-1} \Delta B K_r^{-1} \ddot{q}_m + K_r^{-1} C K_r^{-1} \dot{q}_m + K_r^{-1} g(K_r^{-1} q_m)$$

⇓

$$[K_r^{-1} B_{diag} K_r^{-1}] \ddot{q}_m = \tau_m - d$$

with :

$$d = K_r^{-1} \Delta B K_r^{-1} \ddot{q}_m + K_r^{-1} C K_r^{-1} \dot{q}_m + K_r^{-1} g(K_r^{-1} q_m)$$

Solution (continued)

$$[K_r^{-1} B_{diag} K_r^{-1}]\ddot{q}_m = \tau_m - d$$

with :

$$d = K_r^{-1} \Delta B K_r^{-1} \ddot{q}_m + K_r^{-1} C K_r^{-1} \dot{q}_m + K_r^{-1} g(K_r^{-1} q_m)$$

$$\Downarrow$$

$$\ddot{q}_m = [K_r^{-1} B_{diag} K_r^{-1}]^{-1} (\tau_m - d)$$

with :

$$d = K_r^{-1} \Delta B K_r^{-1} \ddot{q}_m + K_r^{-1} C K_r^{-1} \dot{q}_m + K_r^{-1} g(K_r^{-1} q_m)$$

Solution (continued)

$$\ddot{q}_m = [K_r^{-1} B_{diag} K_r^{-1}]^{-1}(\tau_m - d)$$

with :

$$d = K_r^{-1}\Delta B K_r^{-1}\ddot{q}_m + K_r^{-1} C K_r^{-1}\dot{q}_m + K_r^{-1} g(K_r^{-1} q_m)$$

⇩

$$(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$$

$$\ddot{q}_m = K_r B_{diag}^{-1} K_r (\tau_m - d)$$

with :

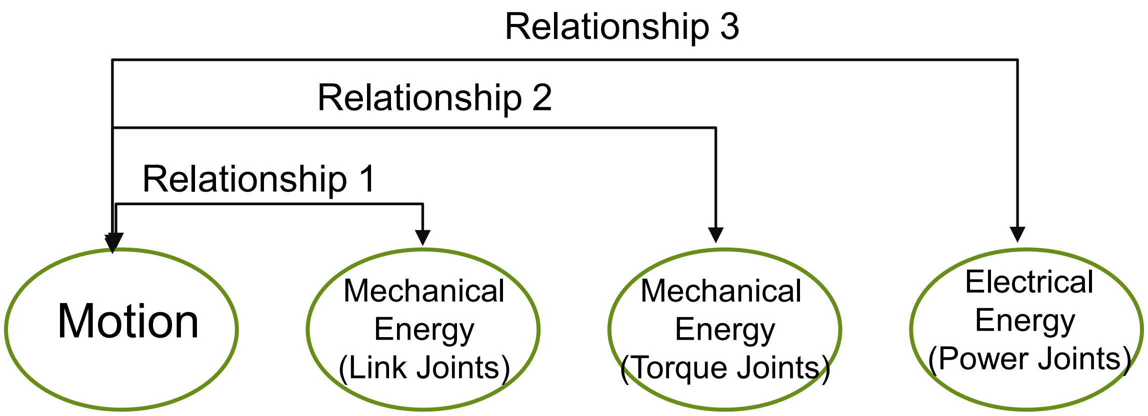
$$d = K_r^{-1}\Delta B K_r^{-1}\ddot{q}_m + K_r^{-1} C K_r^{-1}\dot{q}_m + K_r^{-1} g(K_r^{-1} q_m)$$

Outline of Lecture 1

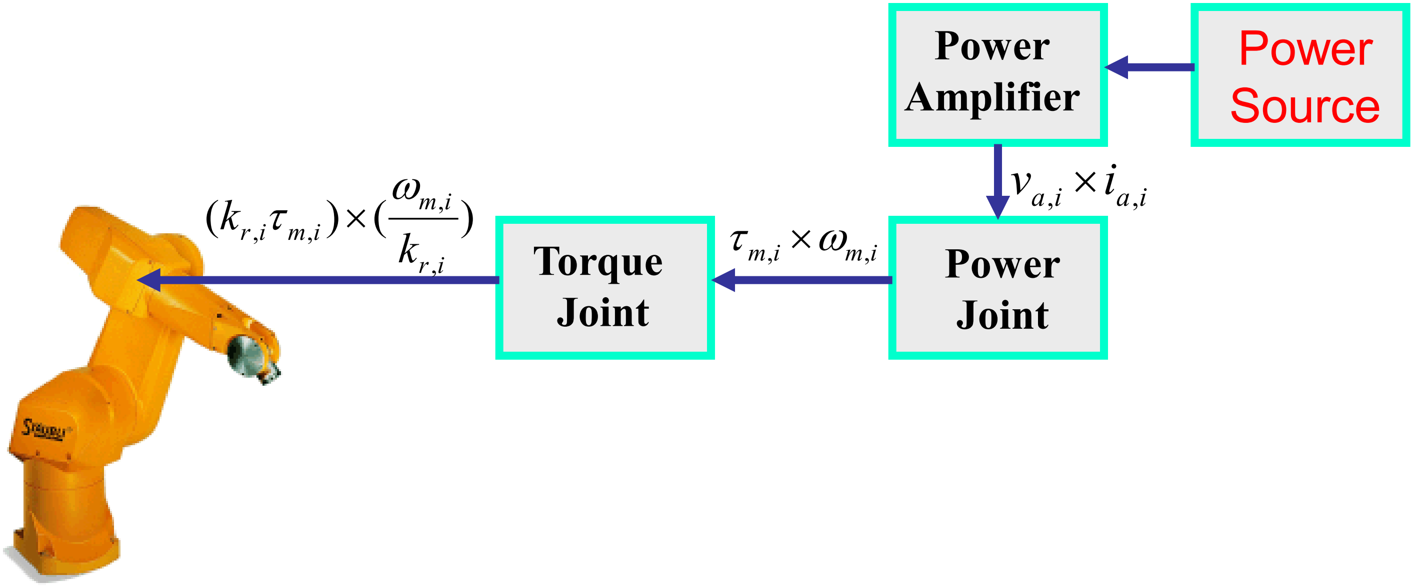
- Three Relationship Between Motion and Energy
- Dynamic Equations of Motion at Link Joints
- Dynamic Equations of Motion at Torque Joints
- Dynamic Equations of Motion at Power Joints
- Dynamic Equations of Motion at Controllers

What is the relationship between motion and electrical energy at power joints?

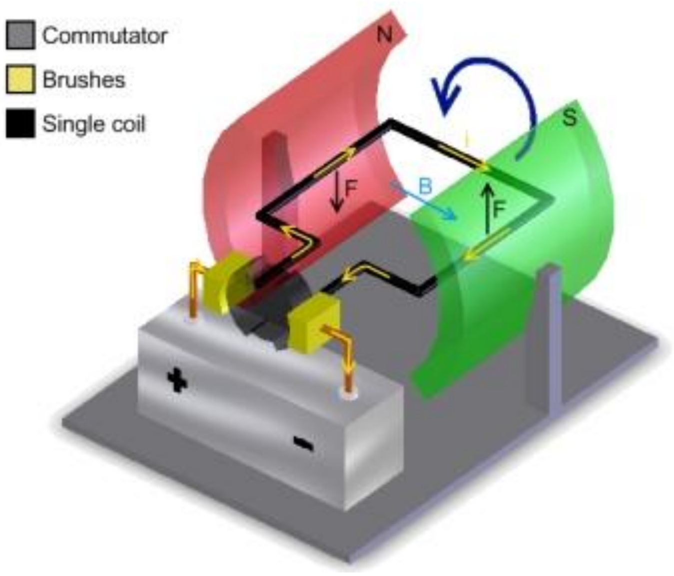
- Motions are related to mechanical energy at link joints
- Motions are related to mechanical energy at torque joints
- Motions are related to electrical energy at power joints

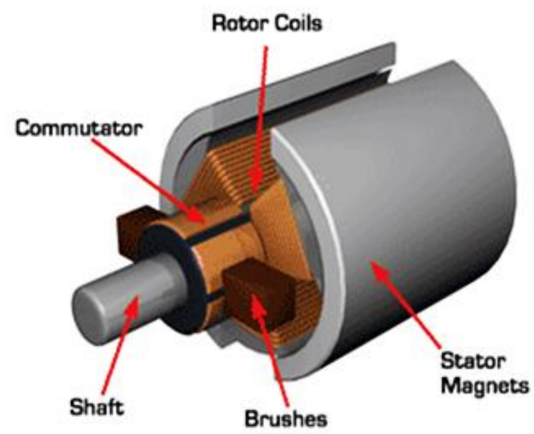


Mechanical energies at output of power joints come from the electrical energies at their inputs

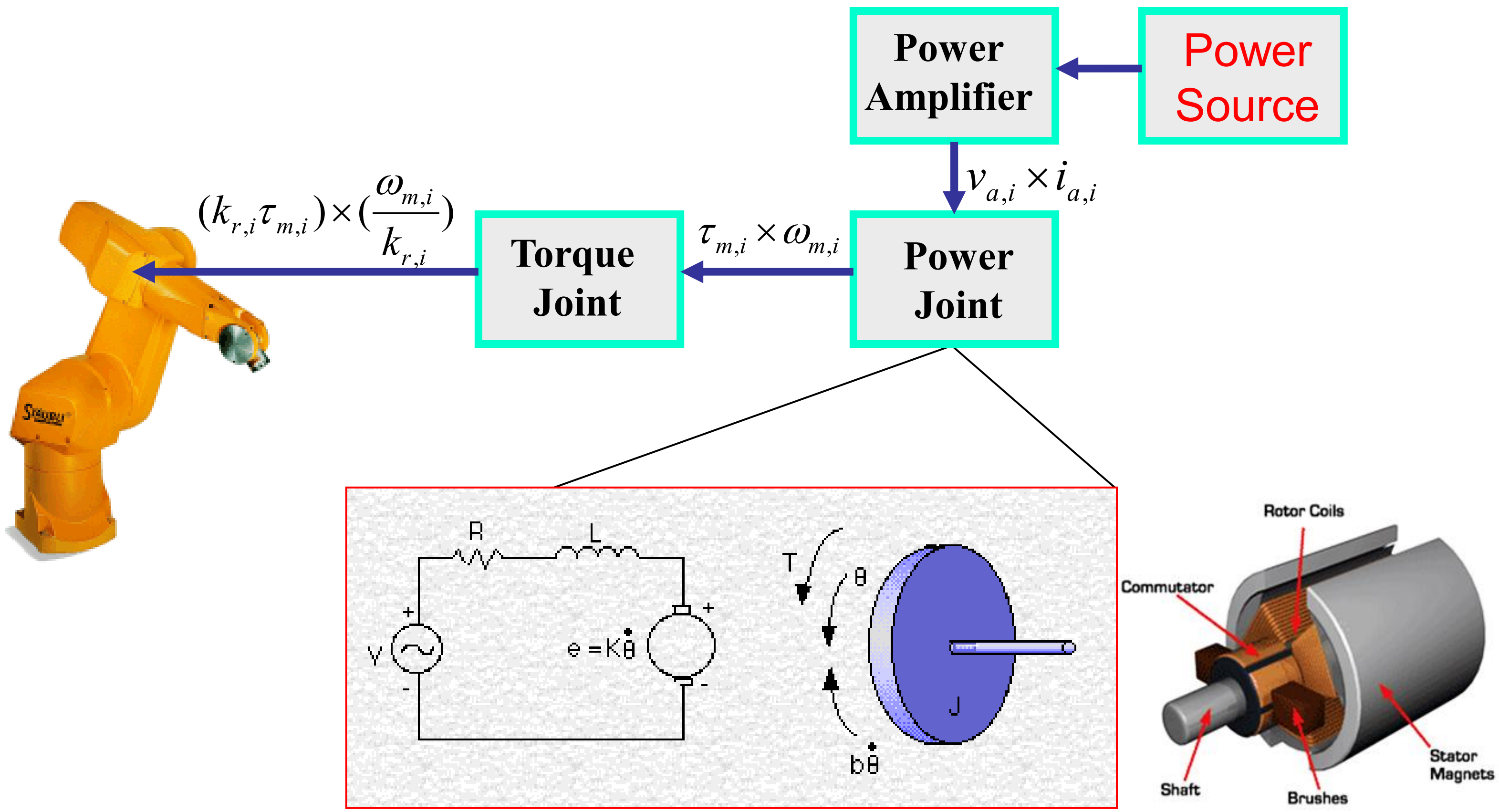


An electrical motor is an electro-mechanical device





Equivalent diagram of electrical motor

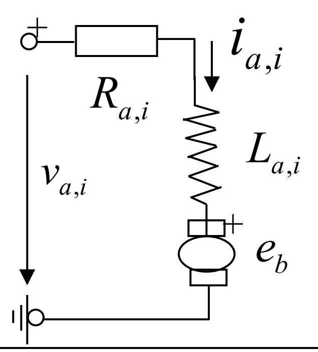


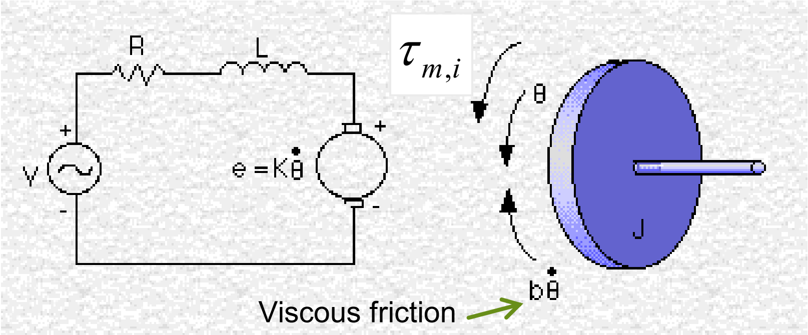
Equation of Power Joints

Electro-motif force

$$v_{a,i} = R_{a,i} i_{a,i} + L_{a,i} \frac{di_{a,i}}{dt} + k_{v,i} \dot{q}_{m,i}$$

$$e_b = k_{v,i} \dot{q}_{m,i}$$



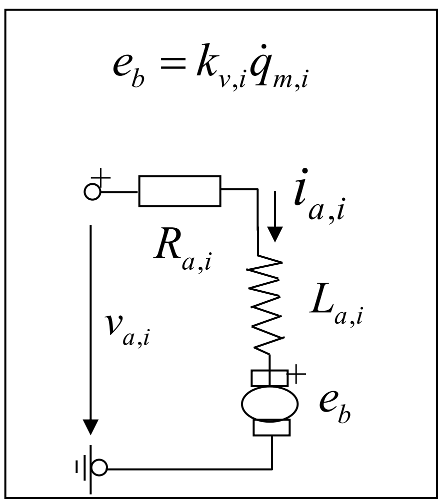


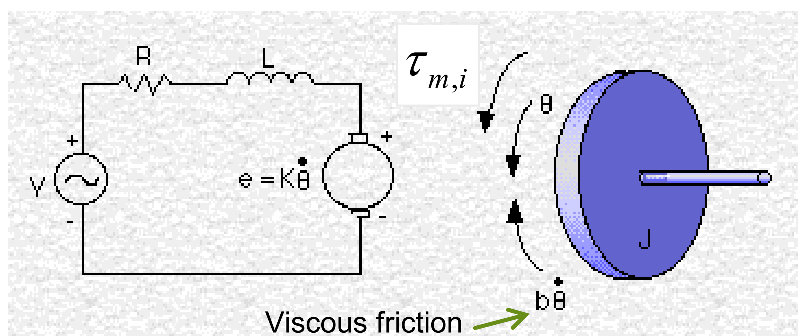
Dynamic Equation 1 of Power Joints (continued)

$$\tau_{m,i} = k_{t,i} \bullet i_{a,i}$$

$$i_{a,i} = \frac{1}{k_{t,i}} \bullet \tau_{m,i}$$

$$\frac{di_{a,i}}{dt} = \frac{1}{k_{t,i}} \bullet \frac{d\tau_{m,i}}{dt}$$





Dynamic Equation 2 of Power Joints (continued)

$$\frac{di_{a,i}}{dt} = \frac{1}{k_{t,i}} \bullet \frac{d\tau_{m,i}}{dt}$$

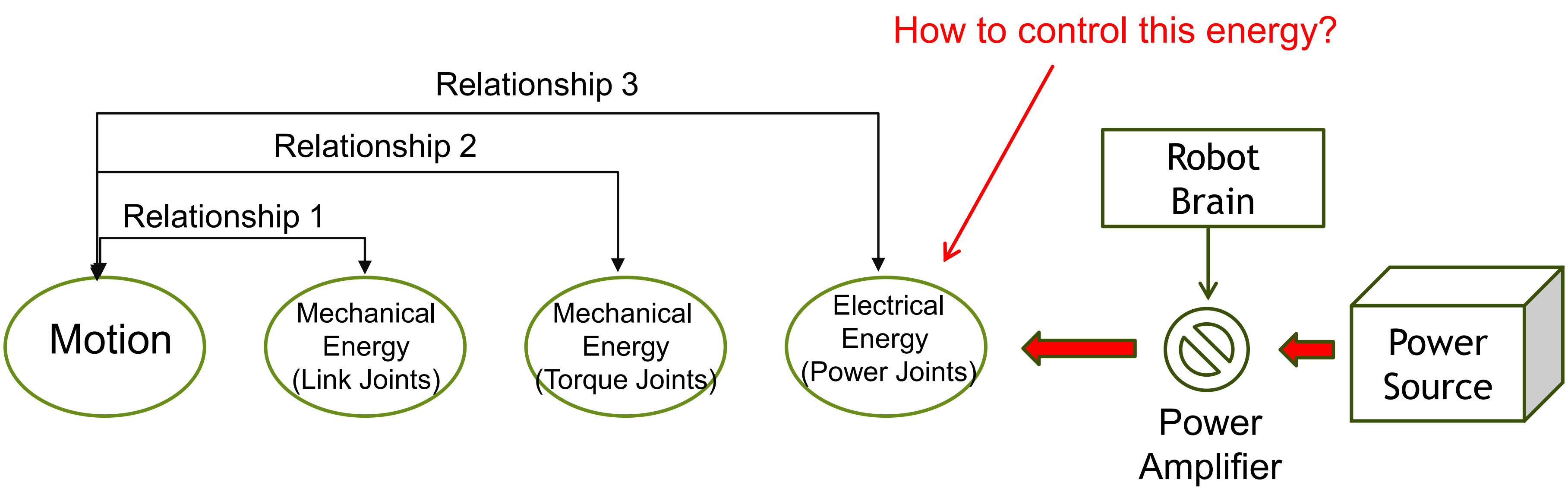
$$v_{a,i} = R_{a,i} i_{a,i} + L_{a,i} \frac{di_{a,i}}{dt} + k_{v,i} \dot{q}_{m,i}$$

$$\tau_{m,i} = k_{t,i} \bullet i_{a,i}$$

$$v_{a,i} = \frac{R_{a,i}}{K_{t,i}} \tau_{m,i} + \frac{L_{a,i}}{K_{t,i}} \frac{d\tau_{m,i}}{dt} + k_{v,i} \dot{q}_{m,i}$$

Next question: What is the relationship between energy and control signal at controllers (i.e., mind and brain)?

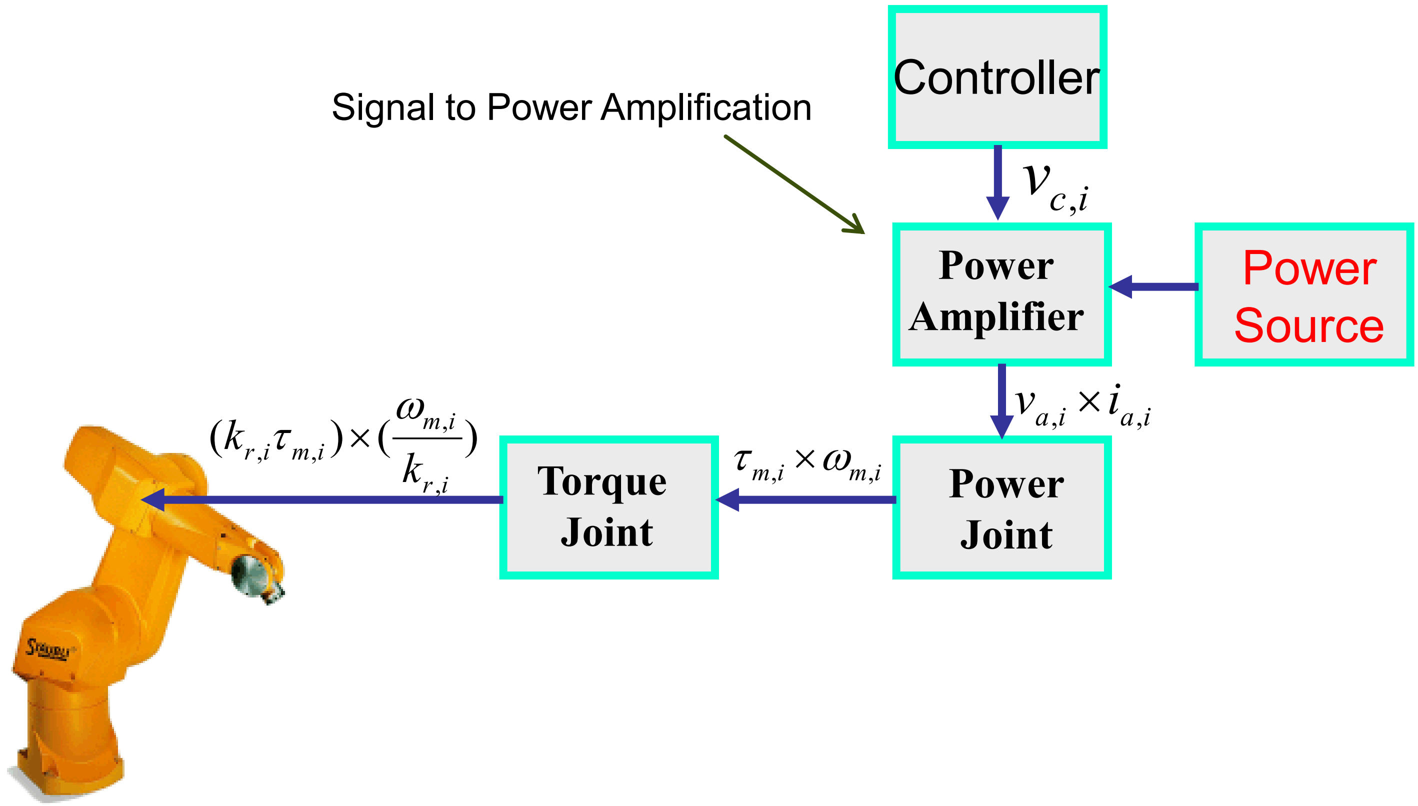
- Motions are related to mechanical energy at link joints
- Motions are related to mechanical energy at torque joints
- Motions are related to electrical energy at power joints



Outline of Lecture 1

- Three Relationship Between Motion and Energy
- Dynamic Equations of Motion at Link Joints
- Dynamic Equations of Motion at Torque Joints
- Dynamic Equations of Motion at Power Joints
- Dynamic Equations of Motion at Controllers

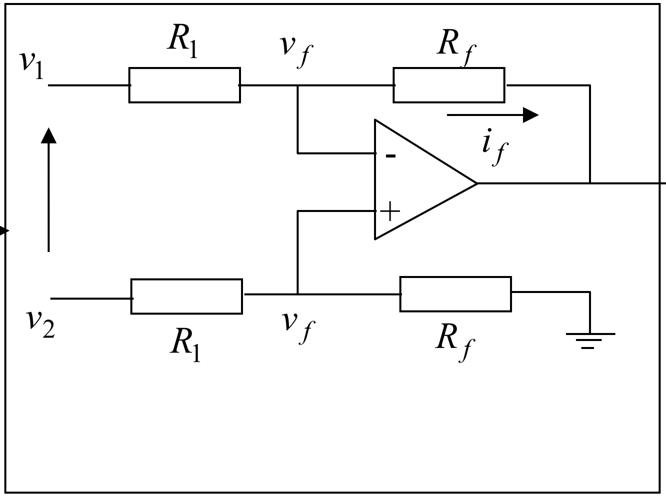
Electrical energies at input of power joints (i.e. motors) are under the control of signals from controllers (i.e., mind and brain) …



Example of Equation of Power Amplifier

$$v_f = \frac{v_2}{R_1 + R_f} R_f \quad \Rightarrow \quad i_f = \frac{v_1 - v_f}{R_1} \quad \Rightarrow \quad v_{a,i} = v_f - R_f \times i_f$$

$$v_{c,i} = v_2 - v_1$$



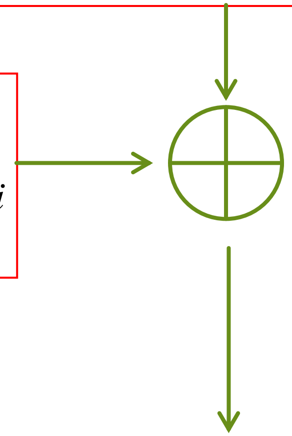
$$v_{a,i}$$

$$v_{a,i} = \frac{R_f}{R_1}(v_2 - v_1) = \frac{R_f}{R_1} v_{c,i}$$

Dynamic Equations at Controllers

$$v_{a,i} = \frac{R_{a,i}}{K_{t,i}} \tau_{m,i} + \frac{L_{a,i}}{K_{t,i}} \frac{d\tau_{m,i}}{dt} + k_{v,i} \dot{q}_{m,i}$$

$$v_{a,i} = \frac{R_f}{R_1}(v_2 - v_1) = \frac{R_f}{R_1} v_{c,i}$$

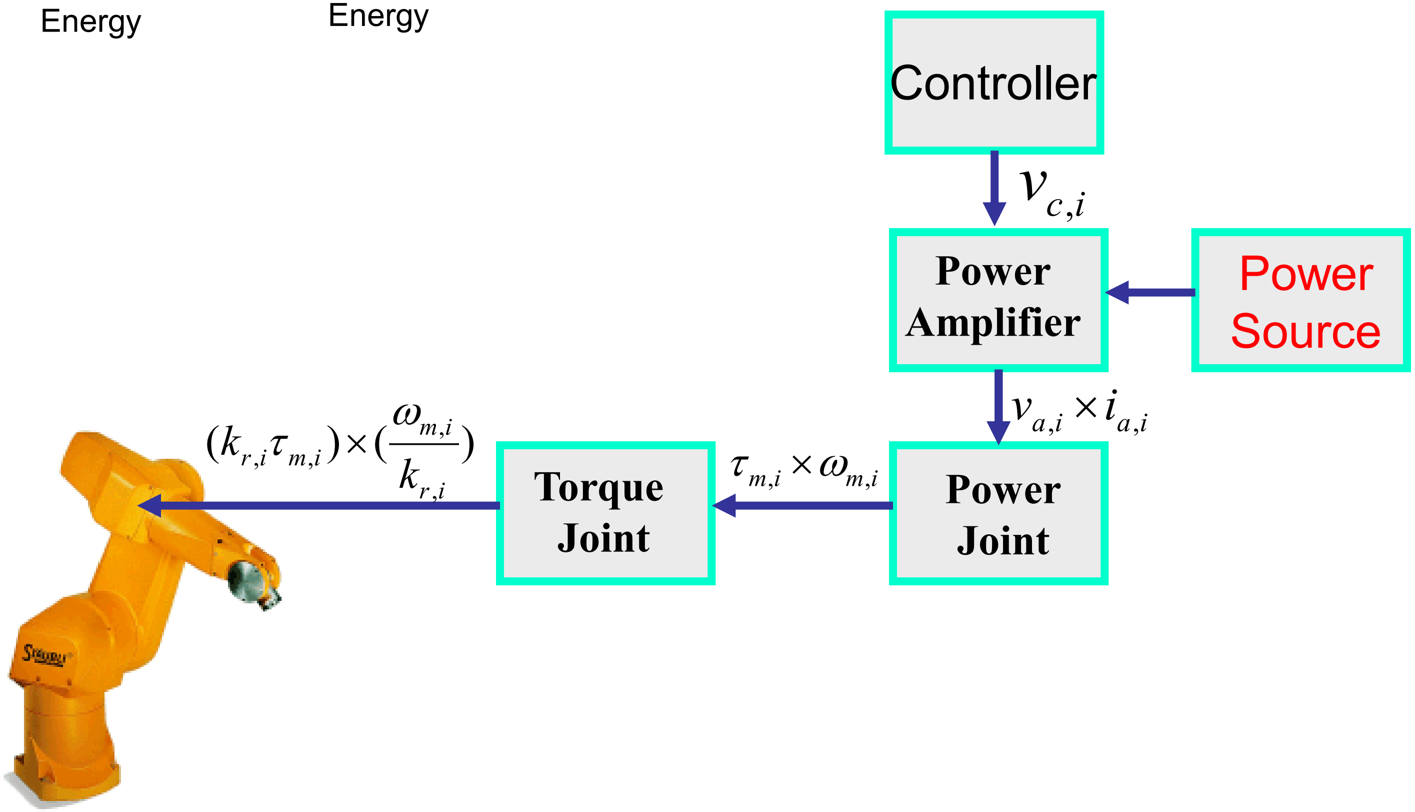


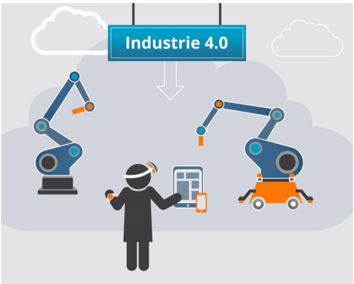
$$v_{c,i} = \frac{R_1}{R_f} \frac{R_{a,i}}{K_{t,i}} \tau_{m,i} + \frac{R_1}{R_f} \frac{L_{a,i}}{K_{t,i}} \frac{d\tau_{m,i}}{dt} + \frac{R_1}{R_f} k_{v,i} \dot{q}_{m,i}$$

Summary of Transforming Signal Flow (Mind and Brain) to Energy Flow (Body) …

$v_{c,i} \longrightarrow v_{a,i} \longrightarrow \tau_{m,i} \longrightarrow \omega_{m,i} \longrightarrow \tau \longrightarrow \omega$

Signals | Electrical Energy | Mechanical Energy | Motions | Motions



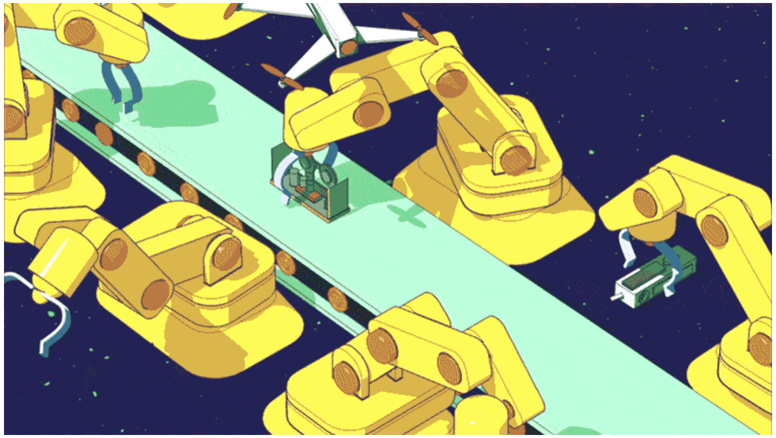


Summary of Lecture 1

- Three Relationship Between Motion and Energy
- Dynamic Equations of Motion at Link Joints
- Dynamic Equations of Motion at Torque Joints
- Dynamic Equations of Motion at Power Joints
- Dynamic Equations of Motion at Controllers

Outline of Module 4

- Dynamics under Control
- Signal Flow Diagram
- Design of Control Systems
- Control in Joint-Space
- Control in Task-Space



Module 4

MA4825 Robotics

Lecture 2

Signal Flow Diagram

Xie Ming, PhD (France)

http://personal.ntu.edu.sg/mmxie

Outline of Lecture 2

- Laplace Transforms
- Transfer Functions
- Signal Flow Diagram
- Signal Flow Diagram of Robot's Dynamics

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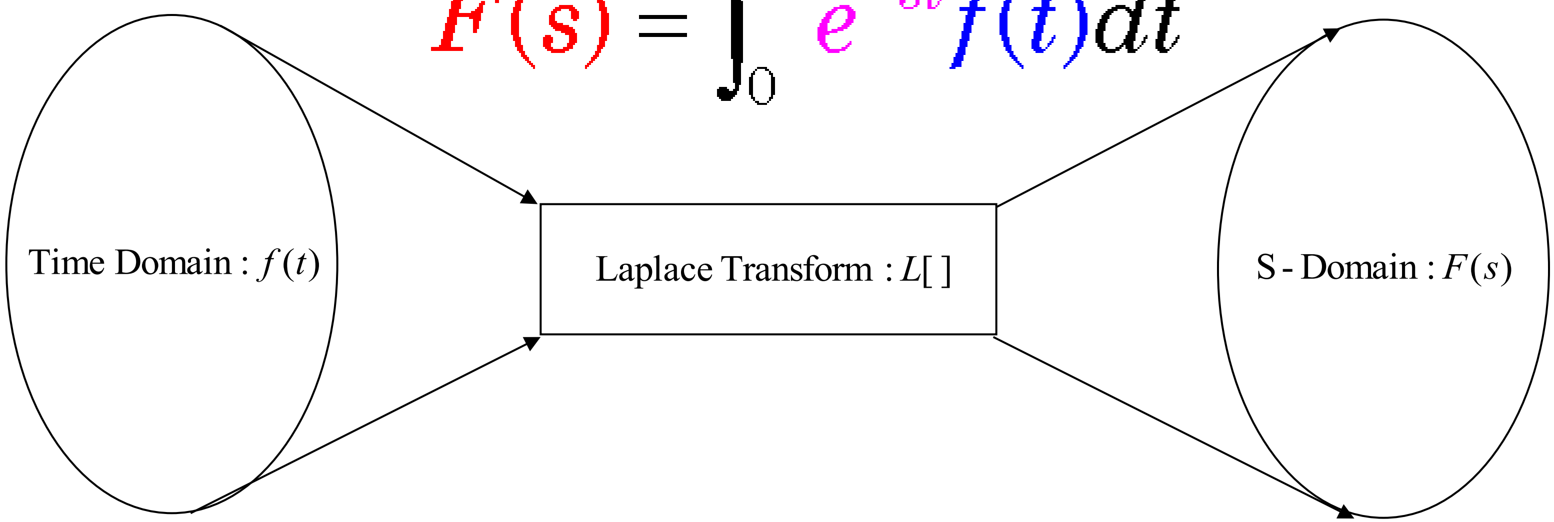
What is Laplace Transform?

- Laplace Transform is an effective way of transforming differential equations into polynomial equations in frequency domain.

$$F(s) = \int_0^\infty e^{-st} f(t)\,dt$$



$$f(t) = \frac{1}{2\pi j}\left[\int_{c-j\infty}^{c+j\infty} F(s)e^{st}\,ds\right]$$

Laplace Transform

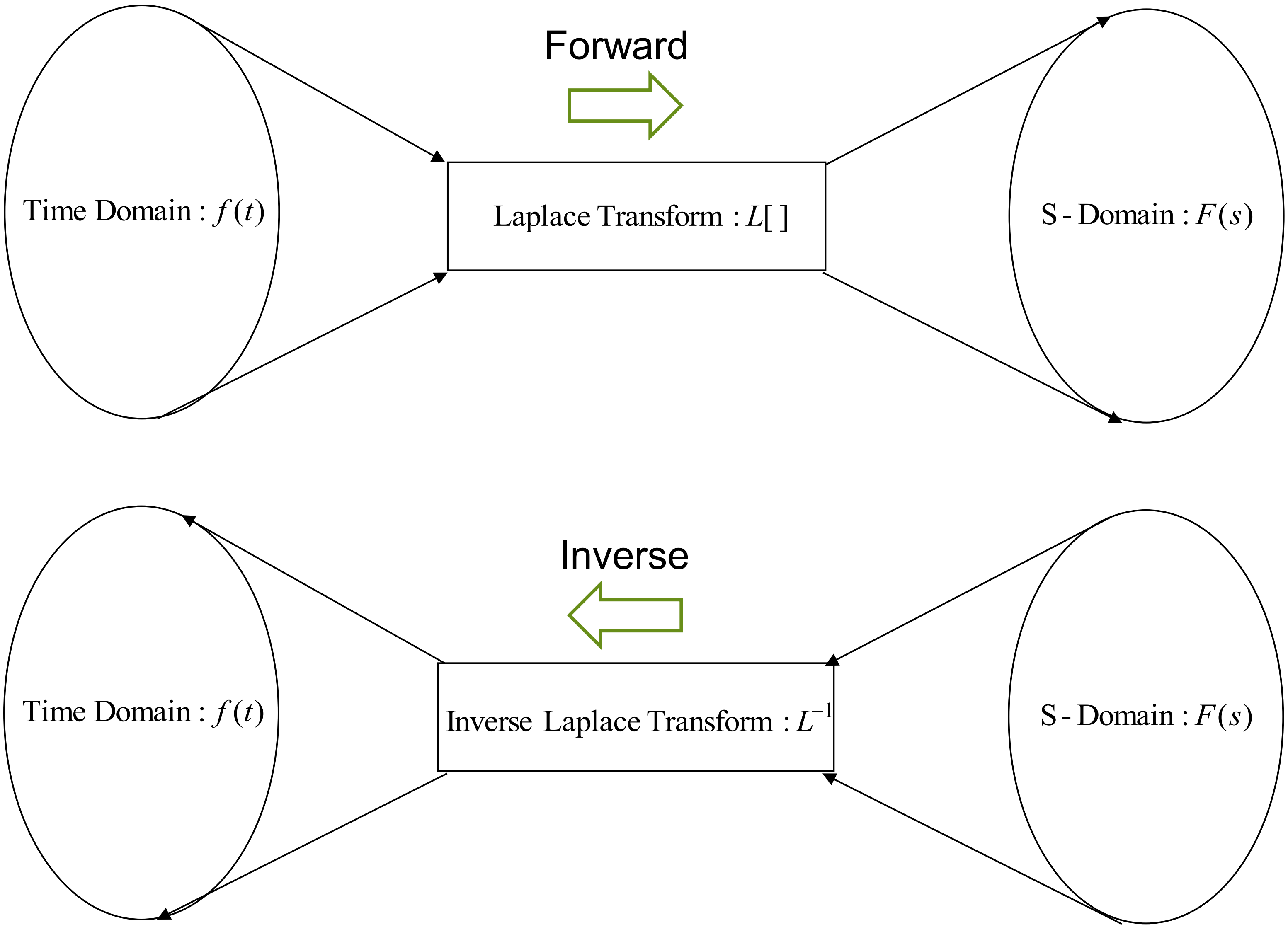
$f(t)$: a function of time t such that $f(t) = 0$ for t < 0
s: a complex variable (i.e. real $+$ j $\times$ imaginary)
$L[\ ]$: an operational symbol indicating Laplace Transformation.
$F(s)$: Laplace transform of f(t).

Forward process of Laplace transform:

$$L[f(t)] = F(s) = \int_0^{\infty} f(t) \bullet e^{-st} dt$$

Inverse process of Laplace transform:

$$L^{-1}[F(s)] = f(t) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} F(s) \bullet e^{st} ds$$

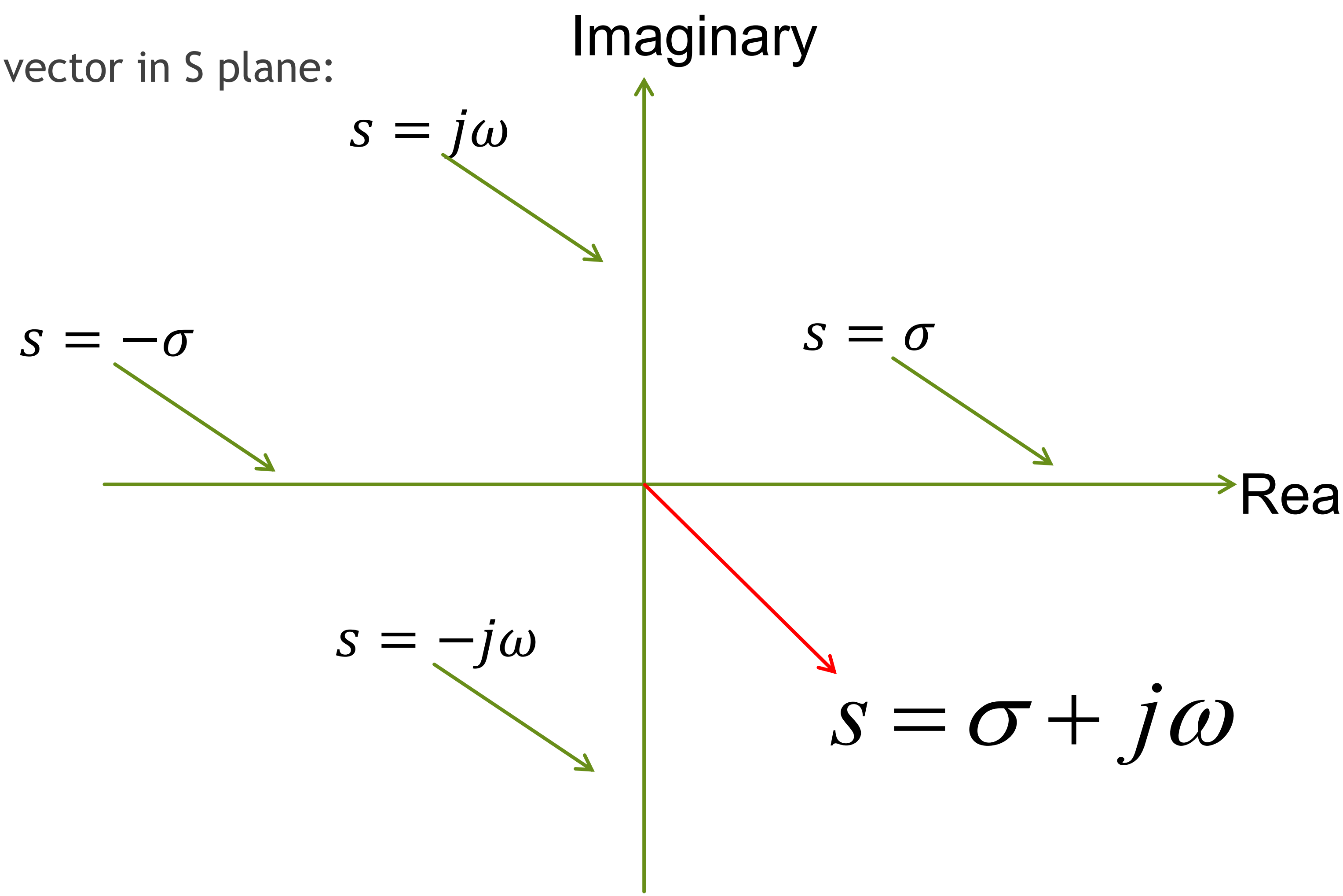

Forward
Time Domain : $f(t)$
Laplace Transform : $L[\ ]$
S - Domain : $F(s)$
Inverse
Time Domain : $f(t)$
Inverse Laplace Transform : L^{-1}
S - Domain : $F(s)$

Basic Functions and Their Laplace Transforms

Time Domain	Frequency Domain
Unit pulse $f(t) = \delta(t)$	$F(s) = 1$
Unit step $f(t) = 1(t)$	$F(s) = \frac{1}{s}$
Ramp $f(t) = A \bullet t$	$F(s) = \frac{A}{s^2}$

What is vector “s” in Laplace Transform?

- S is a vector in S plane:



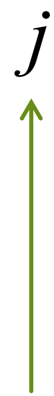
What is S-Plane?

Euler Equation:

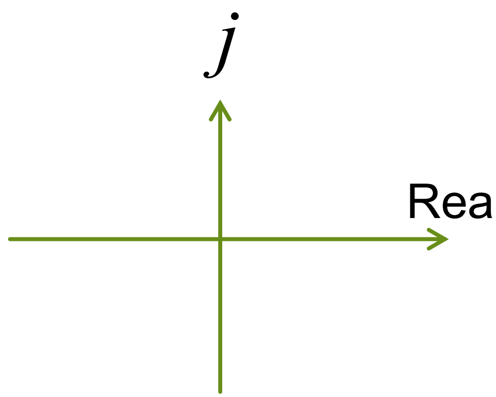
$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

- Real Axis:

- Imaginary Axis:



- Complex Plane:



- Complex Numbers or Vectors: $s = \sigma + j\omega$

Property 1 of Laplace Transform

$$\frac{dx}{dt} \xrightarrow{\text{Forward}} sX(s)$$

$$\frac{d^2x}{dt^2} \xrightarrow{\text{Forward}} s^2X(s)$$

$$\frac{d^nx}{dt^n} \xrightarrow{\text{Forward}} s^nX(s)$$

Property 2 of Laplace Transform

$$\int f(t)dt \xrightarrow{\text{Forward}} \frac{1}{s}F(s)$$

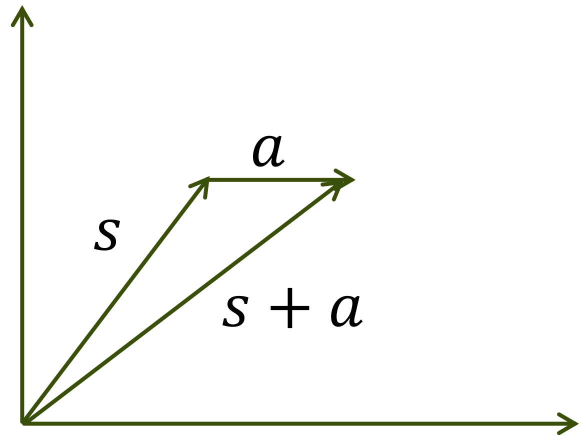
$$\iint f(t)dtdt \xrightarrow{\text{Forward}} \frac{1}{s^2}F(s)$$

$$\iiint f(t)dtdtdt \xrightarrow{\text{Forward}} \frac{1}{s^3}F(s)$$

Property 3 of Laplace Transform

Unit step $f(t) = 1(t)$

$$F(s) = \frac{1}{s}$$



Displacement in Frequency Domain

Exponential $Function$: $f(t) = e^{-at}$

$$F(s) = \frac{1}{s+a}$$

Property 4 of Laplace Transform

$f(t)$	$F(s) = \mathcal{L}[f(t)]$		Formula
$f(t) = 1$	$F(s) = \frac{1}{s}$	$s > 0$	A
$f(t) = e^{at}$	$F(s) = \frac{1}{(s-a)}$	$s > a$	B
$f(t) = t^n$	$F(s) = \frac{n!}{s^{(n+1)}}$	$s > 0$	C
$f(t) = \sin(at)$	$F(s) = \frac{a}{s^2 + a^2}$	$s > 0$	D
$f(t) = \cos(at)$	$F(s) = \frac{s}{s^2 + a^2}$	$s > 0$	E
$f(t) = \sinh(at)$	$F(s) = \frac{a}{s^2 - a^2}$	$s > \|a\|$	F
$f(t) = \cosh(at)$	$F(s) = \frac{s}{s^2 - a^2}$	$s > \|a\|$	G
$f(t) = t^n e^{at}$	$F(s) = \frac{n!}{(s-a)^{(n+1)}}$	$s > a$	H
$f(t) = e^{at}\sin(bt)$	$F(s) = \frac{b}{(s-a)^2 + b^2}$	$s > a$	I
$f(t) = e^{at}\cos(bt)$	$F(s) = \frac{(s-a)}{(s-a)^2 + b^2}$	$s > a$	J
$f(t) = e^{at}\sinh(bt)$	$F(s) = \frac{b}{(s-a)^2 - b^2}$	$s - a > \|b\|$	K
$f(t) = e^{at}\cosh(bt)$	$F(s) = \frac{(s-a)}{(s-a)^2 - b^2}$	$s - a > \|b\|$	L

$$L[t^n] = \frac{n!}{s^{n+1}}$$

$$L[f(t)] = F(s)$$

$$L[f(t)e^{-at}] = F(s+a)$$

Property 5 of Laplace Transform

Sine $f(t) = A \bullet \sin(\omega t)$

$$F(s) = \frac{A \bullet \omega}{s^2 + \omega^2}$$

$$F(s) = \frac{A\omega}{(s + j\omega)(s - j\omega)}$$

$$F(s) = \frac{A}{2}\left(\frac{j}{s + j\omega} - \frac{j}{s - j\omega}\right)$$

Cosine $f(t) = A \bullet \cos(\omega t)$

$$F(s) = \frac{A \bullet s}{s^2 + \omega^2}$$

$$F(s) = \frac{As}{(s + j\omega)(s - j\omega)}$$

$$F(s) = \frac{A}{2}\left(\frac{1}{s + j\omega} + \frac{1}{s - j\omega}\right)$$

One More Example of Laplace Transform

$$f(t) = e^{-at}\sin(\omega t)$$

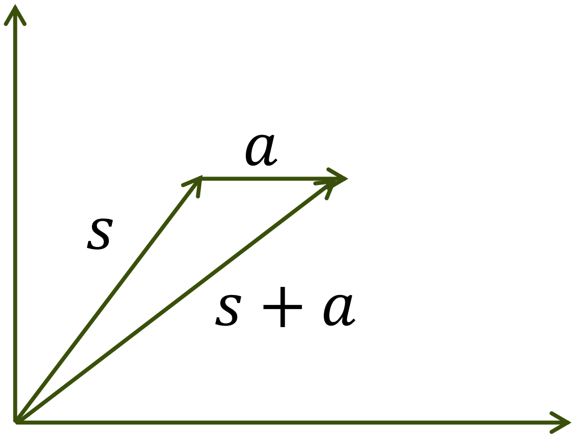
$$F(s) = \frac{\omega}{(s+a)^2 + \omega^2}$$

$$L[f(t)] = F(s)$$

$$L[f(t)e^{-at}] = F(s+a)$$

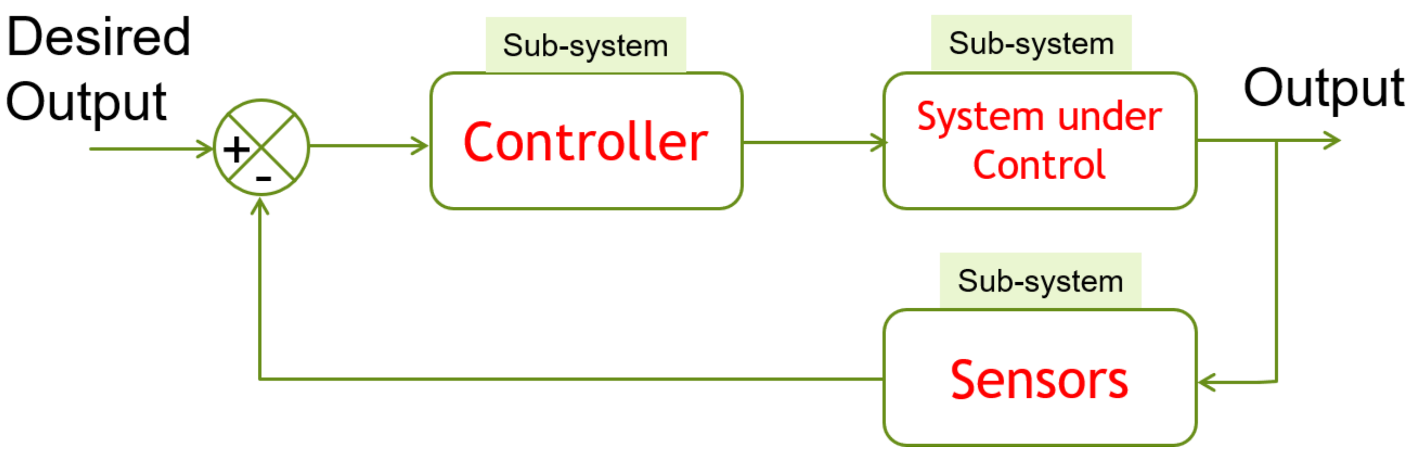
$$f(t) = e^{-at}\cos(\omega t)$$

$$F(s) = \frac{s+a}{(s+a)^2 + \omega^2}$$



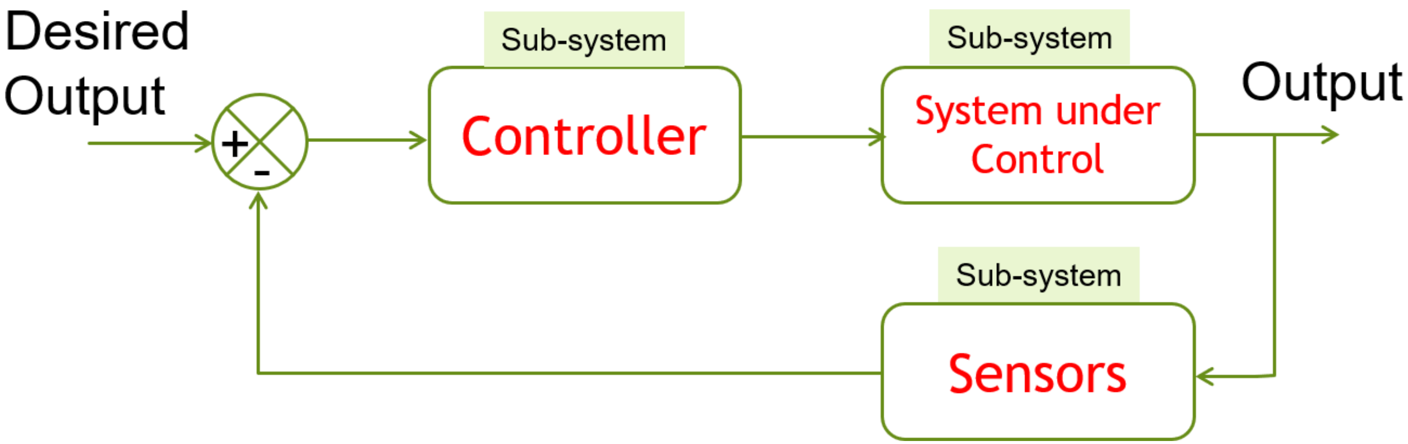
Initial Value Theorem

$$\lim_{t \to 0} y(t) = \lim_{s \to \infty} s \bullet Y(s)$$



Final Value Theorem

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} s \bullet Y(s)$$



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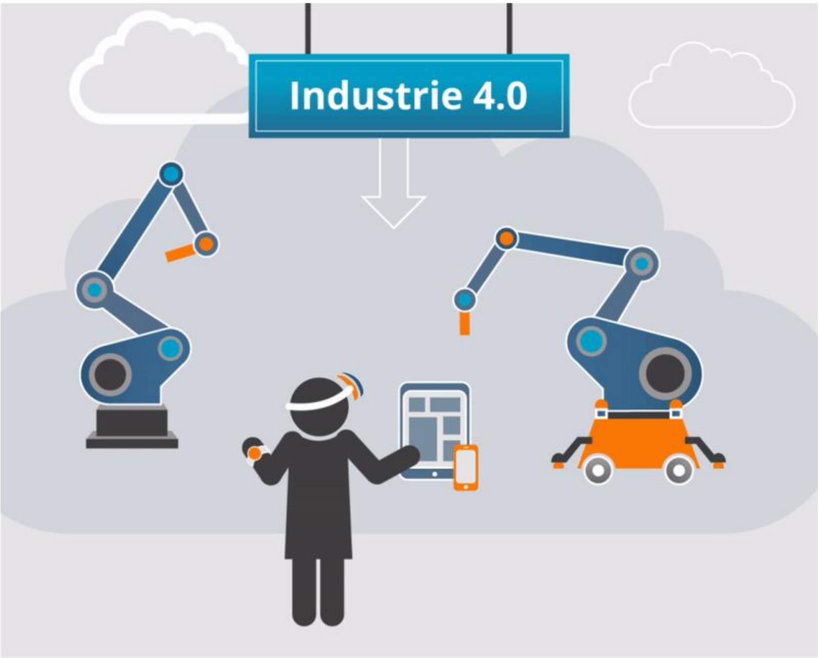
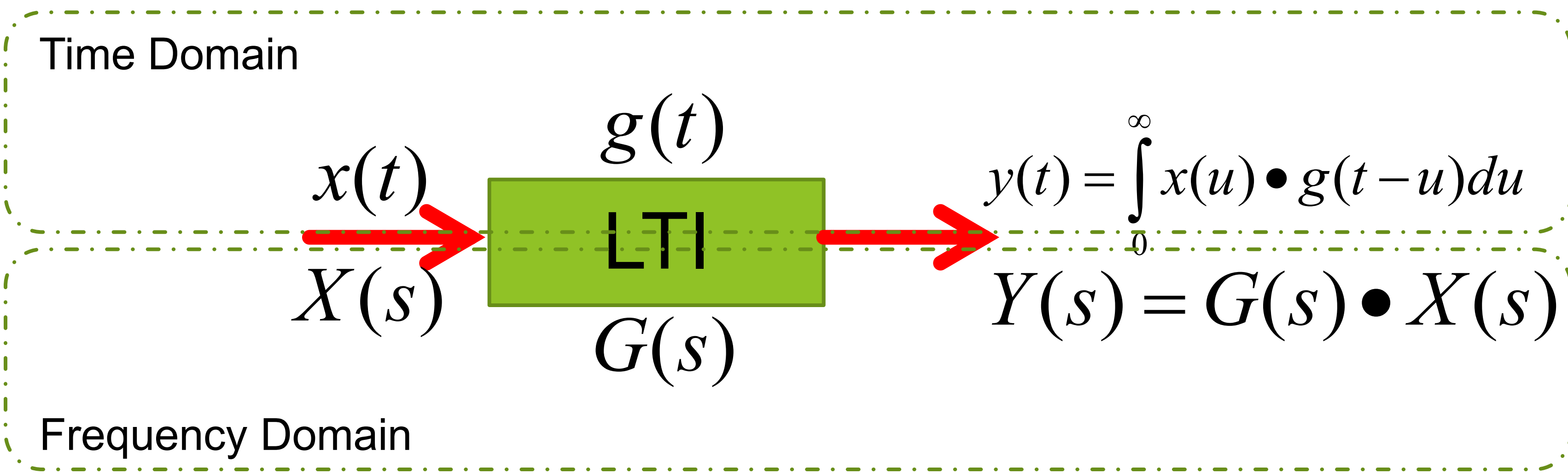
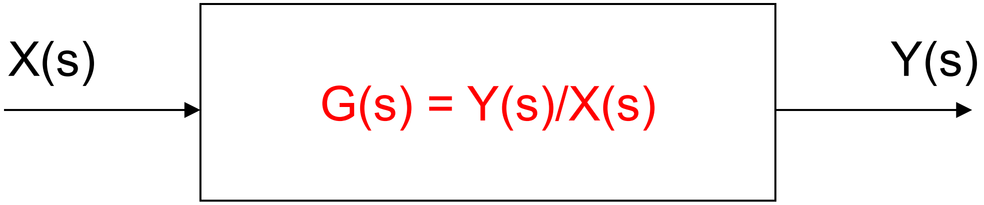


What is transfer function?



Definition of Transfer Function

- A transfer function G(s) is the ratio between the Laplace transform of the output and the Laplace transform of the input.



Example of Transfer Function of Systems without Error Control Loop

Consider the rotational system having a flywheel with inertia J = 100 kg.m^2 and viscous damping coefficient B = 10 N-s/rad shown in Figure 2

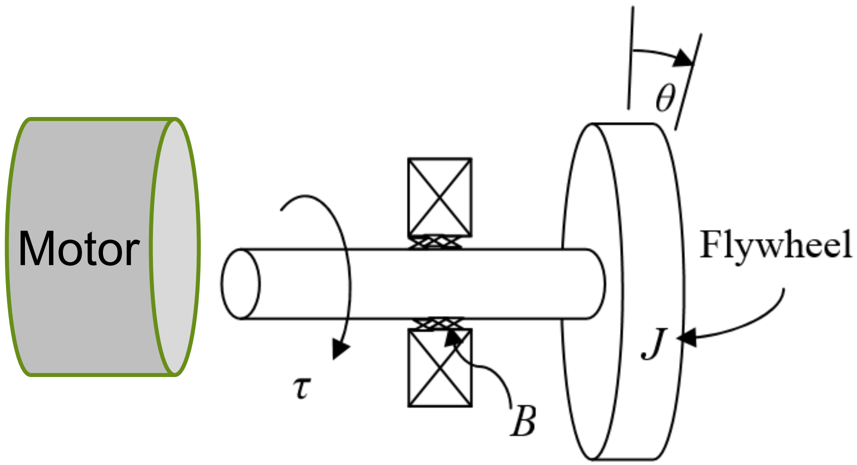


Figure 2: Rotational system

(a) Find the transfer function $\frac{\Omega(s)}{T(s)}$ for input torque τ to angular velocity ω.

(b) Using Laplace Transform find out the ω(t) if a torque of 100 N-m is applied. What is the steady state value for ω(t)?

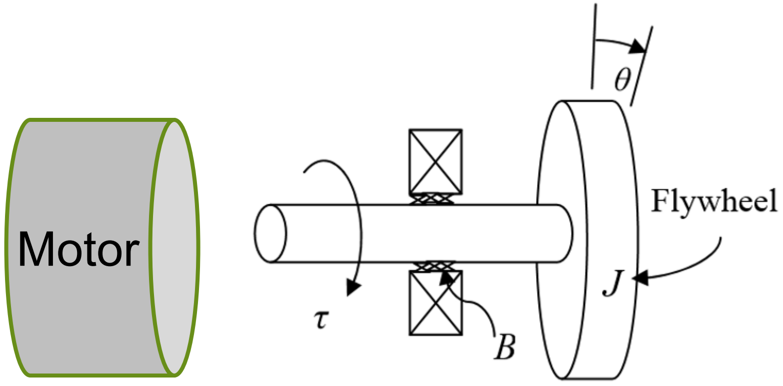
Solution:

- (a) $$J\frac{d}{dt^2}\theta^2 + B\frac{d}{dt}\theta = \tau$$

$$J\dot{\omega} + B\omega = \tau$$

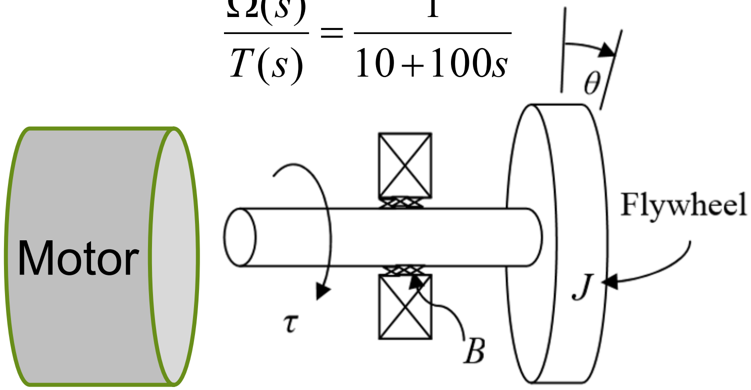
$$Js\Omega(s) + B\Omega(s) = T(s)$$

$$\frac{\Omega(s)}{T(s)} = \frac{1}{10+100s}$$



Solution (continued):

$$\frac{\Omega(s)}{T(s)} = \frac{1}{10+100s}$$



- (b)

$$\tau(t) = 100\ N.m$$

$$T(s) = \frac{100}{s}$$

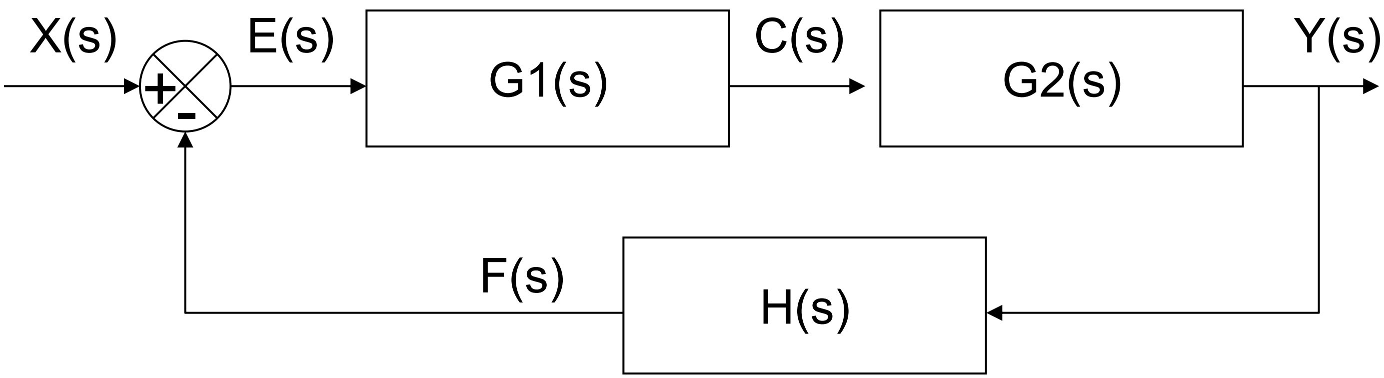
$$\Omega(s) = \frac{1}{10+100s} \bullet \frac{100}{s} = \frac{1}{s} \bullet \frac{1}{s+0.1} = \frac{10}{s} - \frac{10}{s+0.1}$$

$$\omega(t) = 10 - 10e^{-0.1t}$$

$$\lim_{t\to\infty} \omega(t) = \lim_{t\to\infty} 10 - 10e^{-0.1t} = 10\ \text{rad/sec}$$

Transfer Function of Systems with Error Control Loop ...

$$\frac{\text{Output(s)}}{\text{Input(s)}} = \frac{\text{Forward Branch (s)}}{1-\text{Closed Loop (s)}} = \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H(s)}$$



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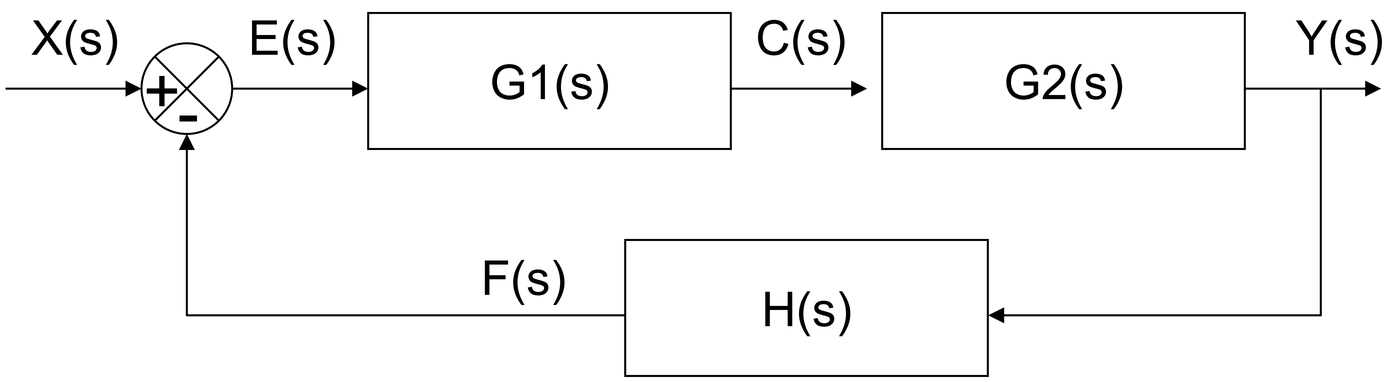
What is a signal flow diagram of a control system?

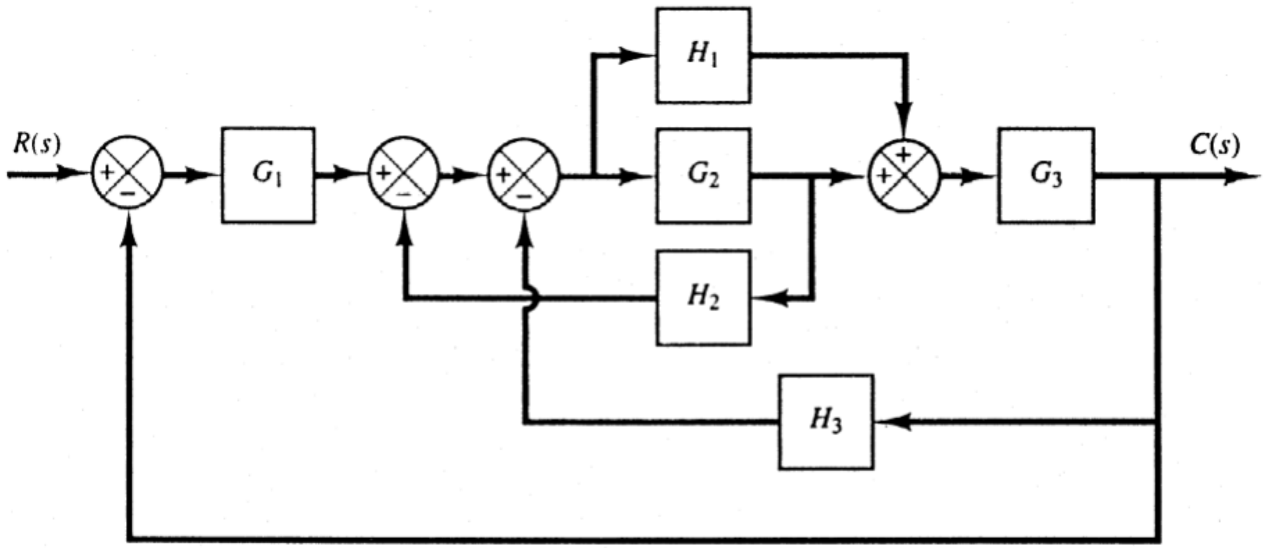
- A signal flow diagram (or Block Diagram) refers to a control system's a flow of signals in frequency domain or Laplace transforms, which are interconnected by transfer functions.



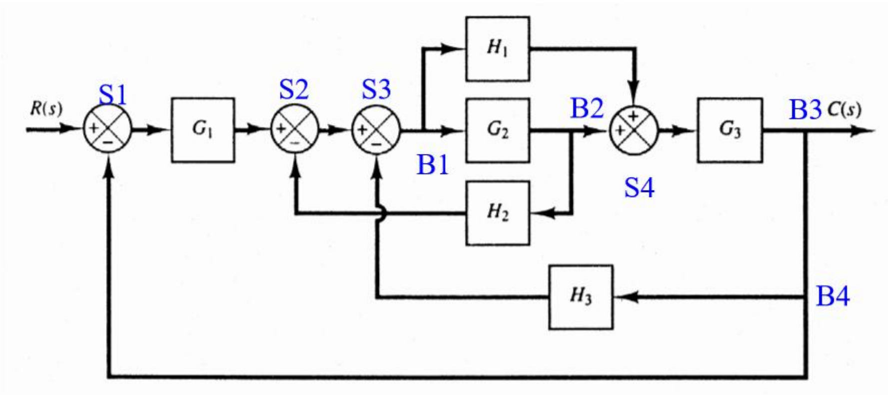
Example

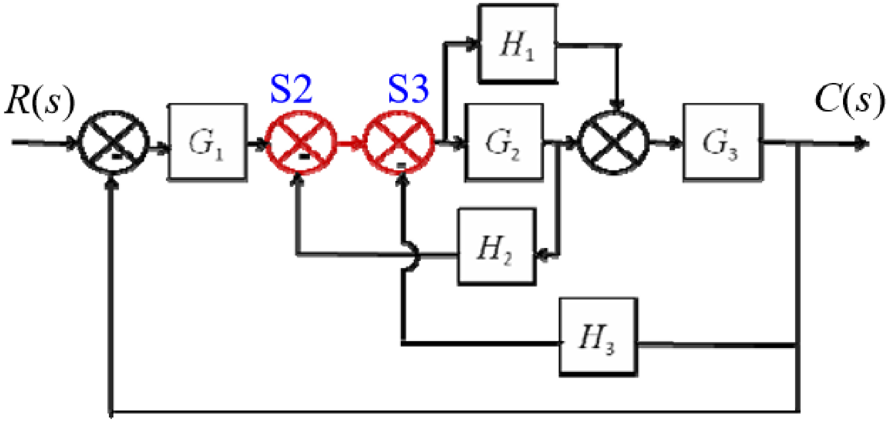
Signal Flow Diagram

Simplify the block diagram shown below and obtain the closed-loop transfer function $\dfrac{C(s)}{R(s)}$.

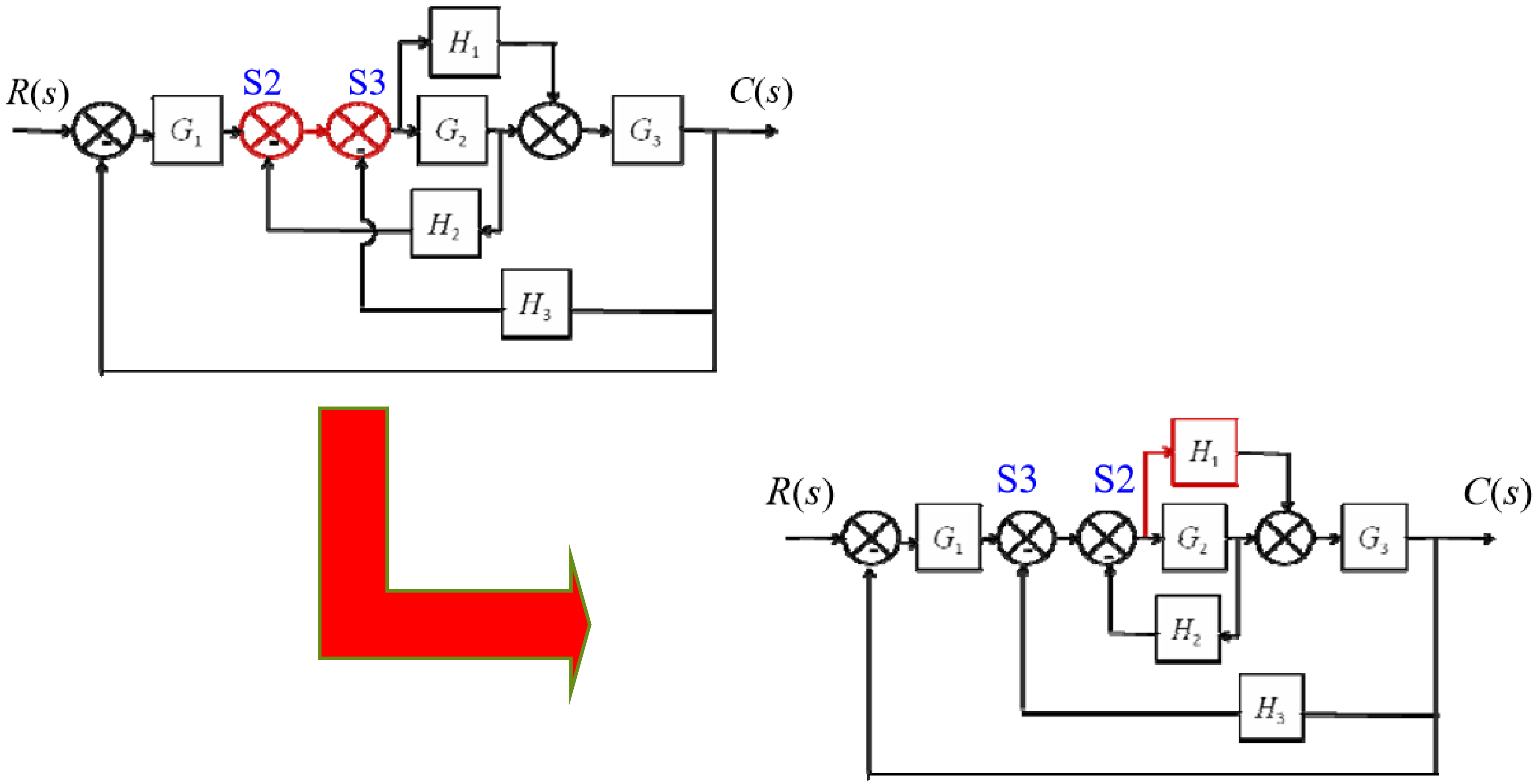


Solution:

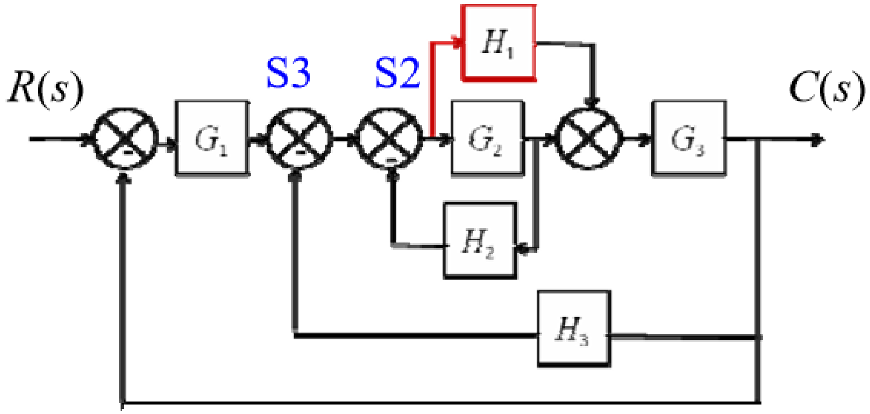

R(s)
S1
G1
S2
S3
H1
G2
B1
B2
S4
H2
G3
B3 C(s)
H3
B4

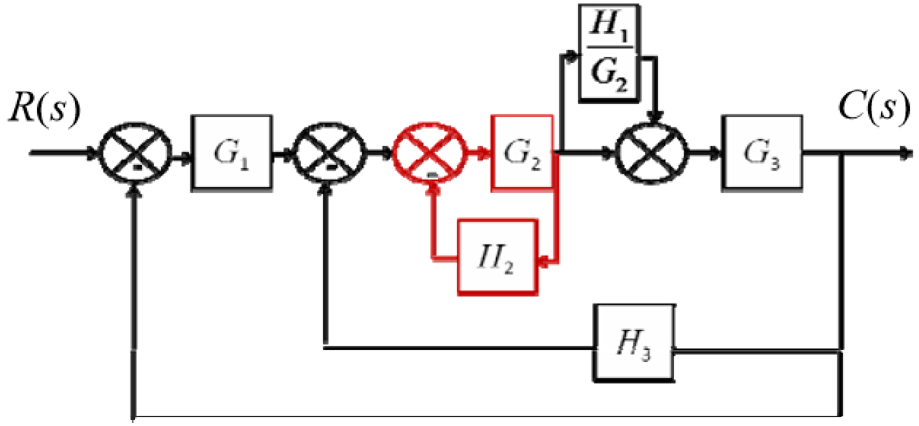

R(s)
G1
S2
S3
H1
G2
H2
G3
C(s)
H3

Solution (continued):

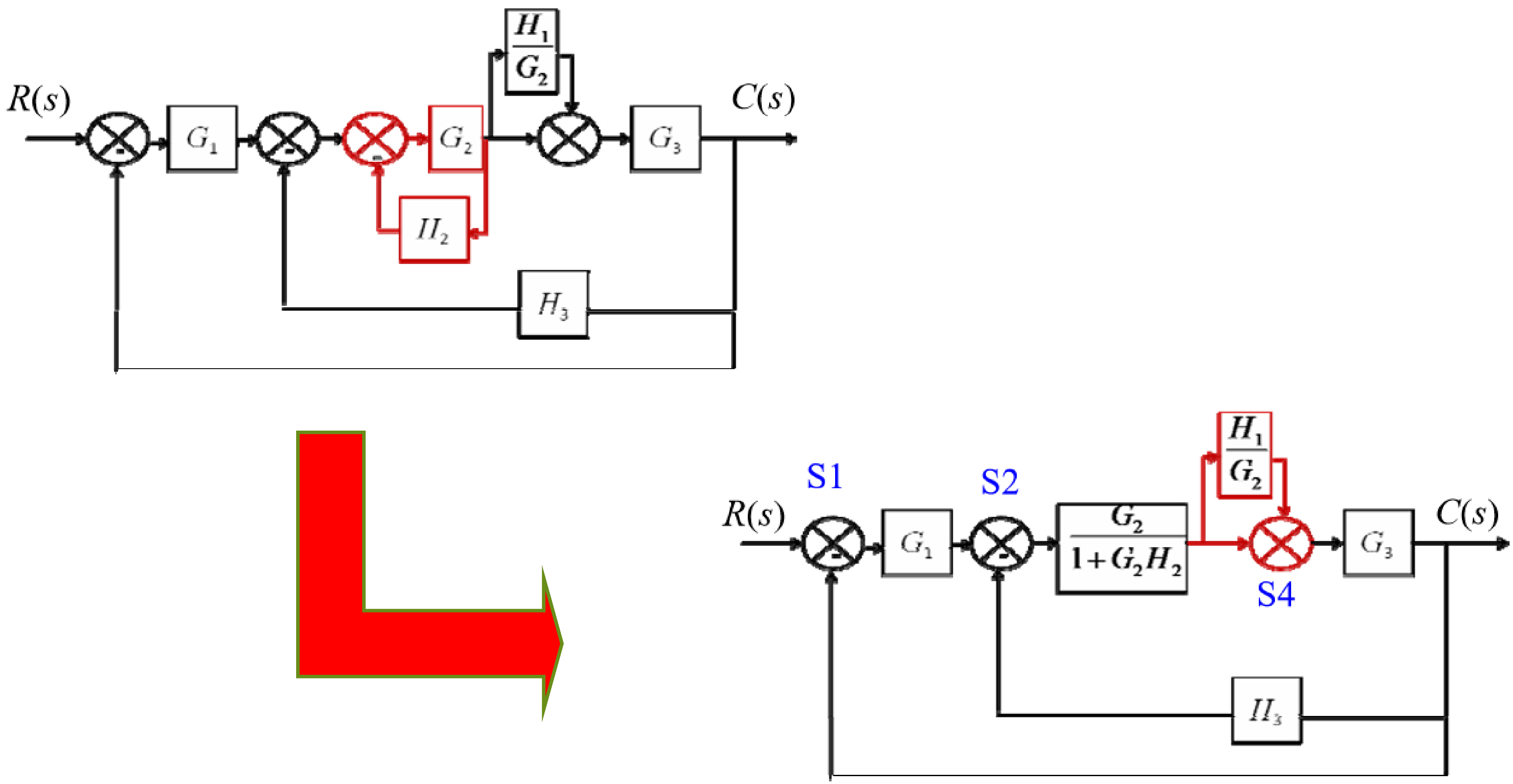


Solution (continued):

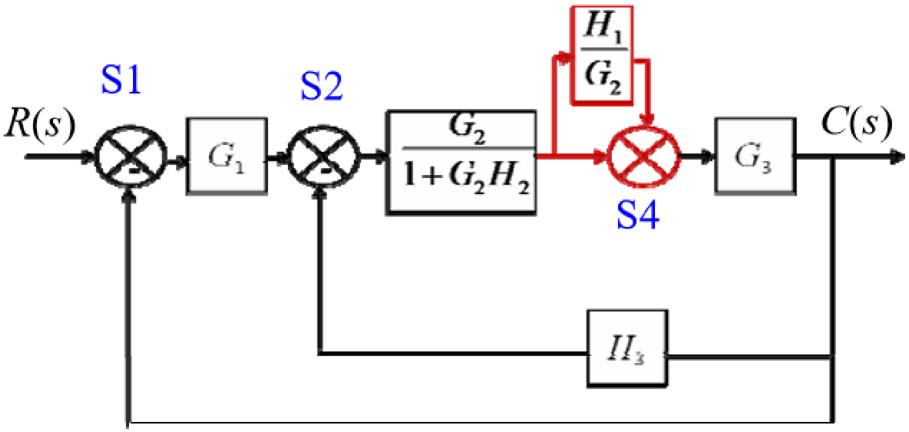


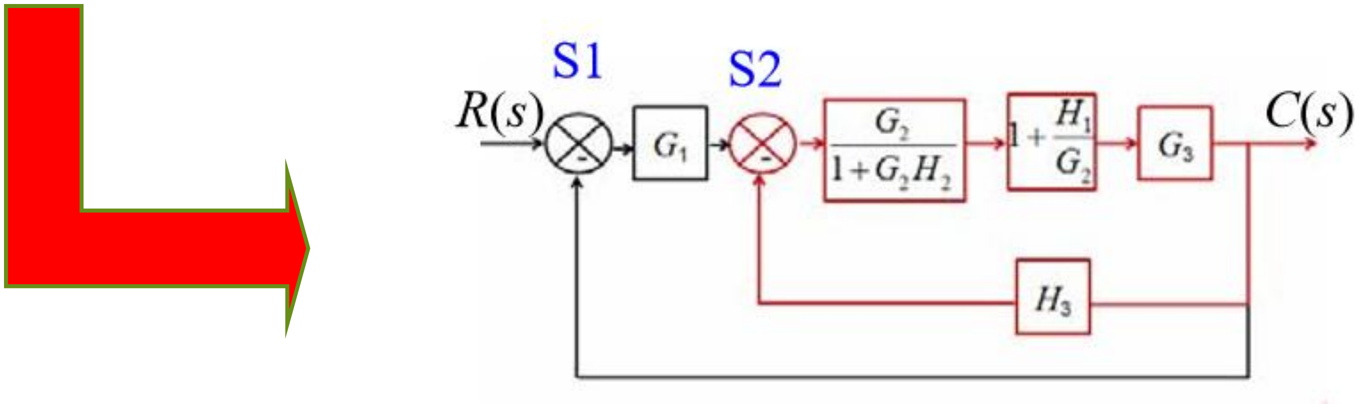


Solution (continued):

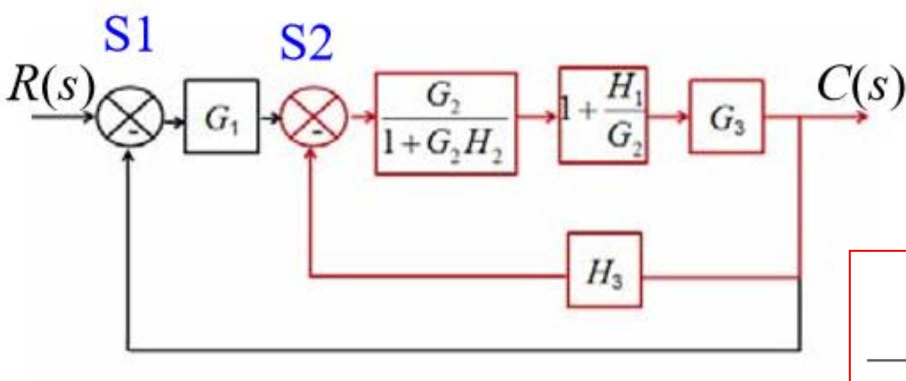


Solution (continued):

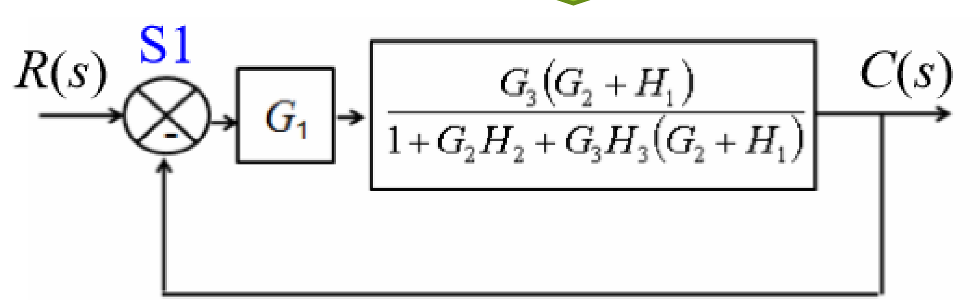




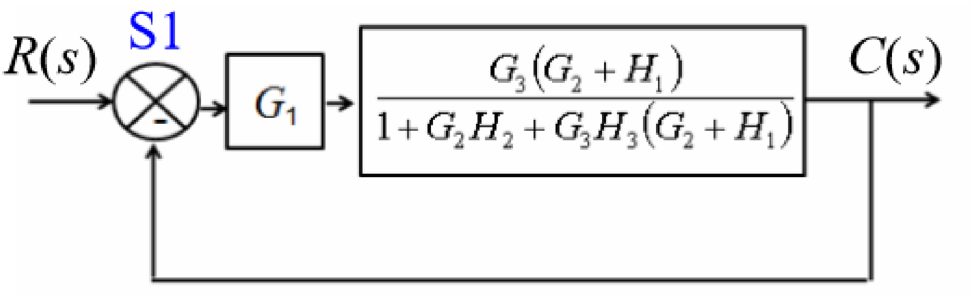
Solution (continued):



$$\frac{\frac{G_2}{1+G_2H_2}\left(1+\frac{H_1}{G_2}\right)G_3}{1+\frac{G_2}{1+G_2H_2}\left(1+\frac{H_1}{G_2}\right)G_3H_3} = \frac{G_3\left(G_2+H_1\right)}{1+G_2H_2+G_3H_3\left(G_2+H_1\right)}$$

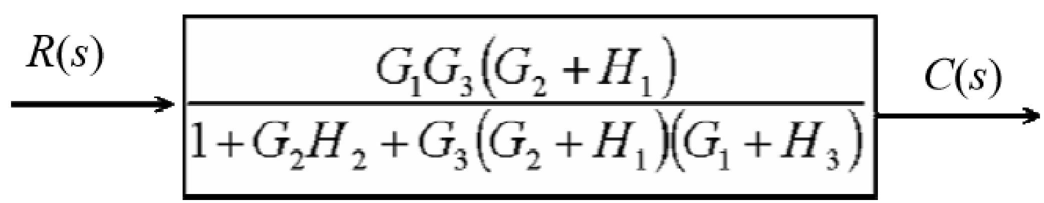


Solution (continued):



$$\frac{\dfrac{G_1G_3(G_2+H_1)}{1+G_2H_2+G_3H_3(G_2+H_1)}}{1+\dfrac{G_1G_3(G_2+H_1)}{1+G_2H_2+G_3H_3(G_2+H_1)}}=\frac{G_1G_3(G_2+H_1)}{1+G_2H_2+G_3H_3(G_2+H_1)+G_1G_3(G_2+H_1)}$$

$$=\frac{G_1G_3(G_2+H_1)}{1+G_2H_2+G_3(G_2+H_1)(G_1+H_3)}$$



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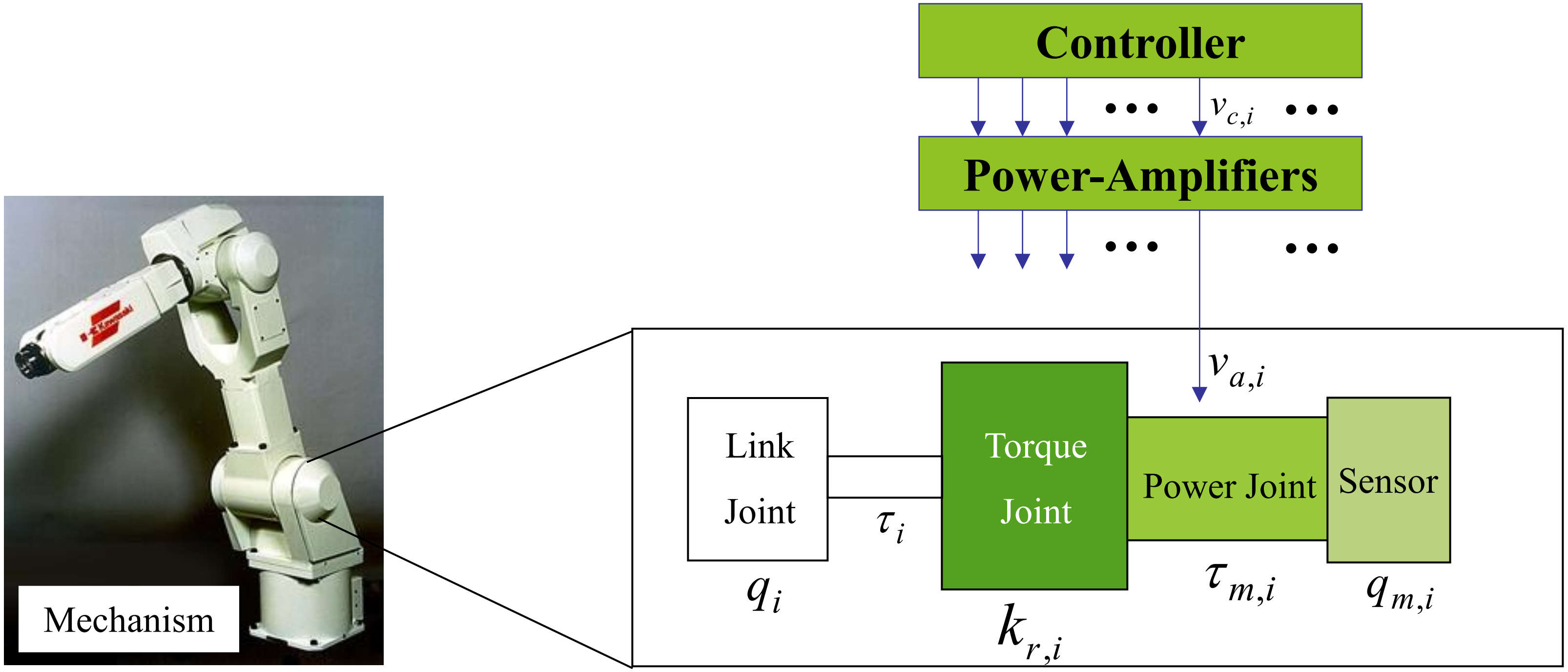
Robot's Control Systems: MIMO



Signal Flow Diagram of Robot Dynamics at Torque Joints …

- What is the signal flow diagram of the following equation?

$$\ddot{q}_m = K_r B_{diag}^{-1} K_r (\tau_m - d)$$

with :

$$d = K_r^{-1} \Delta B K_r^{-1} \ddot{q}_m + K_r^{-1} C K_r^{-1} \dot{q}_m + K_r^{-1} g(K_r^{-1} q_m)$$

Solution

- Step 1: Do Laplace transform

$$\ddot{q}_m = K_r B_{diag}^{-1} K_r (\tau_m - d)$$

with :

$$d = K_r^{-1} \Delta B K_r^{-1} \ddot{q}_m + K_r^{-1} C K_r^{-1} \dot{q}_m + K_r^{-1} g(K_r^{-1} q_m)$$

⇩

$$s^2 q_m(s) = K_r B_{diag}^{-1} K_r (\tau_m(s) - d(s))$$

with:

$$\mathrm{d(s)} = K_r^{-1} \Delta B K_r^{-1} s^2 q_m(s) + K_r^{-1} C K_r^{-1} s q_m + K_r^{-1} g(s) K_r^{-1} q_m(s)$$

$$s^2 q_m(s) = K_r B_{diag}^{-1} K_r(\tau_m(s) - d(s))$$

with

$$d(s) = K_r^{-1} \Delta B K_r^{-1} s^2 q_m(s) + K_r^{-1} C K_r^{-1} s q_m + K_r^{-1} g(s) K_r^{-1} q_m(s)$$

$$q_m(s) = \frac{1}{s^2} K_r B_{diag}^{-1} K_r(\tau_m(s) - d(s))$$

with

$$\mathrm{d}(s) = K_r^{-1} \Delta B K_r^{-1} s^2 q_m(s) + K_r^{-1} C K_r^{-1} s q_m + K_r^{-1} g(s) K_r^{-1} q_m(s)$$

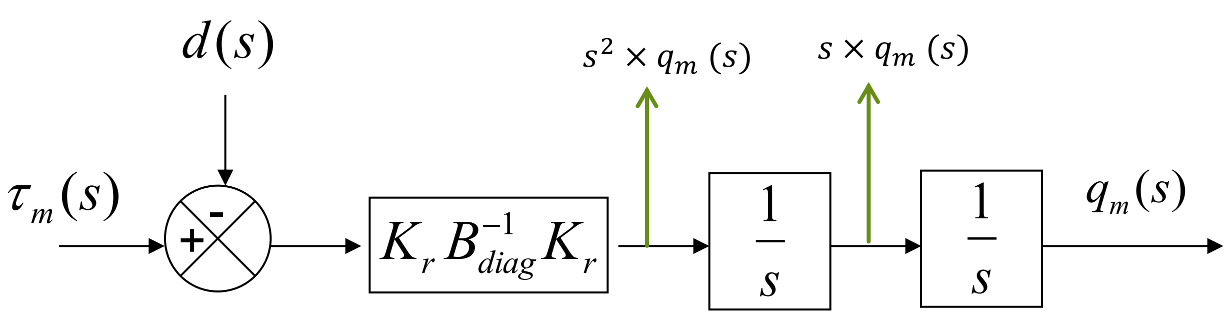
Solution (continued)

- Step 2: Draw signal diagram

$$q_m(s) = \frac{1}{s^2} K_r B_{diag}^{-1} K_r (\tau_m(s) - d(s))$$

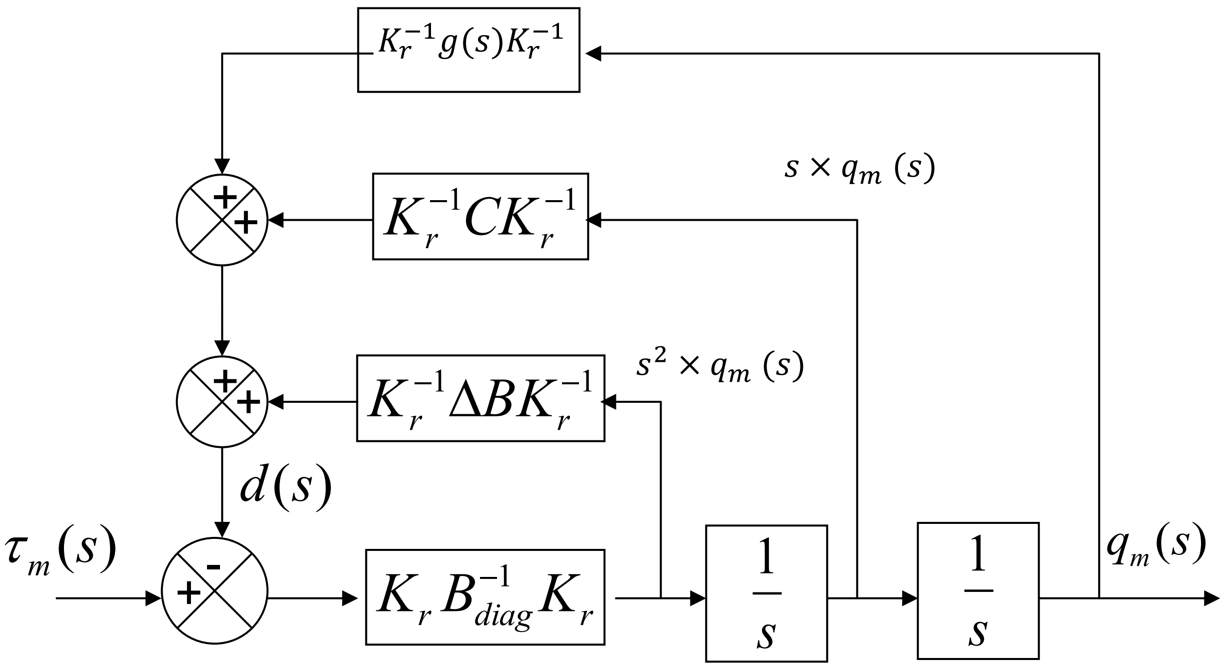
with

$$d(s) = K_r^{-1} \Delta B K_r^{-1} s^2 q_m(s) + K_r^{-1} C K_r^{-1} s q_m + K_r^{-1} g(s) K_r^{-1} q_m(s)$$

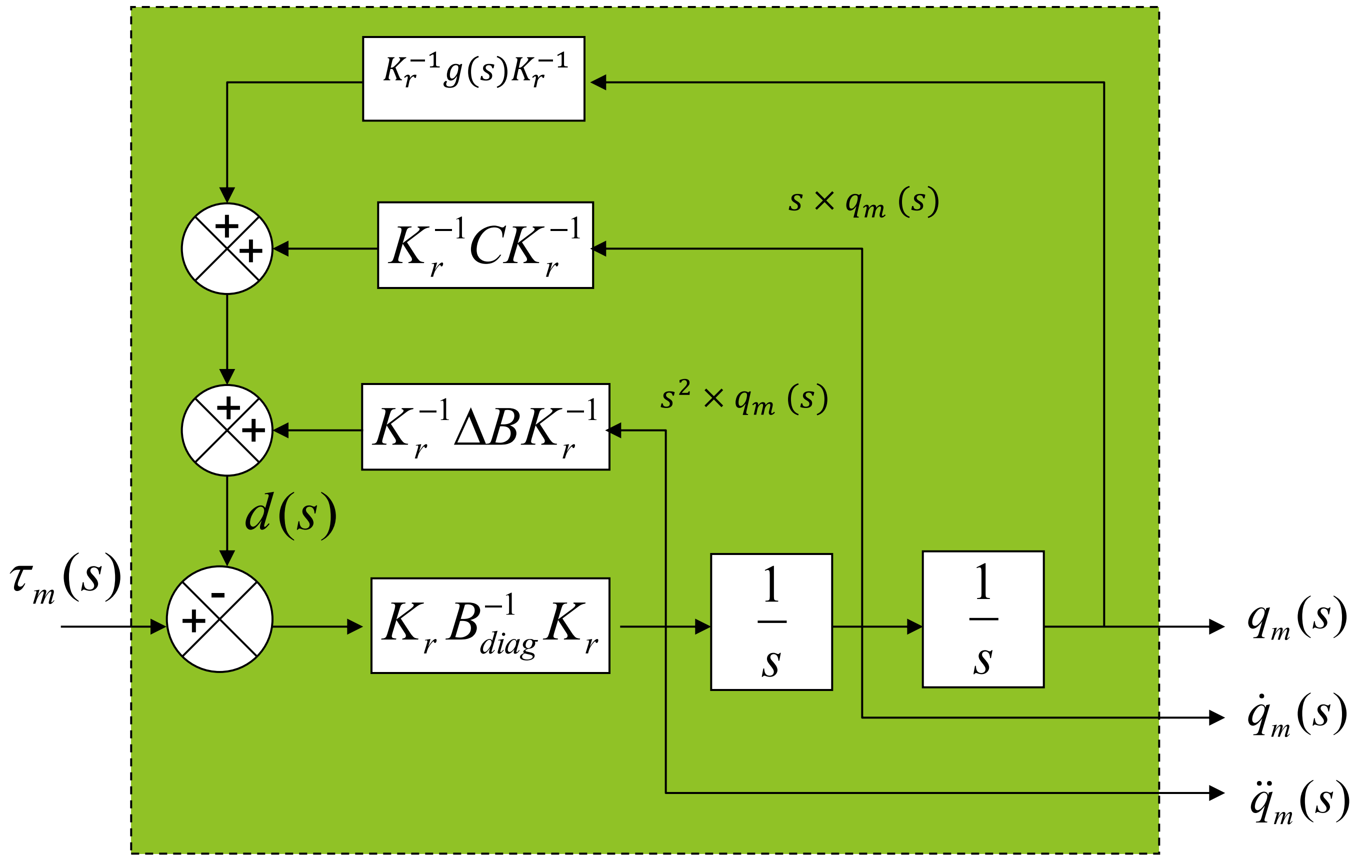


(to continue …)

$$d(s) = K_r^{-1}\Delta B K_r^{-1} s^2 q_m(s) + K_r^{-1} C K_r^{-1} s q_m + K_r^{-1} g(s) K_r^{-1} q_m(s)$$



(to continue …)



$K_r^{-1} g(s) K_r^{-1}$
$s \times q_m(s)$
$K_r^{-1} C K_r^{-1}$
$s^2 \times q_m(s)$
$K_r^{-1} \Delta B K_r^{-1}$
$d(s)$
$\tau_m(s)$
$K_r B_{diag}^{-1} K_r$
$\frac{1}{s}$
$\frac{1}{s}$
$q_m(s)$
$\dot{q}_m(s)$
$\ddot{q}_m(s)$

Signal Flow Diagram of Robot Dynamics at Controllers …

- What is the signal flow diagram of the following system of equations?

$$v_{c,i} = \frac{R_1}{R_f}\frac{R_{a,i}}{K_{t,i}}\tau_{m,i} + \frac{R_1}{R_f}\frac{L_{a,i}}{K_{t,i}}\frac{d\tau_{m,i}}{dt} + \frac{R_1}{R_f}k_{v,i}\dot{q}_{m,i}$$

$$i = 1,2,3,\ldots,n$$

Solution

- Step 1: Do Laplace transform

$$v_{c,i} = \frac{R_1}{R_f}\frac{R_{a,i}}{K_{t,i}}\tau_{m,i} + \frac{R_1}{R_f}\frac{L_{a,i}}{K_{t,i}}\frac{d\tau_{m,i}}{dt} + \frac{R_1}{R_f}k_{v,i}\dot{q}_{m,i}$$

$$i = 1,2,3,\ldots,n$$

⇩

$$v_{c,i}(s) = \frac{R_1}{R_f}\frac{R_{a,i}}{K_{t,i}}\tau_{m,i}(s) + \frac{R_1}{R_f}\frac{L_{a,i}}{K_{t,i}}s\tau_{m,i}(s) + \frac{R_1}{R_f}k_{v,i}sq_{m,i}(s)$$

$$i = 1,2,3,\ldots,n$$

$$v_{c,i}(s) = \frac{R_1}{R_f}\frac{R_{a,i}}{K_{t,i}}\tau_{m,i}(s) + \frac{R_1}{R_f}\frac{L_{a,i}}{K_{t,i}}s\tau_{m,i}(s) + \frac{R_1}{R_f}k_{v,i}sq_{m,i}(s)$$

$$i = 1,2,3,\ldots,n$$

⇓

$$v_{c,i}(s) = (\frac{R_1 R_{a,i}}{R_f K_{t,i}} + \frac{R_1 L_{a,i}}{R_f K_{t,i}}s)\tau_{m,i}(s) + \frac{R_1 k_{v,i}}{R_f}\dot{q}_{m,i}(s)$$

$$i = 1,2,3,\ldots,n$$

(to continue …)

$$v_{c,i}(s) = (\frac{R_1 R_{a,i}}{R_f K_{t,i}} + \frac{R_1 L_{a,i}}{R_f K_{t,i}} s)\tau_{m,i}(s) + \frac{R_1 k_{v,i}}{R_f} \dot{q}_{m,i}(s)$$

$$i = 1,2,3,\ldots,n$$

$$\Downarrow$$

$$\{v_{c,i}(s) - \frac{R_1 k_{v,i}}{R_f} \dot{q}_{m,i}(s)\} = \frac{R_1 R_{a,i}}{R_f K_{t,i}} (1 + \frac{L_{a,i}}{R_{a,i}} s)\tau_{m,i}(s)$$

$$i = 1,2,3,\ldots,n$$

(to continue …)

Define : $K_{e,i} = \dfrac{R_1 k_{v,i}}{R_f}$

Define : $\dfrac{1}{K_{m,i}} = \dfrac{R_1 R_{a,i}}{R_f K_{t,i}}$

Define : $T_{m,i} = \dfrac{L_{a,i}}{R_{a,i}}$

$$\{v_{c,i}(s) - \frac{R_1 k_{v,i}}{R_f}\dot{q}_{m,i}(s)\} = \frac{R_1 R_{a,i}}{R_f K_{t,i}}(1 + \frac{L_{a,i}}{R_{a,i}}s)\tau_{m,i}(s)$$

$$i = 1,2,3,\ldots,n$$

⇓

$$\{v_{c,i}(s) - K_{e,i}\dot{q}_{m,i}(s)\} = \frac{1}{K_{m,i}}(1 + T_{m,i}s)\tau_{m,i}(s)$$

$$i = 1,2,3,\ldots,n$$

(to continue …)

$$\{v_{c,i}(s) - K_{e,i}\dot{q}_{m,i}(s)\} = \frac{1}{K_{m,i}}(1 + T_{m,i}s)\tau_{m,i}(s)$$

$$i = 1,2,3,\ldots,n$$

⇓

$$\text{Define}: \frac{1}{G_{m,i}(s)} = \frac{1}{K_{m,i}}(1 + T_{m,i}s)$$

$$\tau_{m,i}(s) = G_{m,i}(s)\{v_{c,i}(s) - K_{e,i}\dot{q}_{m,i}(s)\}$$

$$i = 1,2,3,\ldots,n$$

(to continue …)

$$\text{Define}: \ \tau_m(s) = \begin{pmatrix} \tau_{m,1}(s) \\ \tau_{m,2}(s) \\ \ldots \\ \tau_{m,n}(s) \end{pmatrix}$$

$$\text{Define}: \ G_m(s) = \begin{pmatrix} G_{m,1}(s) & 0 & \ldots & 0 \\ 0 & G_{m,2}(s) & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & G_{m,n}(s) \end{pmatrix}$$

$$\tau_{m,i}(s) = G_{m,i}(s)\{v_{c,i}(s) - K_{e,i}\dot{q}_{m,i}(s)\}$$

$$i = 1,2,3,\ldots,n$$

(to continue …)

$$\text{Define}: \; K_e = \begin{pmatrix} K_{e,1} & 0 & \dots & 0 \\ 0 & K_{e,2} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & K_{e,n} \end{pmatrix}$$

$$\tau_{m,i}(s) = G_{m,i}(s)\{v_{c,i}(s) - K_{e,i}\dot{q}_{m,i}(s)\}$$

$$i = 1,2,3,\dots,n$$

$$\text{Define}: \; \dot{q}_m(s) = \begin{pmatrix} \dot{q}_{m,1}(s) \\ \dot{q}_{m,2}(s) \\ \dots \\ \dot{q}_{m,n}(s) \end{pmatrix}$$

$$\text{Define}: \; v_c(s) = \begin{pmatrix} v_{c,1}(s) \\ v_{c,2}(s) \\ \dots \\ v_{c,n}(s) \end{pmatrix}$$

(to continue …)

$$\tau_{m,i}(s) = G_{m,i}(s)\{v_{c,i}(s) - K_{e,i}\dot{q}_{m,i}(s)\}$$

$$i = 1,2,3,\ldots,n$$

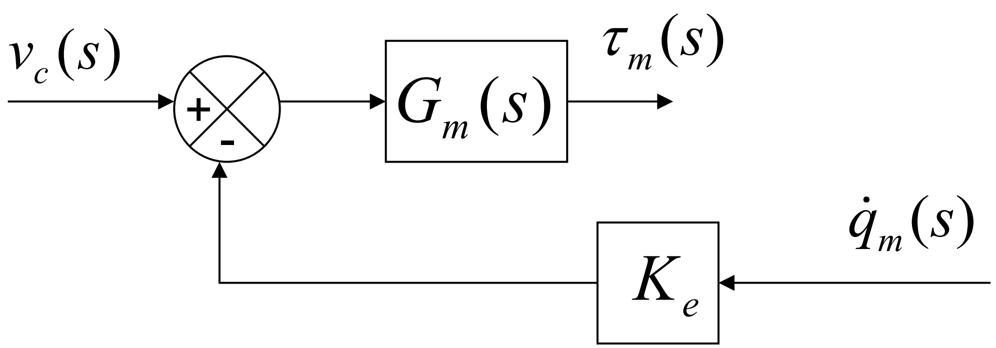
$$\Downarrow$$

$$\tau_m(s) = G_m(s)\{v_c(s) - K_e\dot{q}_m(s)\}$$

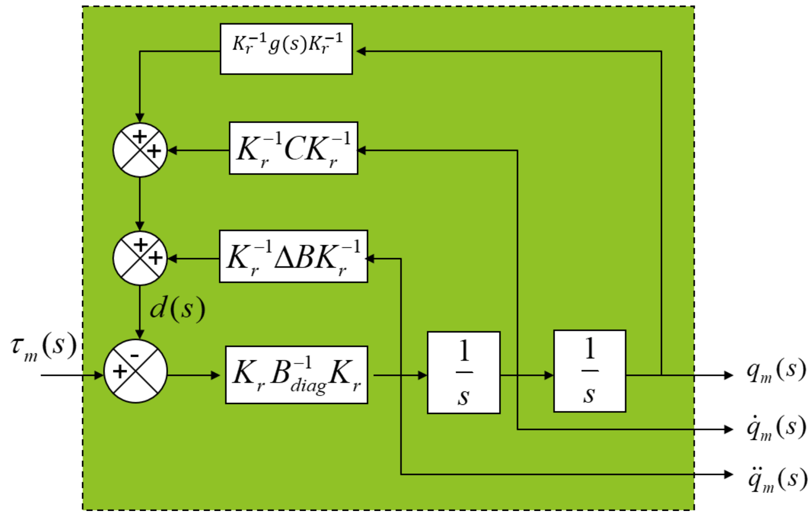
Solution (continued)

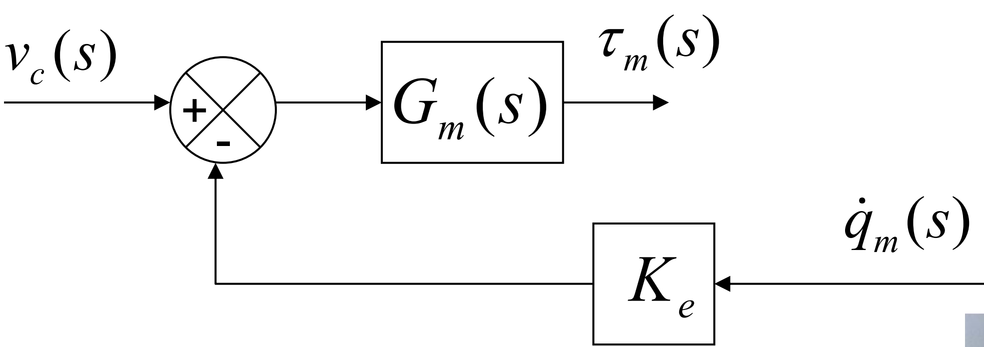
- Step 2: Draw signal diagram

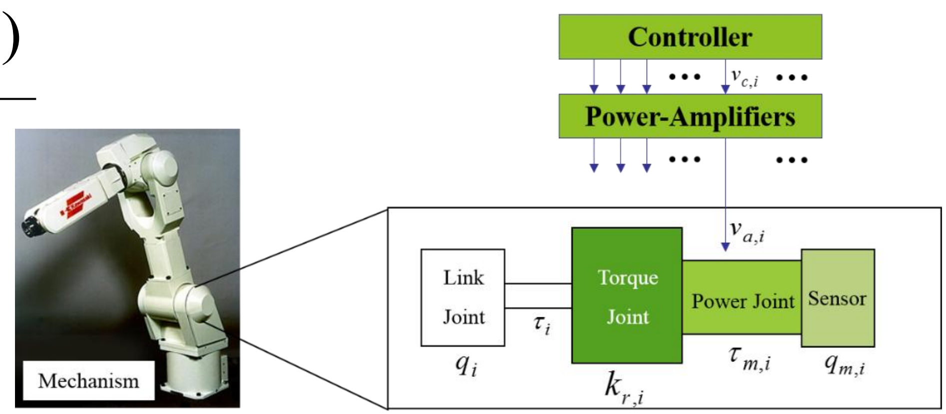
$$\tau_m(s) = G_m(s)\{v_c(s) - K_e \dot{q}_m(s)\}$$

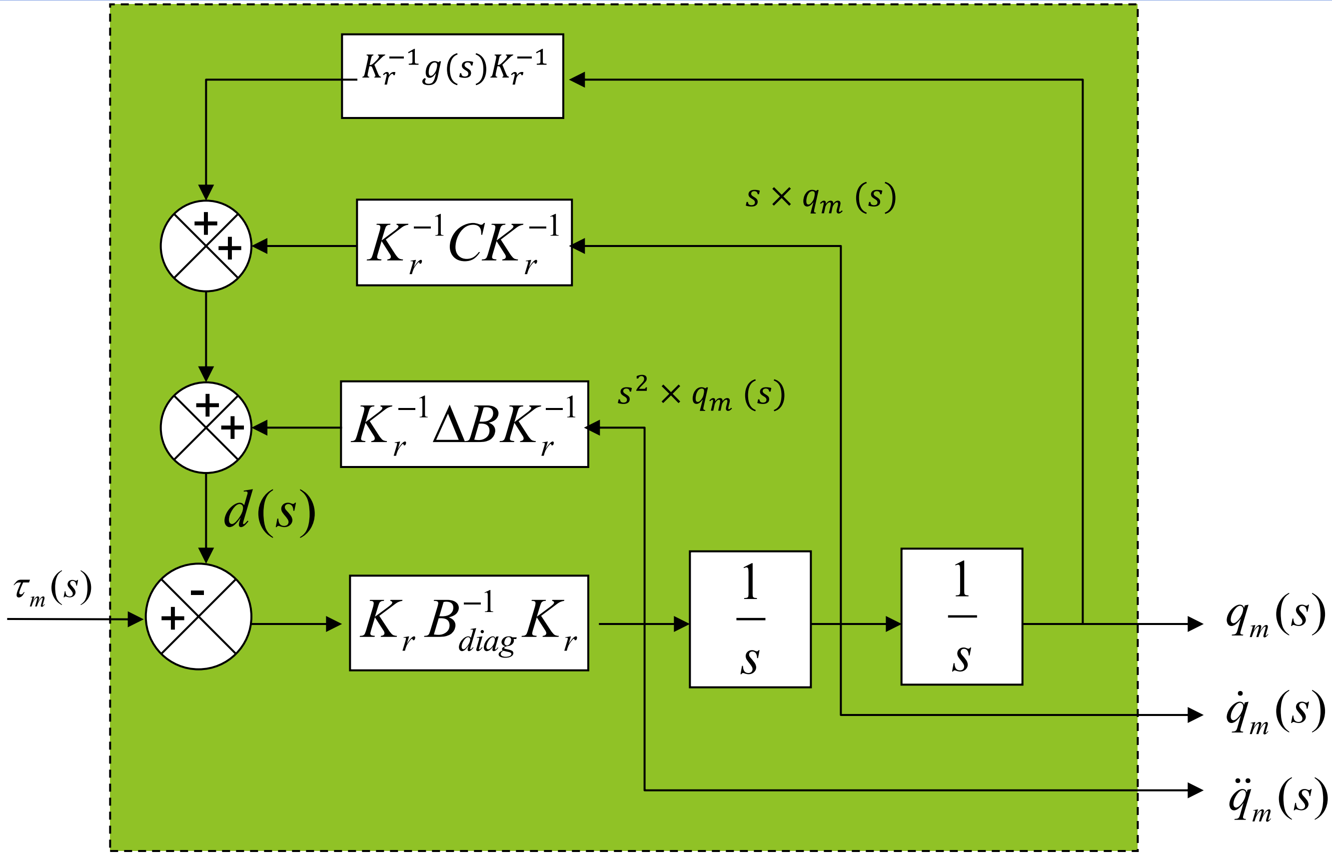


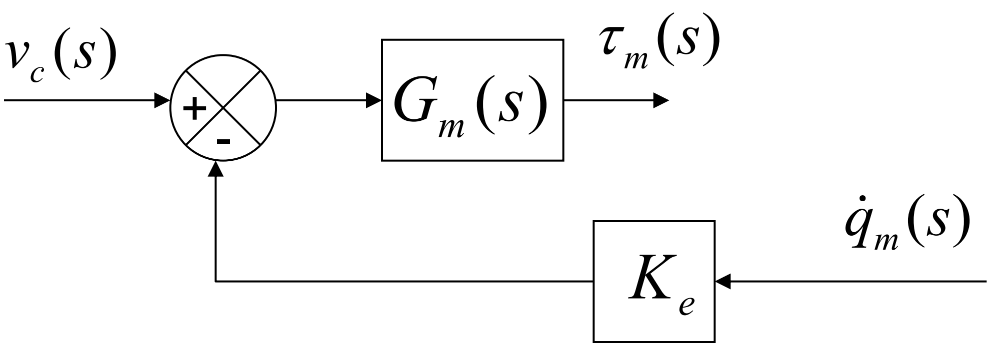
Signal Diagram of Robot's Dynamics at Controllers …





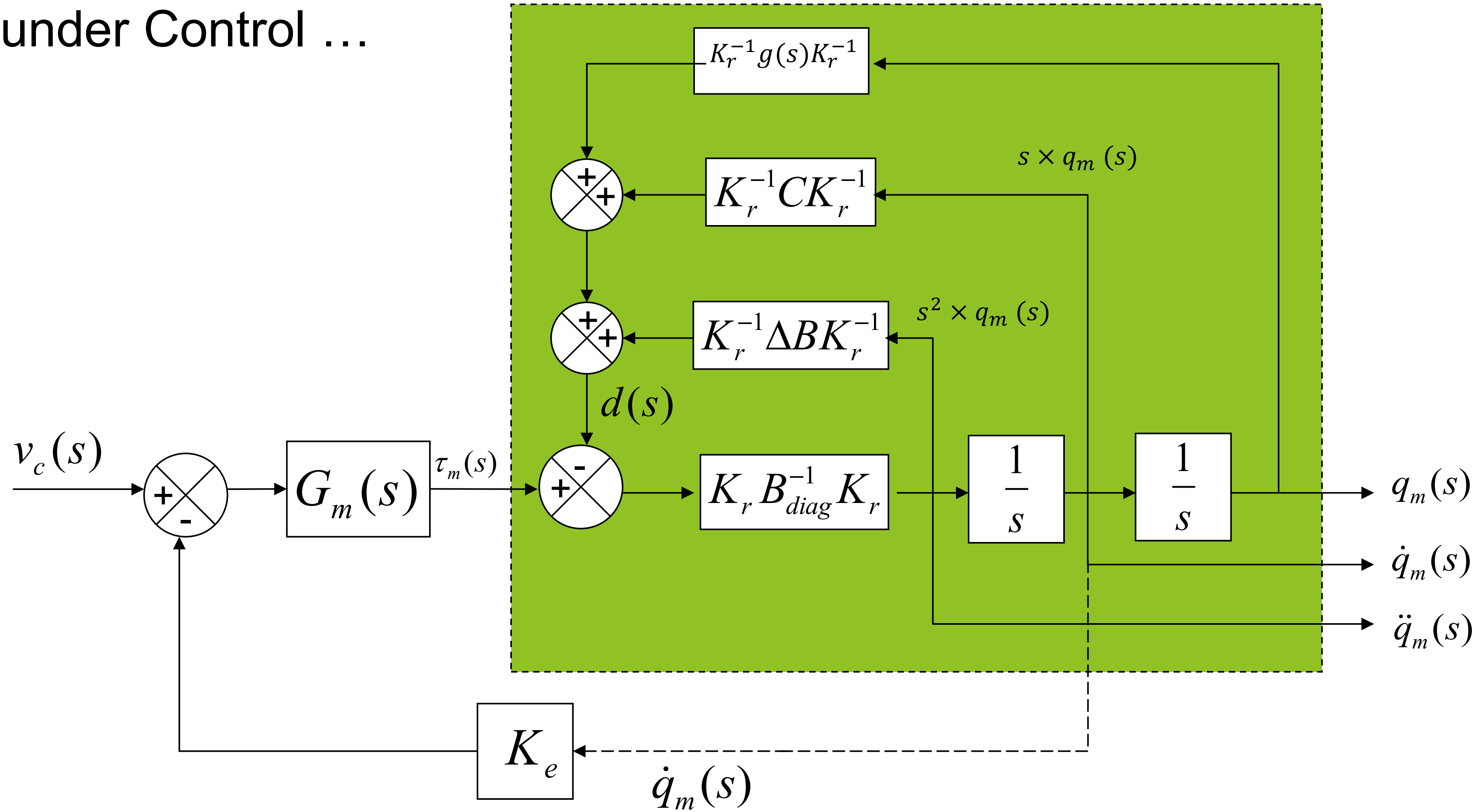



$K_r^{-1} g(s) K_r^{-1}$
$s \times q_m(s)$
$K_r^{-1} C K_r^{-1}$
$s^2 \times q_m(s)$
$K_r^{-1} \Delta B K_r^{-1}$
$d(s)$
$\tau_m(s)$
$K_r B_{diag}^{-1} K_r$
$\frac{1}{s}$
$\frac{1}{s}$
$q_m(s)$
$\dot{q}_m(s)$
$\ddot{q}_m(s)$


$v_c(s)$
$G_m(s)$
$\tau_m(s)$
$\dot{q}_m(s)$
K_e

(to continue …)

This is the Signal Flow Diagram of Robot's Dynamics under Control …



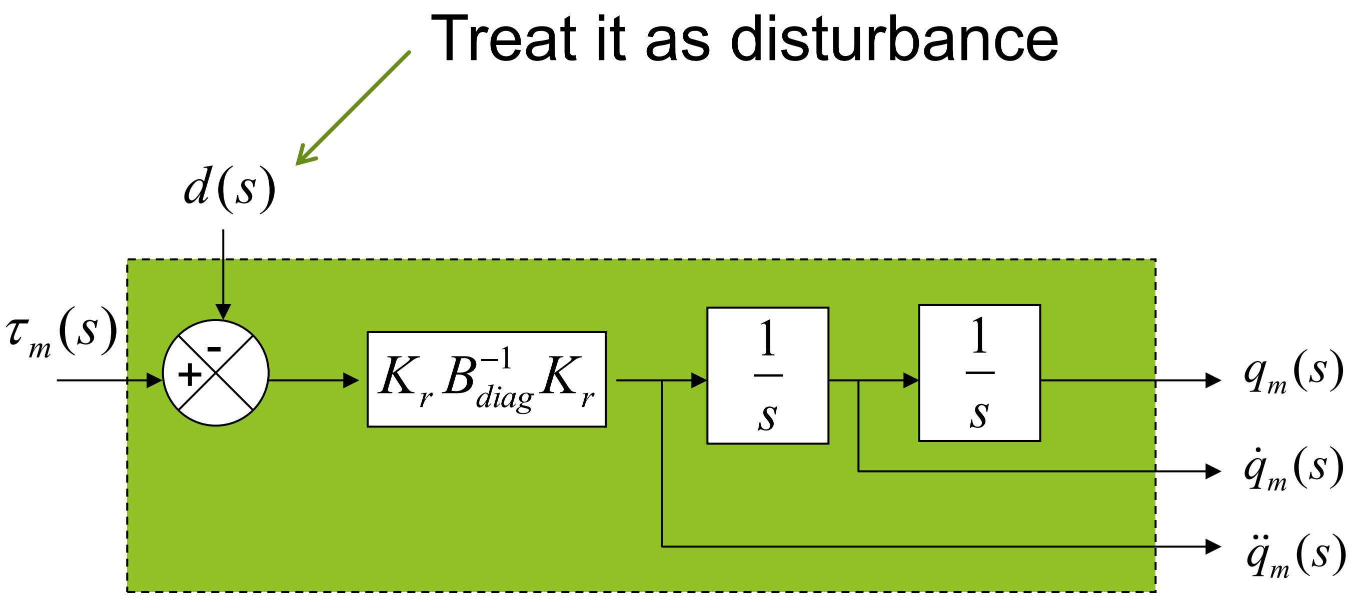
How to make it to become LTI systems?

A Background Knowledge: Multiplication of Diagonal Matrices Also Results In Diagonal Matrix.

$$K_r = \begin{bmatrix} k_{r,1} & 0 & \dots & 0 \\ 0 & k_{r,2} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & k_{r,n} \end{bmatrix} \quad B_{diag}^{-1} = \begin{bmatrix} \frac{1}{b_{11}} & 0 & \dots & 0 \\ 0 & \frac{1}{b_{22}} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \frac{1}{b_{nn}} \end{bmatrix} \quad K_r = \begin{bmatrix} k_{r,1} & 0 & \dots & 0 \\ 0 & k_{r,2} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & k_{r,n} \end{bmatrix}$$

$$K_r B_{diag}^{-1} K_r = \begin{bmatrix} \frac{k_{r,1}^2}{b_{11}} & 0 & \dots & 0 \\ 0 & \frac{k_{r,2}^2}{b_{22}} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \frac{k_{r,n}^2}{b_{nn}} \end{bmatrix}$$

Necessary and Sufficient Condition of Making a Robot to Become an Ideal Dynamic System: The Reduction Ratios Are Big Enough!



An Important Property of Ideal Dynamic System:

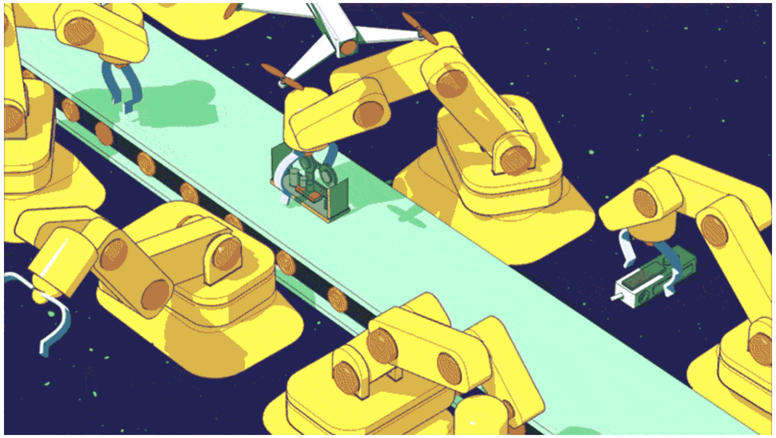
MIMO = Sum of SISOs

Summary of Lecture 2

- Laplace Transforms
- Transfer Functions
- Signal Flow Diagram
- Signal Flow Diagram of Robot's Dynamics

Outline of Module 4

- Dynamics under Control
- Signal Flow Diagram
- Design of Control Systems
- Control in Joint-Space
- Control in Task-Space



Module 4

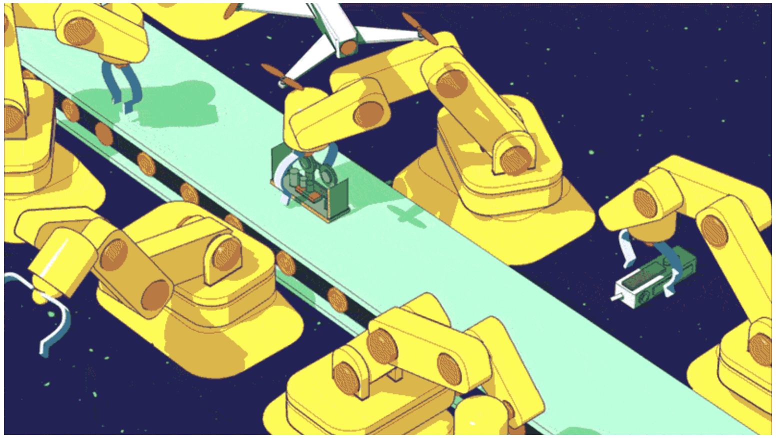
MA4825 Robotics

Lecture 3

Design of Control Systems

Xie Ming, PhD (France)
http://personal.ntu.edu.sg/mmxie

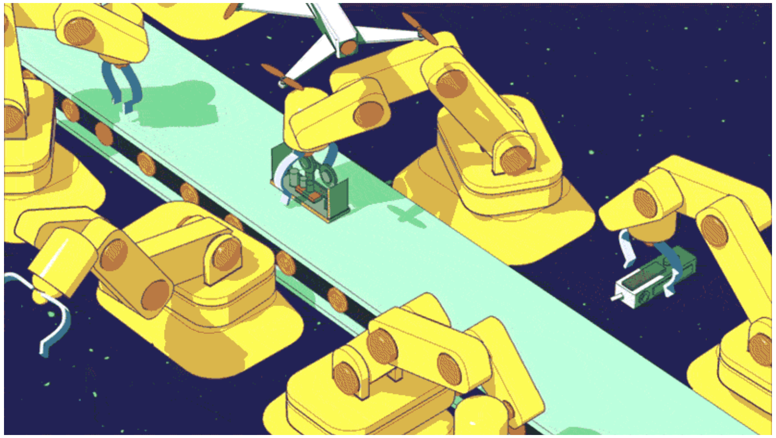
Outline of Lecture 3



- Basics of Systems
- Basics of Control Systems
- Design Solutions

Outline of Lecture 3

- Basics of Systems
- Basics of Control Systems
- Design Solutions



What is a system?

- A system consists of a set of elements or modules, which act and interact together for the purpose of achieving some common goals.

What is the best system in the world?

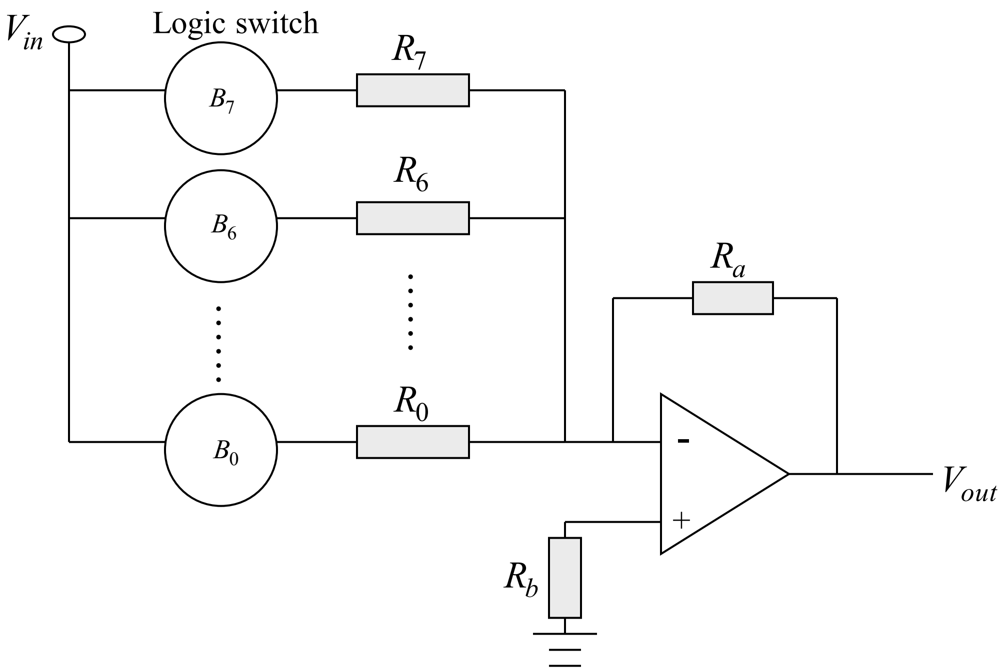
- Answer:
  - Static Systems!

- Any system, in which the input and output relationship is independent of time, is a static system.

- No transient responses
- Steady-state responses

Example of Static Systems

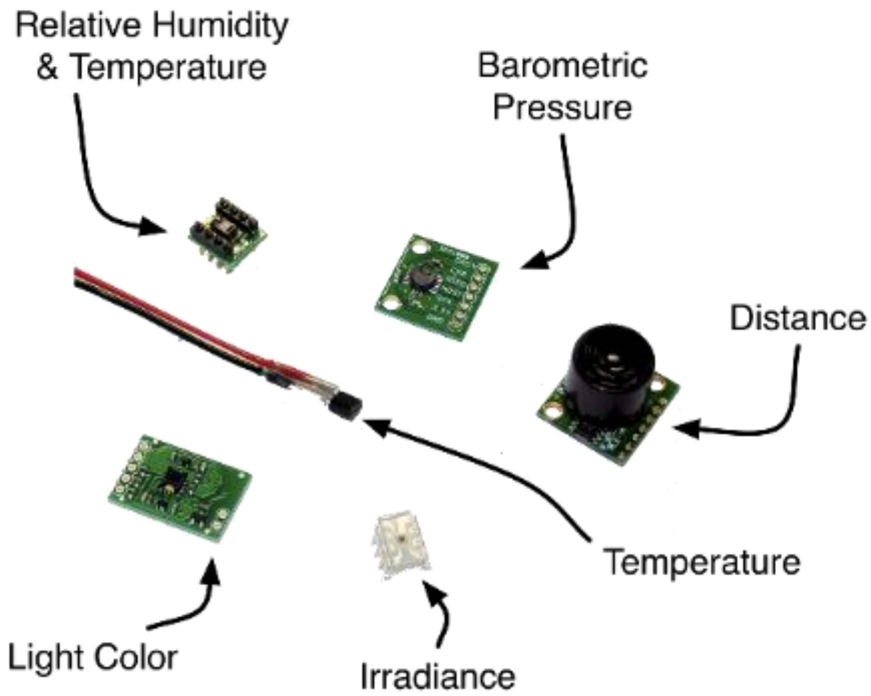
- Digital to Analogue Conversion (DAC) Systems



$$V_{out} = B \times V_{in} + c$$

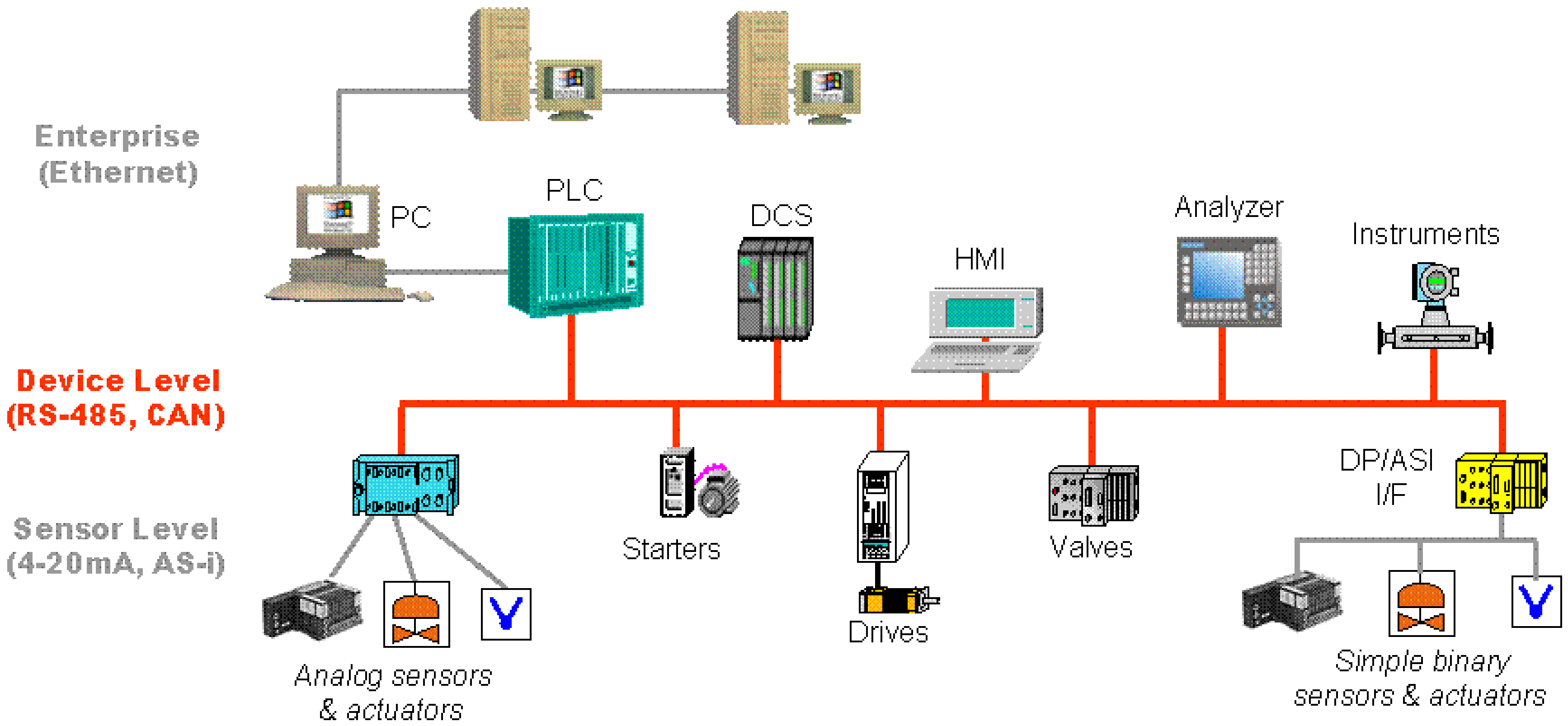
Example of Static Systems

- Sensing Systems



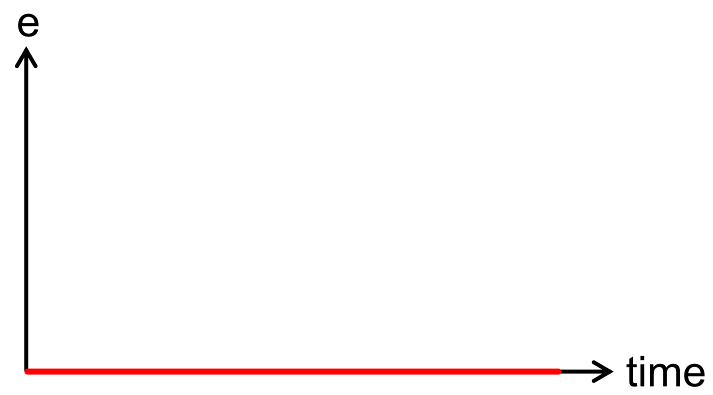
Example of Static Systems

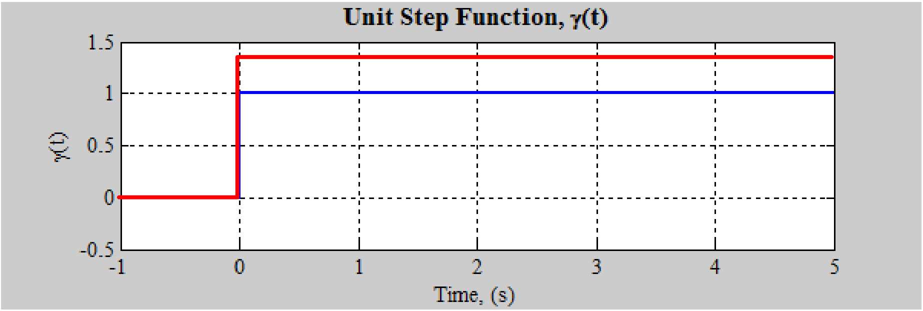
- Communication Systems



Property of Static Systems

- A static system only has steady-state responses and steady-state errors.





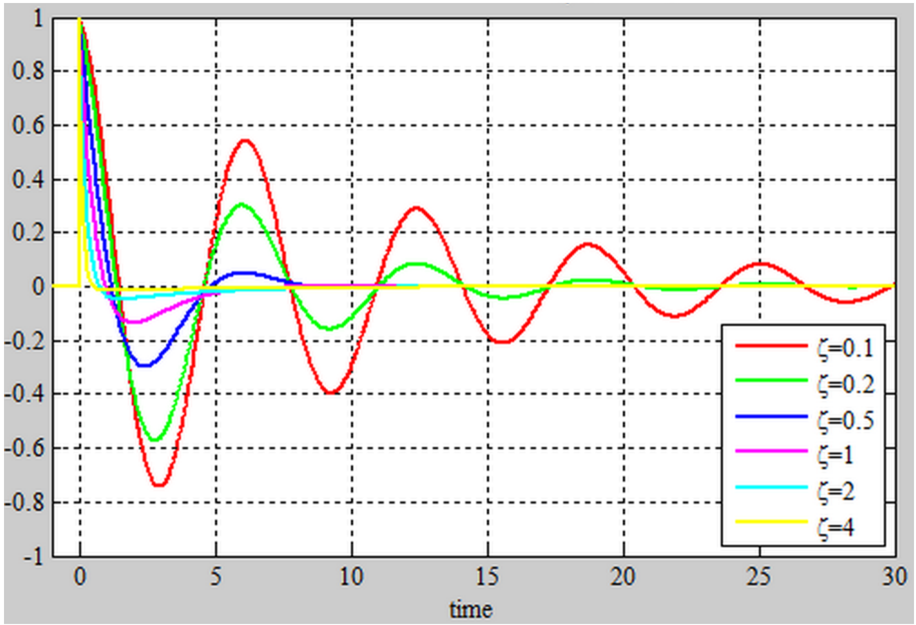
What are dynamic systems?

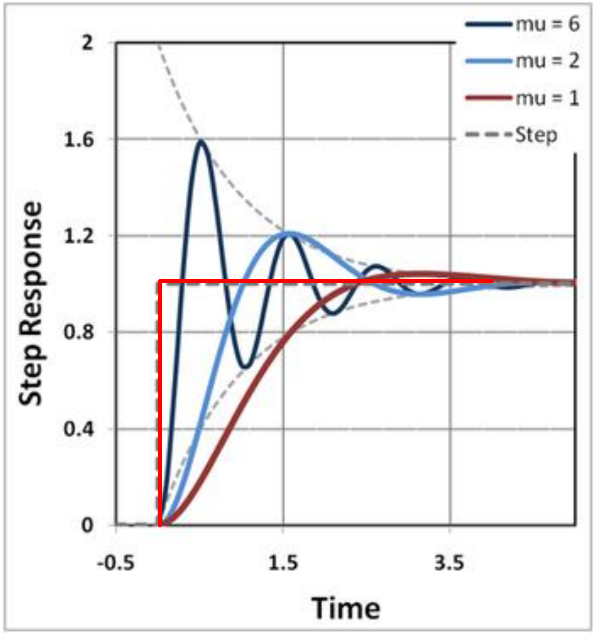
- Any system, in which the input and output relationship is not independent of time, is a dynamic system.

- Transient responses
- Steady-state responses

Property of Dynamic Systems

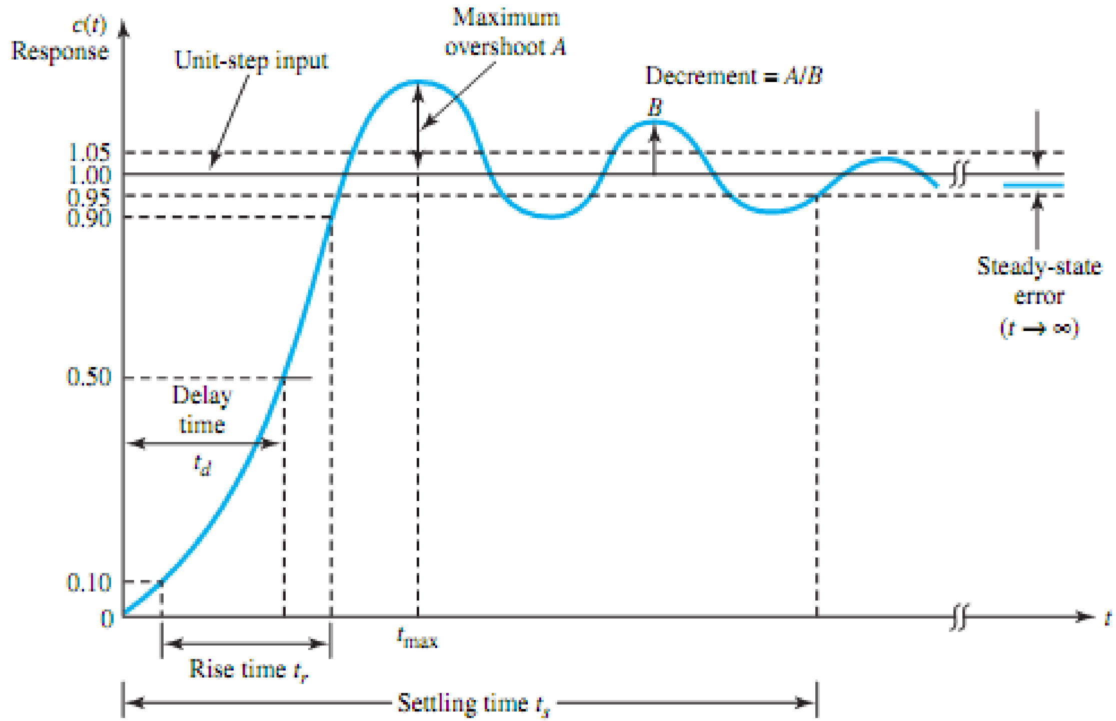
- A dynamic system has both transient and steady-state responses.





How to judge the performance of a dynamic system?

- Three performance indicators:
  - Stability
  - Response time
  - Response accuracy



Typical unit-step response of a control system.

How to make a dynamic system to approach the best system?

Static Systems

- Stability
  - 100%
- Response Time
  - 0 s
- Response Accuracy
  - 100%

Dynamic Systems

- Stability
  - 100% (possible)
- Response Time
  - 0 s (not possible)
- Response Accuracy
  - 100% (possible)

What are the ideal dynamic systems in the world?

- Answer:
  - Linear time-invariant (LTI) systems!

- What are the properties of linear time-invariant (LTI) systems?

Property 1 of Ideal Dynamic Systems

- Time Invariance

Relationship is independent of date and time

Property 2 of Ideal Dynamic Systems

- Homogeneity

Property 3 of Ideal Dynamic Systems

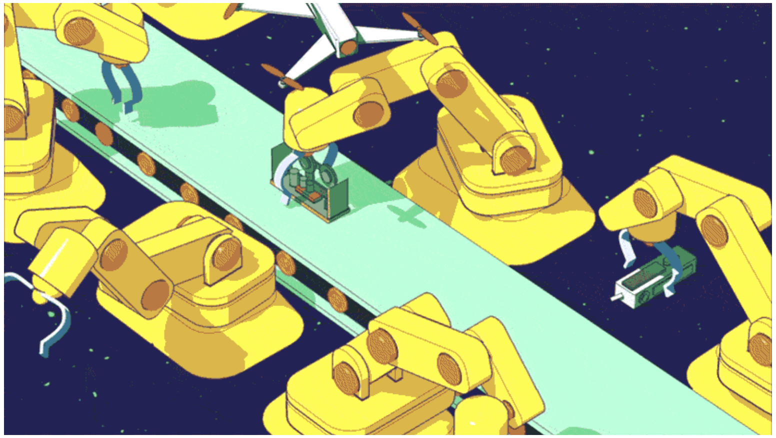
- Superposition

$$x_1(t) + x_2(t) \qquad\qquad y_1(t) + y_2(t)$$

MIMO = Sum of SISOs

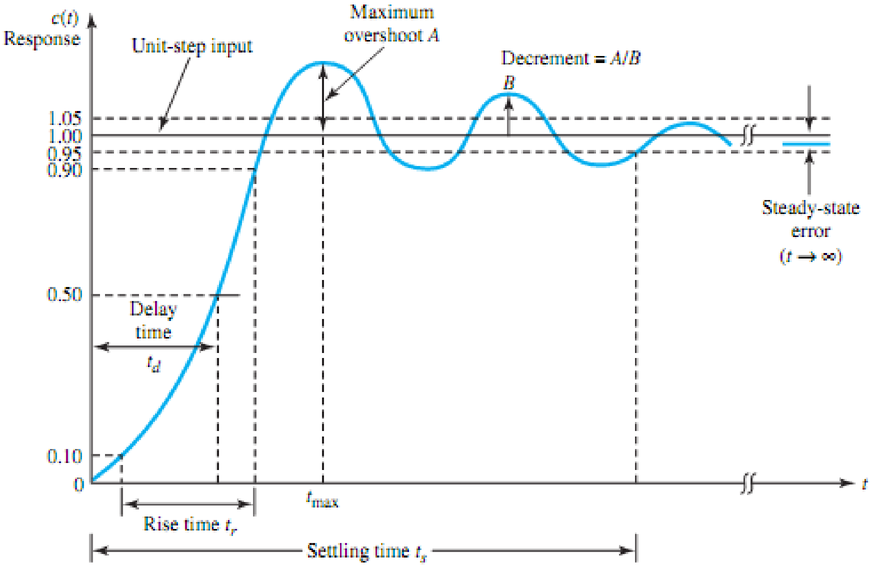
Outline of Lecture 3

- Basics of Systems
- Basics of Control Systems
- Design Solutions



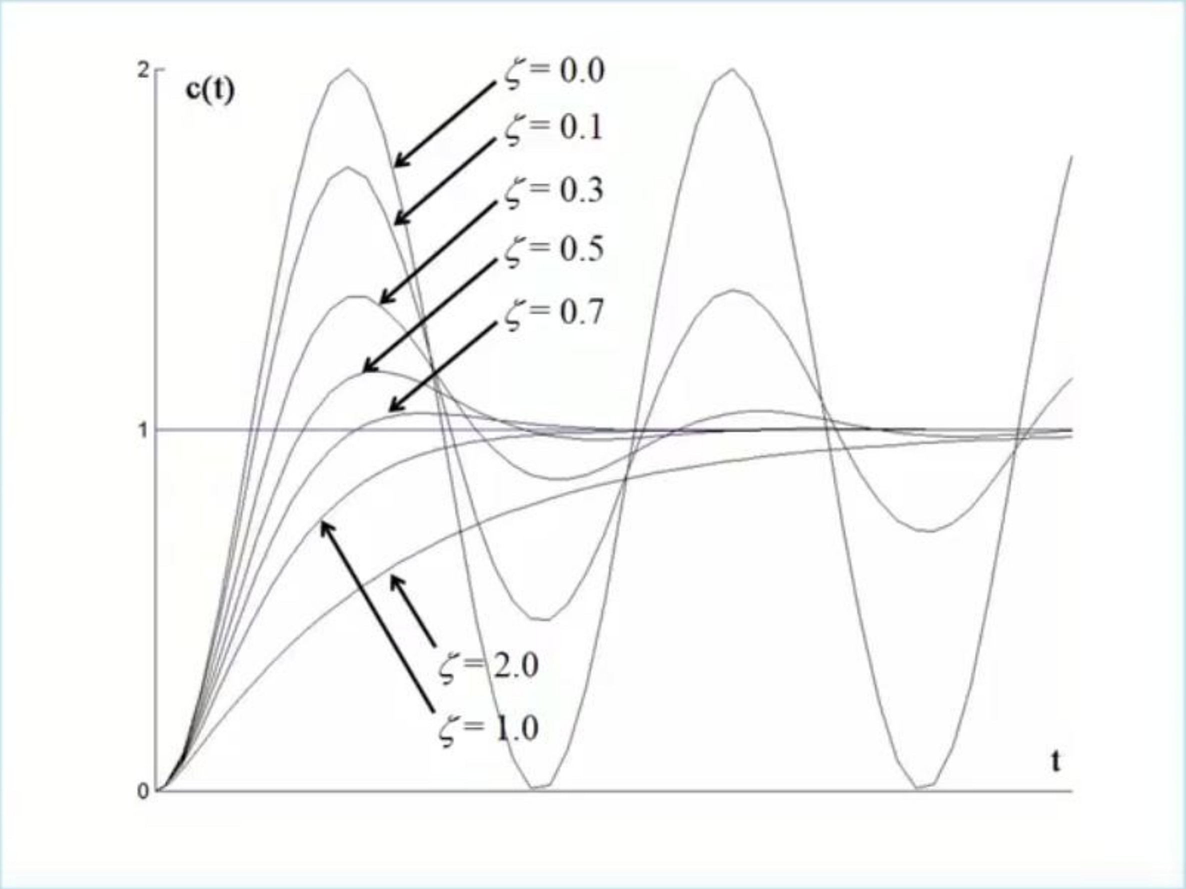
What is the purpose of controlling a dynamic system?

- To achieve the desired transient responses and steady-state responses in terms of:
  - Stability: 100% (possible)
  - Response time: as fast as possible
  - Response accuracy: 100% (possible)



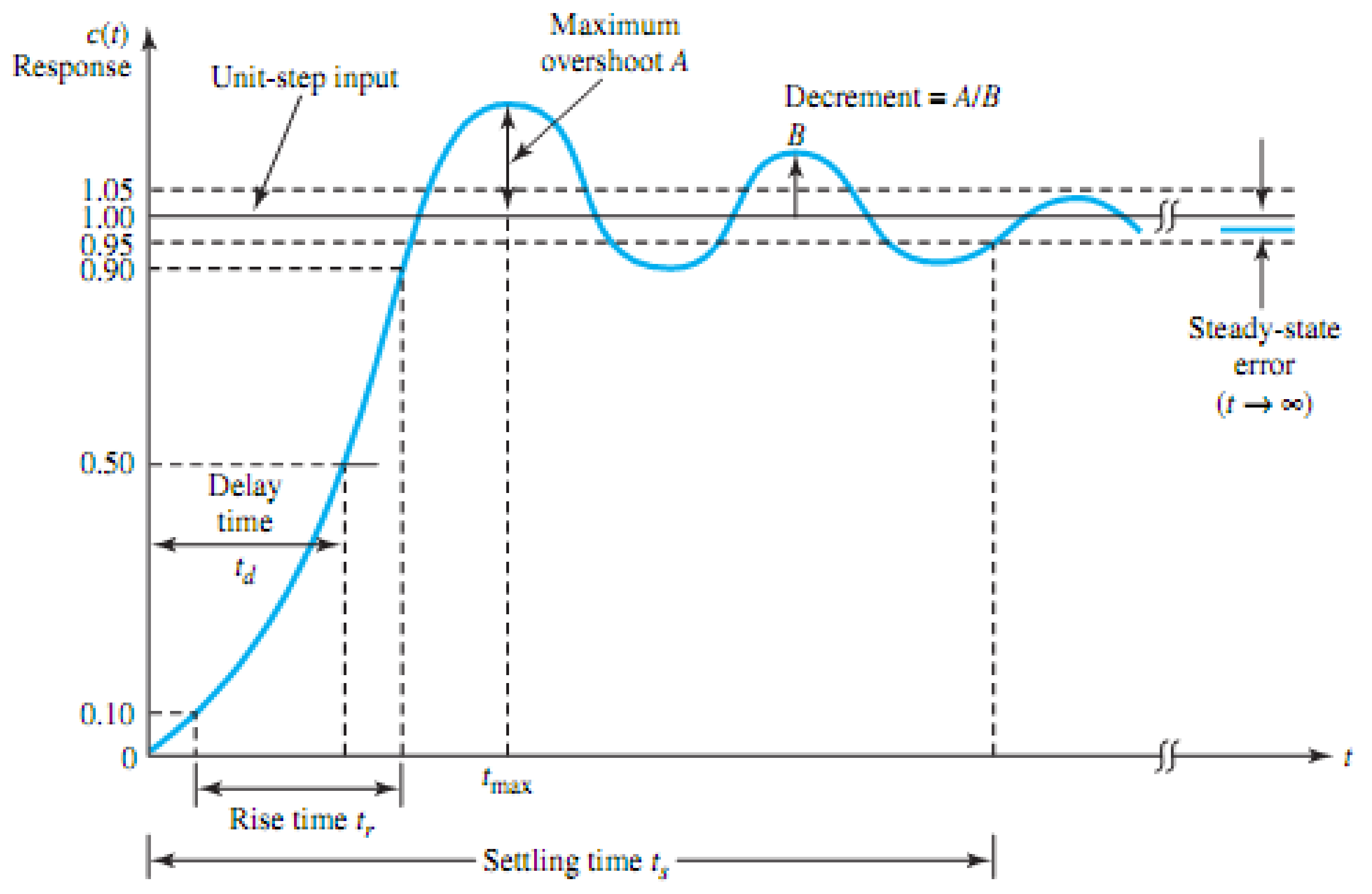
Typical unit-step response of a control system.

Example of Stability



Variation of unit-step response with respect to the damping ratio

Example of Response Time



Typical unit-step response of a control system.

Example of Response Accuracy

$$\lim_{t\to\infty} c(t) = \lim_{s\to 0} s \bullet C(s) = \lim_{s\to 0} s \bullet G(s) \bullet R(s)$$

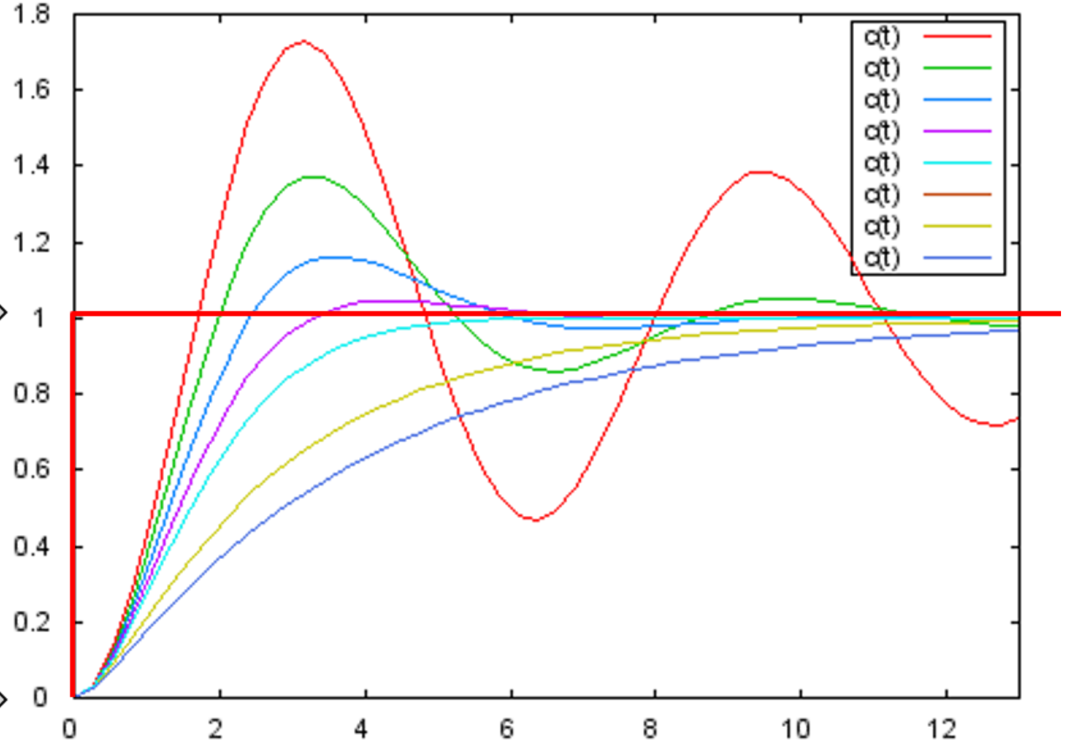
Desired Output

Actual Output

Transfer Function of Closed-loop System

Goal i+1

Goal i

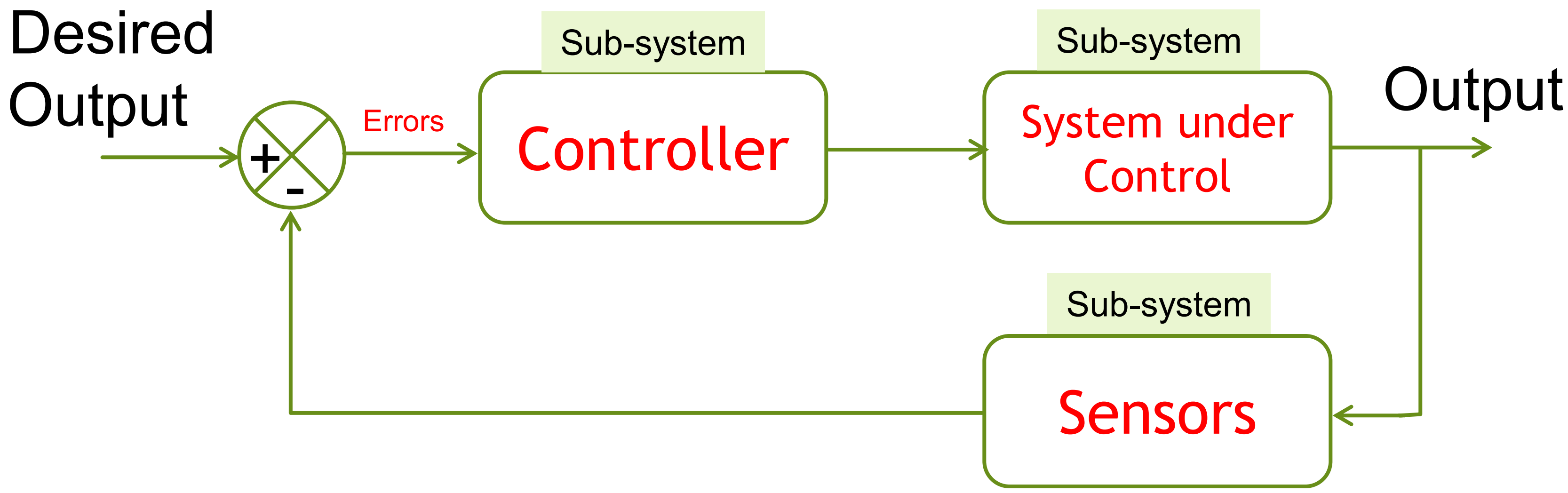


How to achieve the desired responses from a given dynamic system?

- Two Steps:
  - To construct a system with error control loop
  - To design the control law inside the error control loop

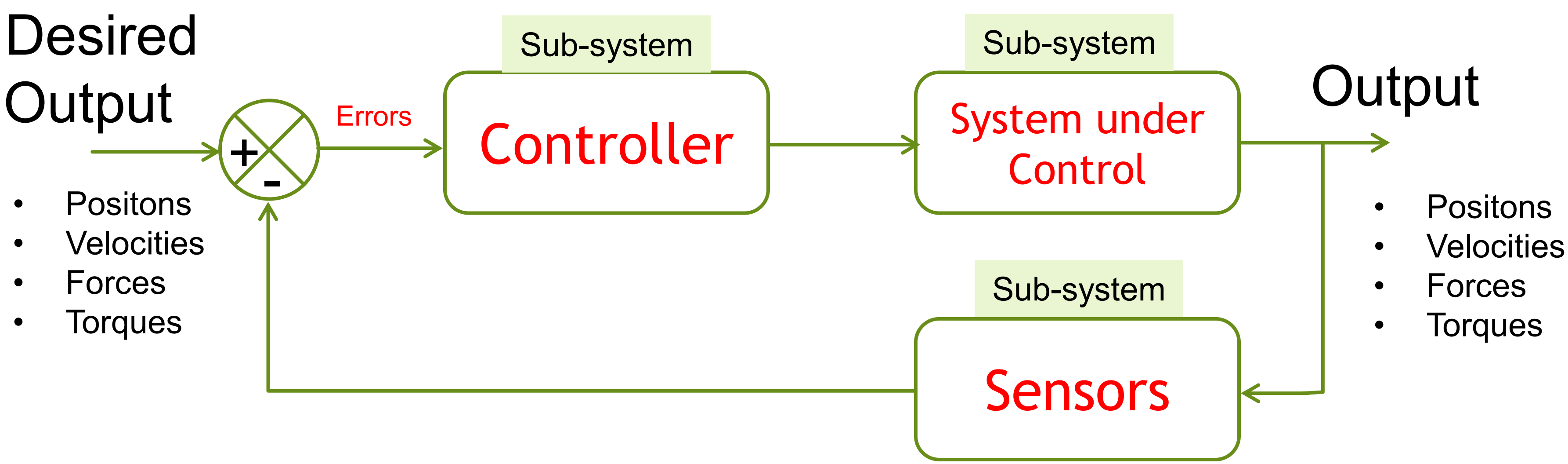
What is a system with error control loop?

- Any system, which responds to errors that is the difference between desired output and actual output, is a system with error control loop.
- A system with error control loop has three basic building blocks:



Example of Motion Control Systems

- Any system, which responds to errors between desired output and actual output, is a system with error control loop.



A System with Error Control Loop

Properties of Systems with Error Control Loops …

- Outputs of such systems strongly depend on errors between desired outputs and actual outputs.

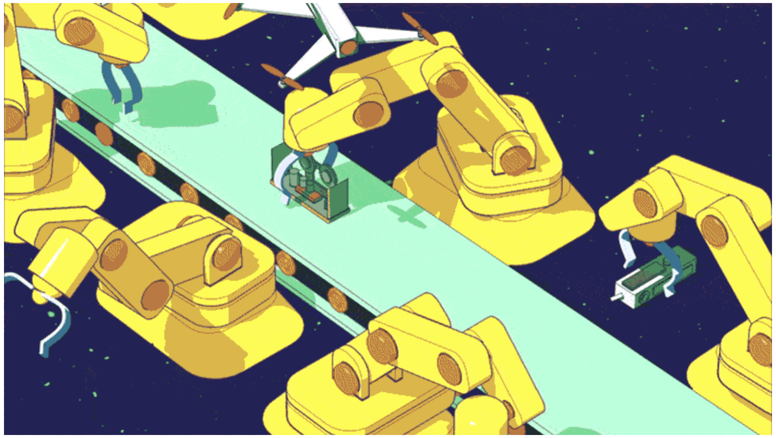
- Outputs of such systems weakly depend on the dynamic constraints of the internal processes.

- Outputs of such systems weakly depend on the static attributes of the internal processes.

How to design a robot's motion control systems?

- There are two methodologies:
  - Apply Design Methods in Time Domain
  - Apply Design Methods in Frequency Domain

Outline of Lecture 3



- Basics of Systems
- Basics of Control Systems
- Design of Control Systems

Dynamics of Single-wheeled Robot

- Equation of Dynamic Behaviors: State Vector

$$F_{net} = m \bullet \ddot{x}$$

$$\ddot{x} = \frac{F_{net}}{m}$$

$$\ddot{x} = \frac{F_{net}}{m} = u$$

$$\ddot{x} = u$$

$$X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x \\ \dot{x} \end{pmatrix}$$

$$\begin{pmatrix} \dot{x} \\ \ddot{x} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ \dot{x} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$$

$$F_{net} = \frac{\tau_{\text{wheel}}}{r_{\text{wheel}}}$$

Dynamics of Single-wheeled Robot

- Equation of Dynamic Behaviors: State-Vector Equations

$$\begin{pmatrix} \dot{x} \\ \ddot{x} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ \dot{x} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$$

$$Y = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ \dot{x} \end{pmatrix}$$

$$\dot{X} = AX + Bu$$

$$Y = CX$$

$$F_{net} = \frac{\tau_{\text{wheel}}}{r_{\text{wheel}}}$$

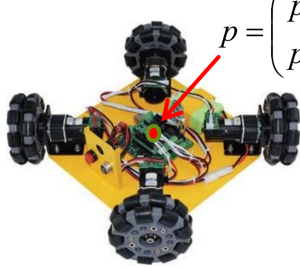
Dynamics of Omni-wheeled Robot

- Equations of Dynamic Behaviours: State Vector

$$\begin{pmatrix} \ddot{p}_x \\ \ddot{p}_y \end{pmatrix} = \begin{pmatrix} u_x \\ u_y \end{pmatrix}$$

$$\begin{pmatrix} \dot{p}_x \\ \ddot{p}_x \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} p_x \\ \dot{p}_x \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u_x$$

$$\begin{pmatrix} \dot{p}_y \\ \ddot{p}_y \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} p_y \\ \dot{p}_y \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u_y$$



$$p = \begin{pmatrix} p_x \\ p_y \end{pmatrix}$$

$$Y = \begin{pmatrix} p_x \\ p_y \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} p_x \\ \dot{p}_x \\ p_y \\ \dot{p}_y \end{pmatrix}$$

Dynamics of Omni-wheeled Robot

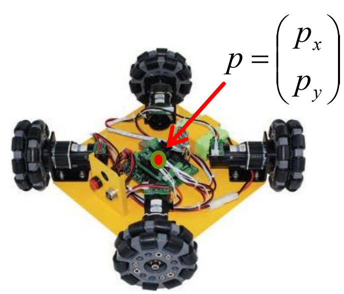
- Equation of Dynamic Behaviors: State-vector Equations

$$\begin{pmatrix} \dot{p}_x \\ \ddot{p}_x \\ \dot{p}_y \\ \ddot{p}_y \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} p_x \\ \dot{p}_x \\ p_y \\ \dot{p}_y \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u_x \\ u_y \end{pmatrix} \Rightarrow$$

$$\dot{X} = AX + Bu$$

$$Y = CX$$

Y



$$Y = \begin{pmatrix} p_x \\ p_y \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} p_x \\ \dot{p}_x \\ p_y \\ \dot{p}_y \end{pmatrix}$$

X

Two Domains for Us to Do Design …

Time Domain	Frequency Domain
$\dot{x} = kx$	$Y(s) = \frac{1}{s-k}$
⇕	⇕
$x(t) = e^{k \times t}$	$y(t) = e^{k \times t}$

More in next slide

Use of Eigenvalues and Eigenvectors …

$$\dot{x} = kx \quad \Rightarrow \quad x(t) = e^{k\times t}$$

$$\dot{X} = AX \qquad A = V \times D \times V^{-1}$$

$$\dot{X} = VDV^{-1}X$$

$$(V^{-1}\dot{X}) = D \times (V^{-1}\,X)$$

$$V^{-1} \times X(t) = e^{D\times t}$$

$$X(t) = V \times e^{D\times t}$$

This is called Diagonalization!

$$D = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \dots & \\ & & & \lambda_n \end{bmatrix}$$

$$V = [v_1 \quad v_2 \quad \dots \quad v_n]$$

$$A\vec{\mathbf{v}} = \lambda\vec{\mathbf{v}}$$

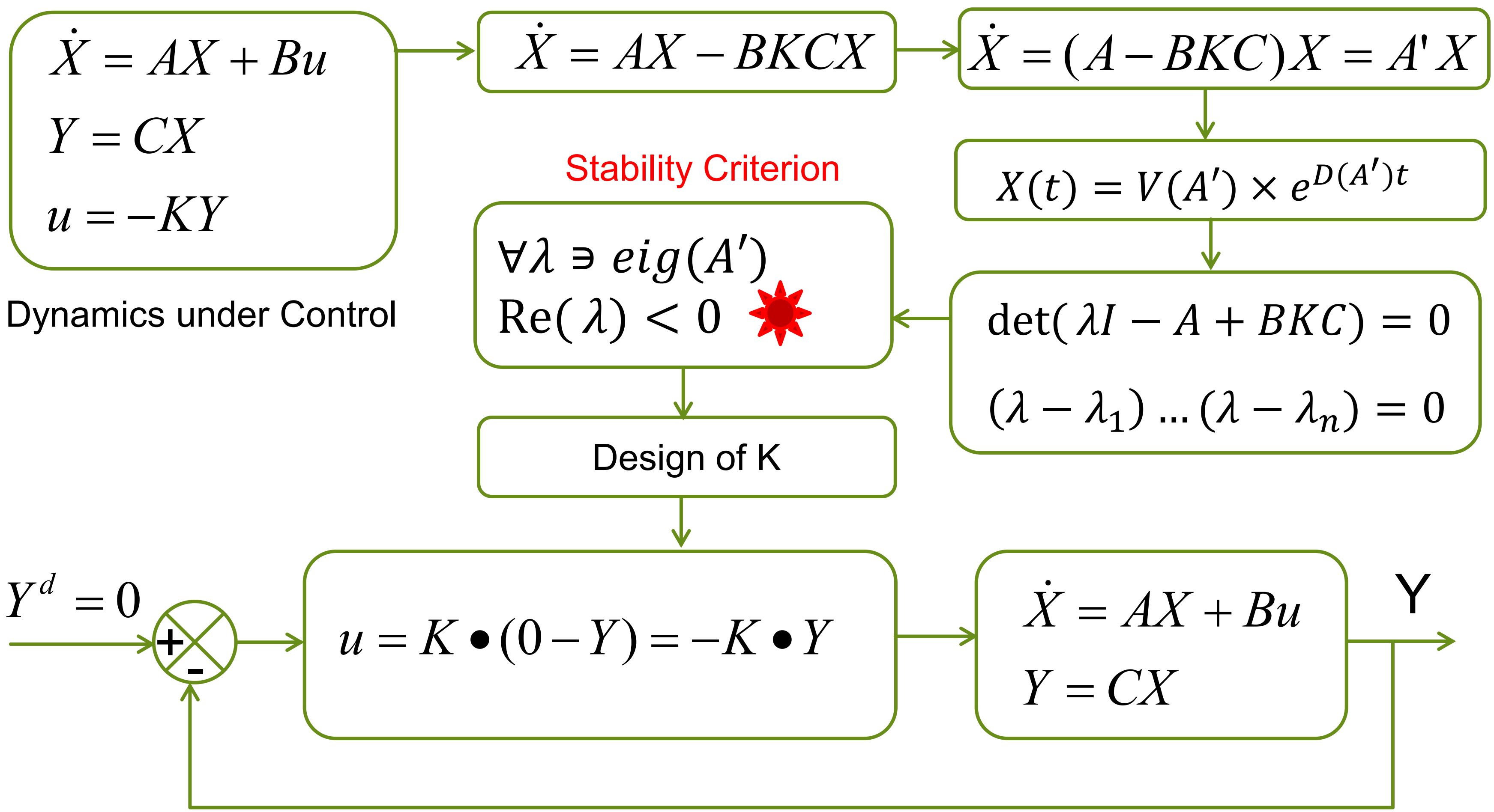
$$A\vec{\mathbf{v}} - \lambda I\vec{\mathbf{v}} = 0$$

$$(A - \lambda I)\vec{\mathbf{v}} = 0$$

$$\det(A - \lambda I) = 0$$

Design Methods in Time Domain

- Use of State-vector Equations:



Design Method 1 in Frequency Domain

- Use of Routh-Hurwitz Criterion

$$G(s) = \frac{C(s)}{R(s)} = \frac{Z(s)}{P(s)} = \frac{Z(s)}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0}$$

Routh Array

s^n	a_n	a_{n-2}	a_{n-4}	$\cdots$
s^{n-1}	a_{n-1}	a_{n-3}	a_{n-5}	$\cdots$
s^{n-2}		A		
...				
s^0				

The system is stable if the number of change of signs in the first column is zero.

$$A = \frac{a_{n-1} \bullet a_{n-4} - a_n \bullet a_{n-5}}{a_{n-1}}$$

Example

A feedback control system has a characteristic equation

$$s^3 + (1 + K)s^2 + 10s + (5 + 15K) = 0$$

The parameter K must be positive. What is the maximum value K can assume before the system becomes unstable ? When K is equal to the maximum value, the system oscillates. Determine the frequency of oscillation.

Solution

- Routh Array:

$$s^3 + (1+K)s^2 + 10s + (5+15K) = 0$$

$$\begin{array}{lcc} s^3 & 1 & 10 \\ s^2 & 1+K & 5+15K \\ s^1 & \dfrac{5-5K}{1+K} & 0 \\ s^0 & 5+15K & \end{array}$$

$$1+K>0$$

$$5-5K>0$$

$$5+15K>0$$

$$1 > \mathrm{K} > -\frac{1}{3}$$

If K is positive,
Then:

$$1 > \mathrm{K} > 0$$

Solution （continued):

- When K = 1, the Routh Array is:

$$s^3+(1+K)s^2+10s+(5+15K)=0$$

$$\begin{array}{lcc} s^3 & 1 & 10 \\ s^2 & 1+K & 5+15K \\ s^1 & \dfrac{5-5K}{1+K} & 0 \\ s^0 & 5+15K & \end{array}$$

$$s^3+2s^2+10s+20=0$$

$$\begin{array}{lcc} s^3 & 1 & 10 \\ s^2 & 2 & 20 \\ s^1 & 0 & 0 \\ s^0 & & \end{array}$$

Auxiliary Polynomials

Characteristic Equation = Polynomials x Auxiliary Polynomials

Solution （continued):

Characteristic Equation = Polynomials x Auxiliary Polynomials

- When K = 1, the auxiliary polynomial equation is: $P_a(s) = 2s^2 + 20 = 0$

$$\frac{dP(s)}{ds} = 4s + 0 = 0$$

$$s^3 + 2s^2 + 10s + 20 = 0$$

s^3	1	10
s^2	2	20
s^1	4	0
s^0	20	

$$P_a(s) = 2s^2 + 20 = 0$$

$$s = \pm j\sqrt{10}$$

Oscillation with: $\omega = \sqrt{10}$ rad/s

Oscillation frequence is : $f = \frac{\omega}{2\pi} = \frac{\sqrt{10}}{2\pi}$

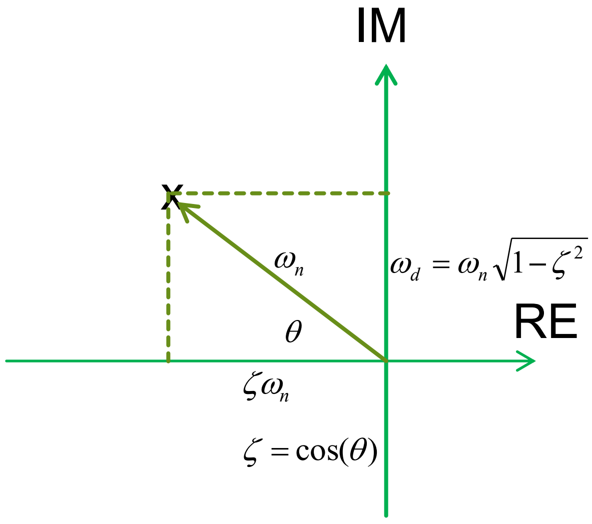
Design Method 2 in Frequency Domain

- Use of Specifications in Time Domain

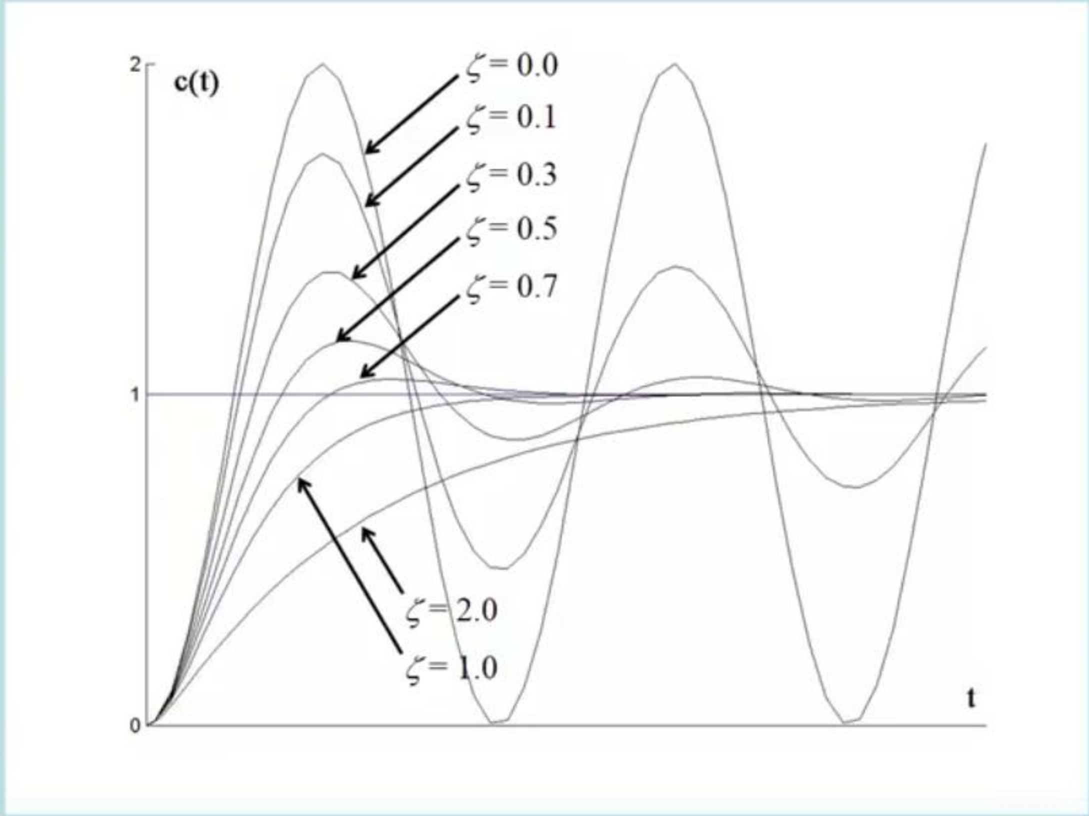
$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$= \frac{\omega_n^2}{s^2+2\zeta\omega_n s+(\zeta\omega_n)^2-(\zeta\omega_n)^2+\omega_n^2}$$

$$= \frac{\omega_n^2}{(s+\zeta\omega_n)^2+(\omega_n\sqrt{1-\zeta^2})^2}$$



Typical Outputs



Variation of unit-step response with respect to the damping ratio

Design Specifications in Time Domain

- Maximum Overshoot: $$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$$
- Time of Maximum Overshoot: $$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$
- Settling Time with 2% of Error Band: $$t_s = \frac{4}{\zeta\omega_n}$$
- Settling Time with 5% of Error Band: $$t_s = \frac{3}{\zeta\omega_n}$$

Example

Consider the closed-loop system given by

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Determine the values of ζ and ω_n so that the system responds to a step input with approximately 5% overshoot and with a 2% settling time of 2 sec.

Solution

- Overshoot of 5%:

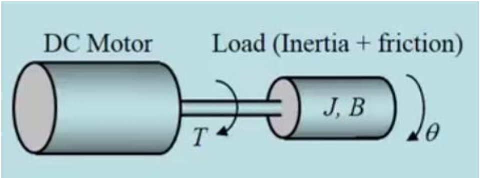
$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 0.05 \Rightarrow -\pi\zeta/\sqrt{1-\zeta^2} = \log_e(0.05) = -2.9957$$

$$\zeta^2 = \left(\frac{2.9957}{\pi}\right)^2(1-\zeta^2) \Rightarrow \zeta = 0.69$$

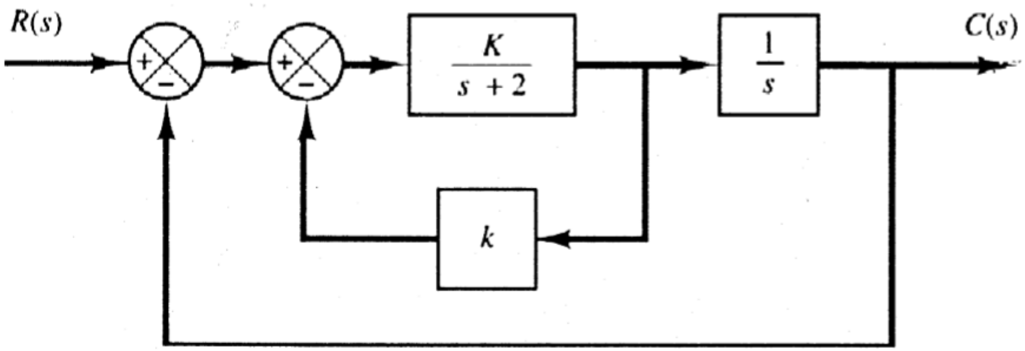
- 2% Settling Time of 2 seconds:

$$t_s = \frac{4}{\zeta\omega_n} = 2 \Rightarrow \omega_n = \frac{2}{\zeta} = \frac{2}{0.69} = 2.8986\,\text{rad/s}$$

Example

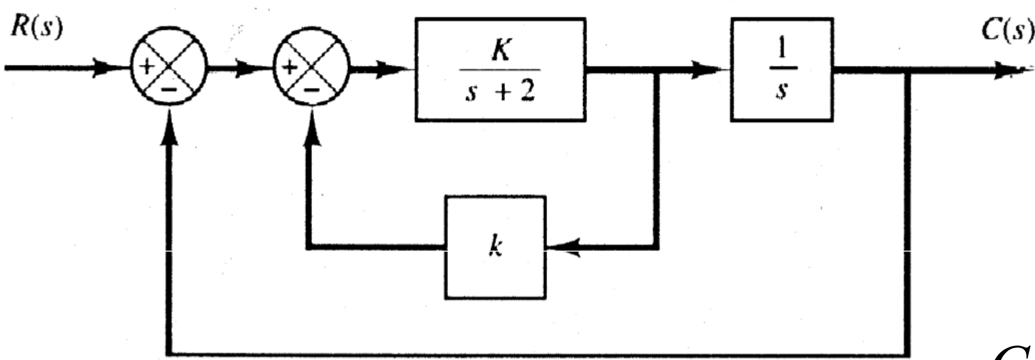


Referring to the system shown below, determine the values of K and k such that the system has a damping ratio ζ of 0.7 and an undamped natural frequency ω_n of 4 rad/sec.

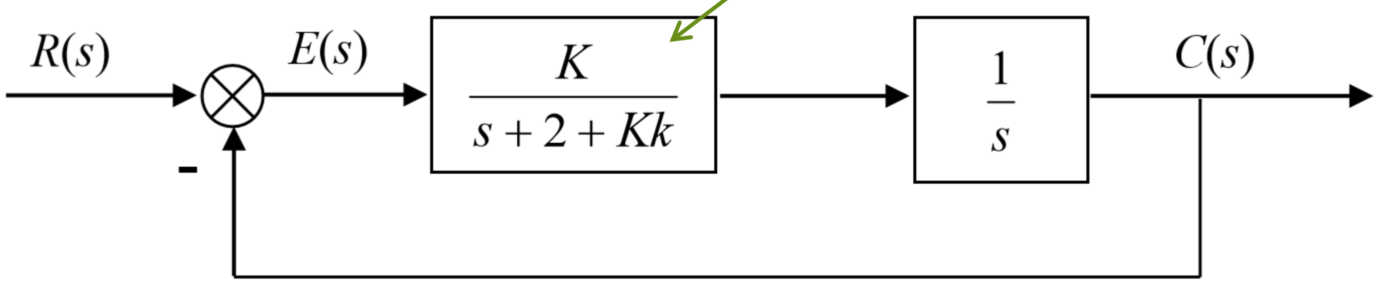


Typical Closed-loop Error Control System for Motors

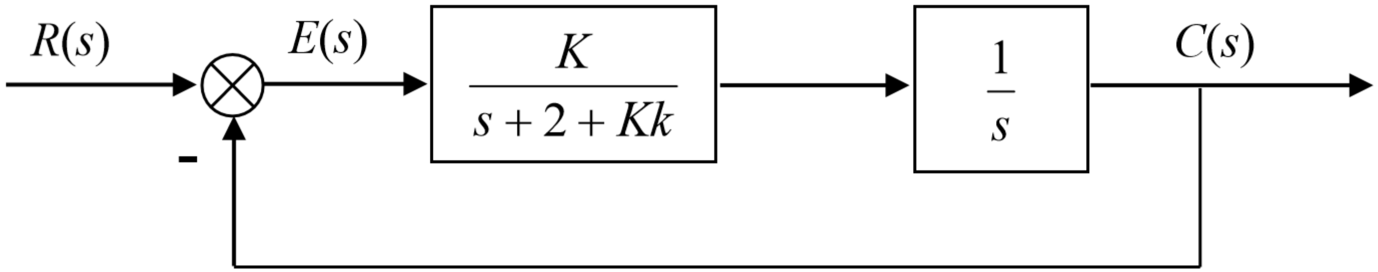
Solution



$$\frac{C(s)}{R(s)} = \frac{\frac{K}{s+2}}{1+\frac{Kk}{s+2}} = \frac{K}{s+2+Kk}$$



Solution (continued)



$$\frac{C(s)}{R(s)} = \frac{\dfrac{K}{s+2+Kk}\dfrac{1}{s}}{1+\dfrac{K}{s+2+Kk}\dfrac{1}{s}} = \frac{K}{(s+2+Kk)s+K} = \frac{K}{s^2+(2+Kk)s+K}$$

Solution (continued)

$$\frac{C(s)}{R(s)} = \frac{\frac{K}{s+2+Kk}\frac{1}{s}}{1+\frac{K}{s+2+Kk}\frac{1}{s}} = \frac{K}{(s+2+Kk)s+K} = \frac{K}{s^2+(2+Kk)s+K}$$

$$\omega_n^2 = K$$

$$2\zeta\omega_n = 2+Kk$$

$$K = 4^2 = 16$$

$$k = \frac{2\zeta\omega_n - 2}{K} = \frac{2\times 0.7\times 4-2}{16} = 0.225$$

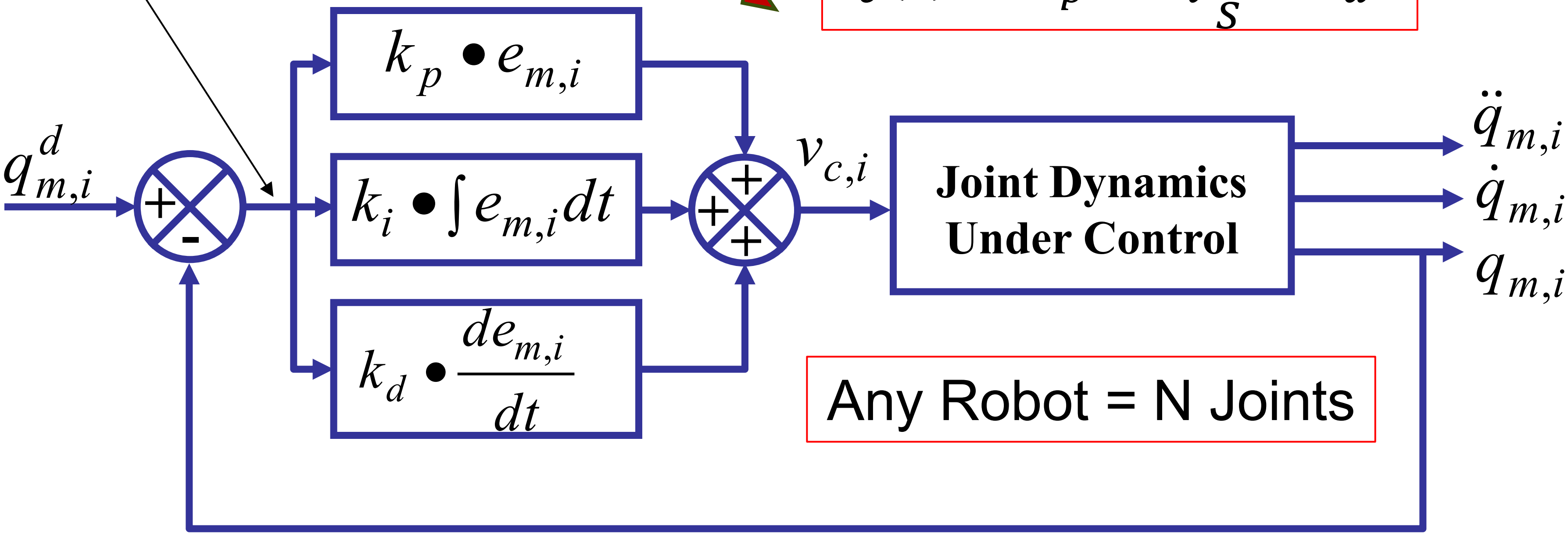
Design Method 3 in Frequency Domain

- Use of PID Transfer Function (i.e., PID Control Laws):

$$e_{m,i} = q^d_{m,i} - q_{m,i}$$

PID Transfer Function

$$G_c(s) = k_p + k_i \frac{1}{s} + k_d s$$

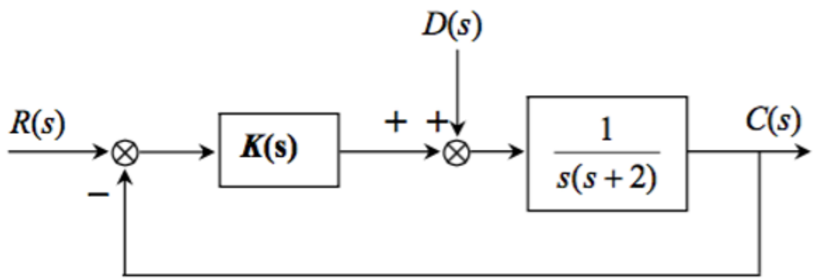


Any Robot = N Joints

$$i = 1,2,\ldots,n$$

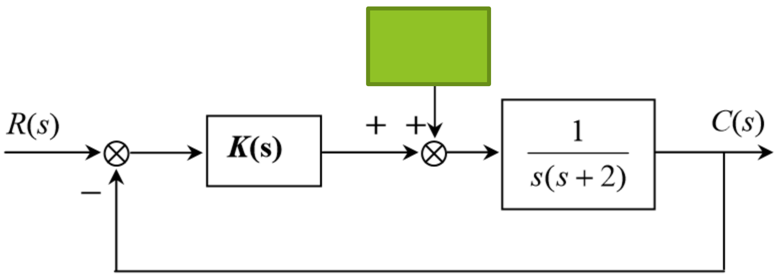
Example

Consider the following system with controller $K(s)$, input $R(s)$, and disturbance $D(s)$.



1. Suppose that $K(s)$ is a proportional controller, i.e. $K(s) = K$. For which value of K the response to unit-ramp input has a steady-state error of 0.1?
2. If we want the response to unit-ramp input to have zero steady-state error, what form should the controller $K(s)$ take?
3. If we want the response to unit-step *disturbance* to be zero at steady-state, what form should the controller $K(s)$ take?

Solution

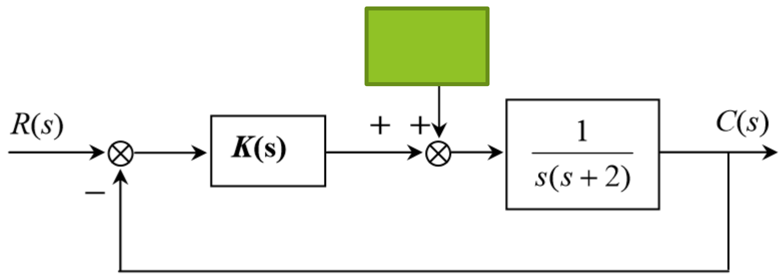


- Transfer Functions of Closed-loop Control System:

$$\frac{C(s)}{R(s)} = \frac{\dfrac{K}{s(s+2)}}{1+\dfrac{K}{s(s+2)}} = \frac{K}{s^2+2s+K}$$

$$\frac{E(s)}{R(s)} = 1 - \frac{C(s)}{R(s)} = 1 - \frac{K}{s^2+2s+K} = \frac{s^2+2s}{s^2+2s+K}$$

Solution (continued)



- a) Steady-state Error to Unit-ramp Input:

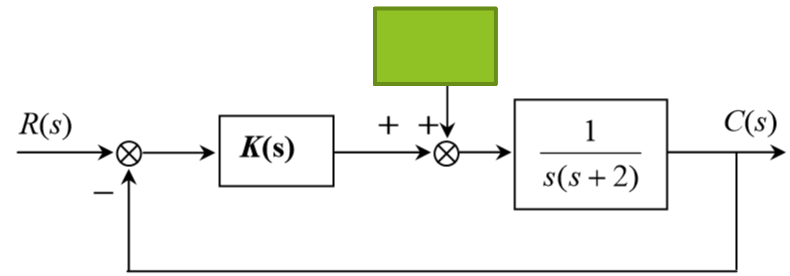
$$R(s) = \frac{1}{s^2}$$

$$e(\infty) = \lim_{s\to 0} s\,E(s) = \lim_{s\to 0} s \bullet \frac{s^2 + 2s}{s^2 + 2s + K(s)} \bullet \frac{1}{s^2} = \lim_{s\to 0} \frac{2}{K(s)}$$

$$\frac{2}{K} = 0.1$$

$$K = 20$$

Solution



- b) Steady-state Error to be zero:

$$e(\infty) = \lim_{s\to 0} s\,E(s) = \lim_{s\to 0} s \bullet \frac{s^2+2s}{s^2+2s+K(s)} \bullet \frac{1}{s^2} = \lim_{s\to 0} \frac{2}{K(s)}$$

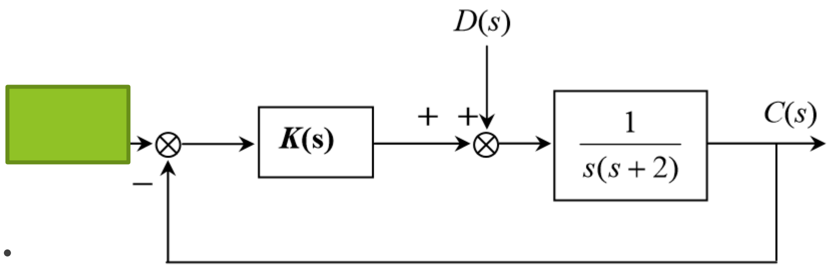
In this case, the controller should include an integrator.

Hence, the controller should be in the following form (or a similar form):

$$K(s) = K_1 + \frac{K_2}{s} = (K_1 s + K_2)\frac{1}{s}$$

$$e(\infty) = \lim_{s\to 0} 2\frac{s}{K_1 s + K_2} = 0$$

Solution (continued)



- c) Transfer Function between D(s) and C(s):

$$\frac{C(s)}{D(s)} = \frac{\frac{1}{s(s+2)}}{1+\frac{K(s)}{s(s+2)}} = \frac{1}{s^2+2s+K(s)}$$

Steady-state Response to Unit-step Input of D(s):

$$c(\infty) = \lim_{s\to 0} s \bullet \frac{1}{s^2+2s+K(s)} \bullet \frac{1}{s} = \lim_{s\to 0} \frac{1}{K(s)}$$

$$K(s) = K_1 + \frac{K_2}{s}$$

$$K(s) = K_1 + \frac{K_2}{s} = (K_1 s + K_2)\frac{1}{s}$$

$$c(\infty) = \lim_{s\to 0} \frac{s}{K_1 s + K_2} = 0$$

Hence, K(s) must include an integrator in order to make the response to be zero.

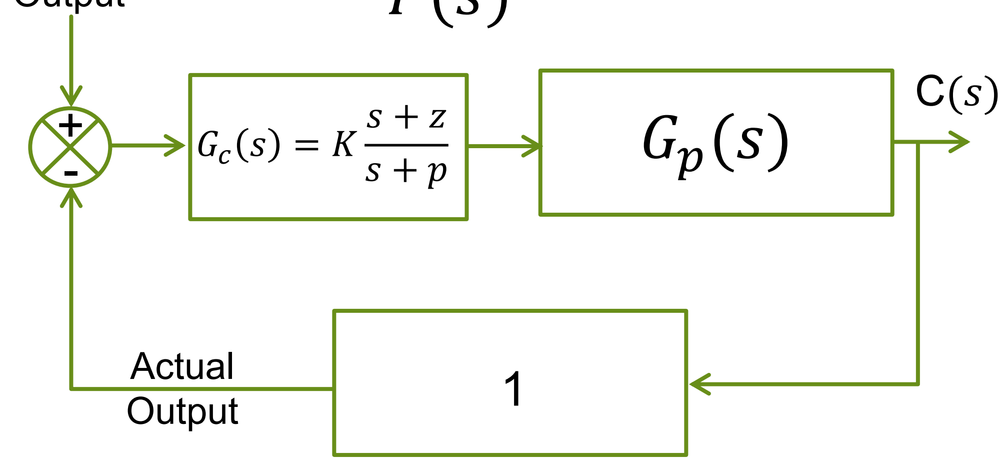
Design Method 4 in Frequency Domain

- Use of Phase-Compensation Transfer Function (i.e., Phase Compensator):

$$\frac{C(s)}{R(s)} = \frac{K \times G(s)}{1 + K \times G(s)}$$

$$1 + K \times G(s) = 0$$

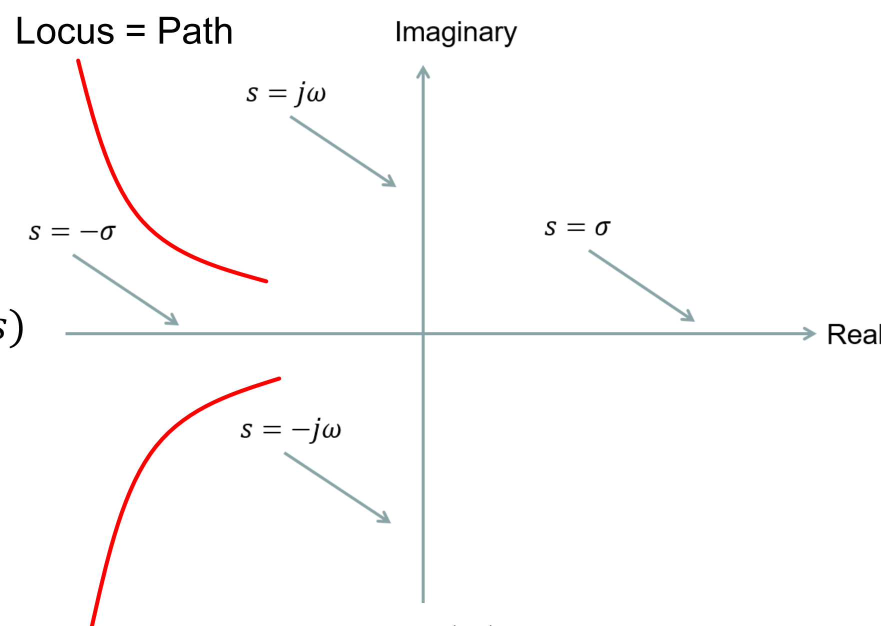
$$1 + K \times \frac{Z(s)}{P(s)} = 0$$



Equation of Root Locus

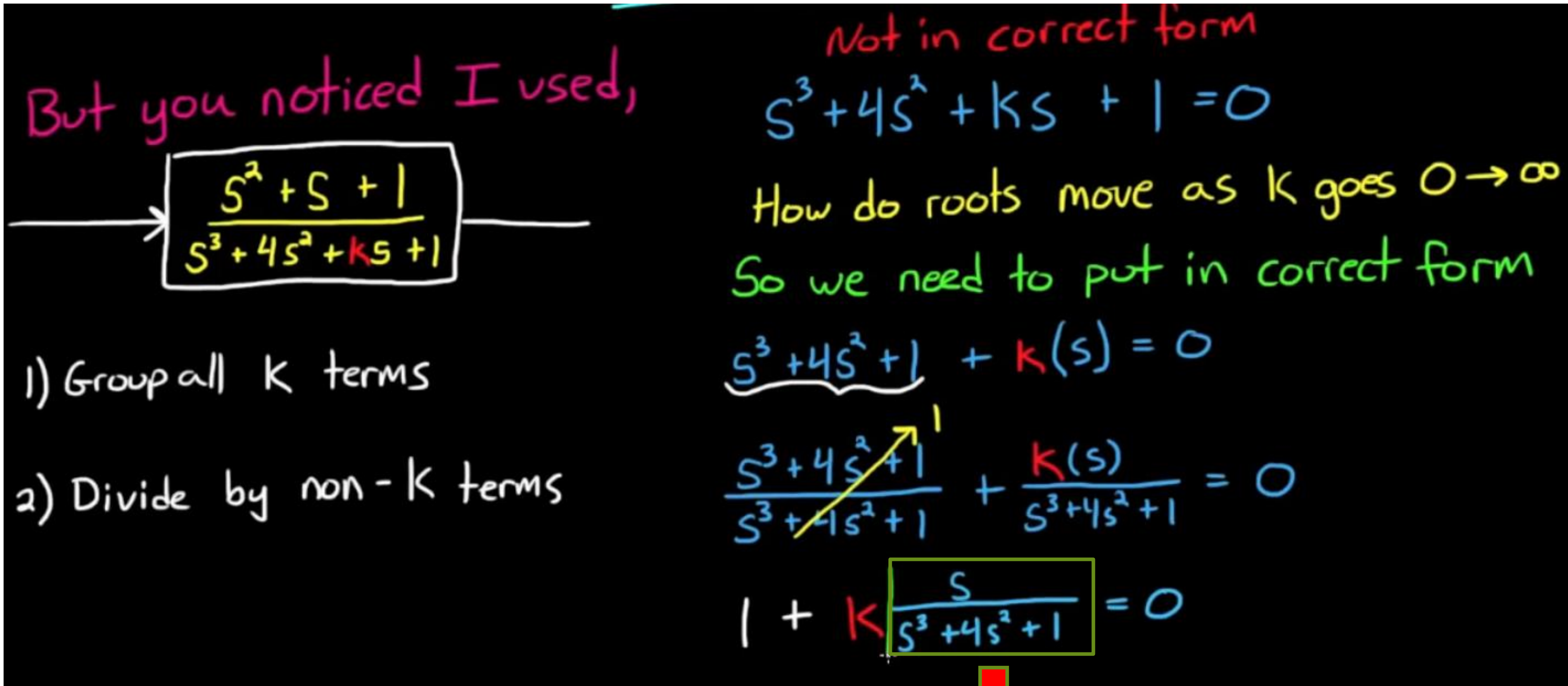
$$\mathrm{P}(s) + K \times \mathrm{Z}(s) = 0$$

When K varies from 0 to infinity, we have:



$$G(s) = G_c(s)G_p(s)$$

How to Determine Equation of Root Locus?



Equation of Root Locus $1 + K \bullet G(s) = 0$

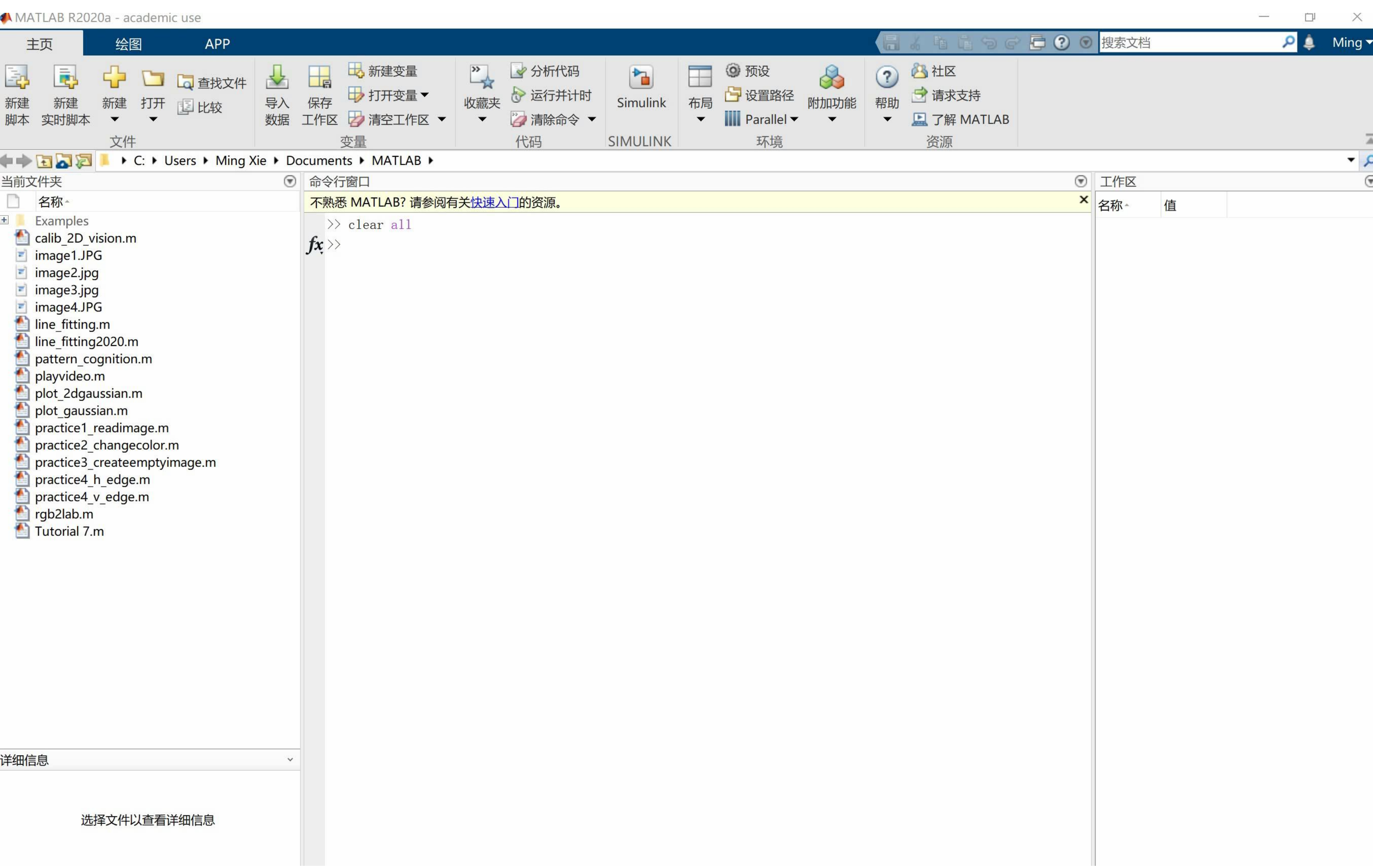

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image4.JPG
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line_fitting2020.m
pattern_cognition.m
playvideo.m
plot_2dgaussian.m
plot_gaussian.m
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practice2_changecolor.m
practice3_createemptyimage.m
practice4_h_edge.m
practice4_v_edge.m
rgb2lab.m
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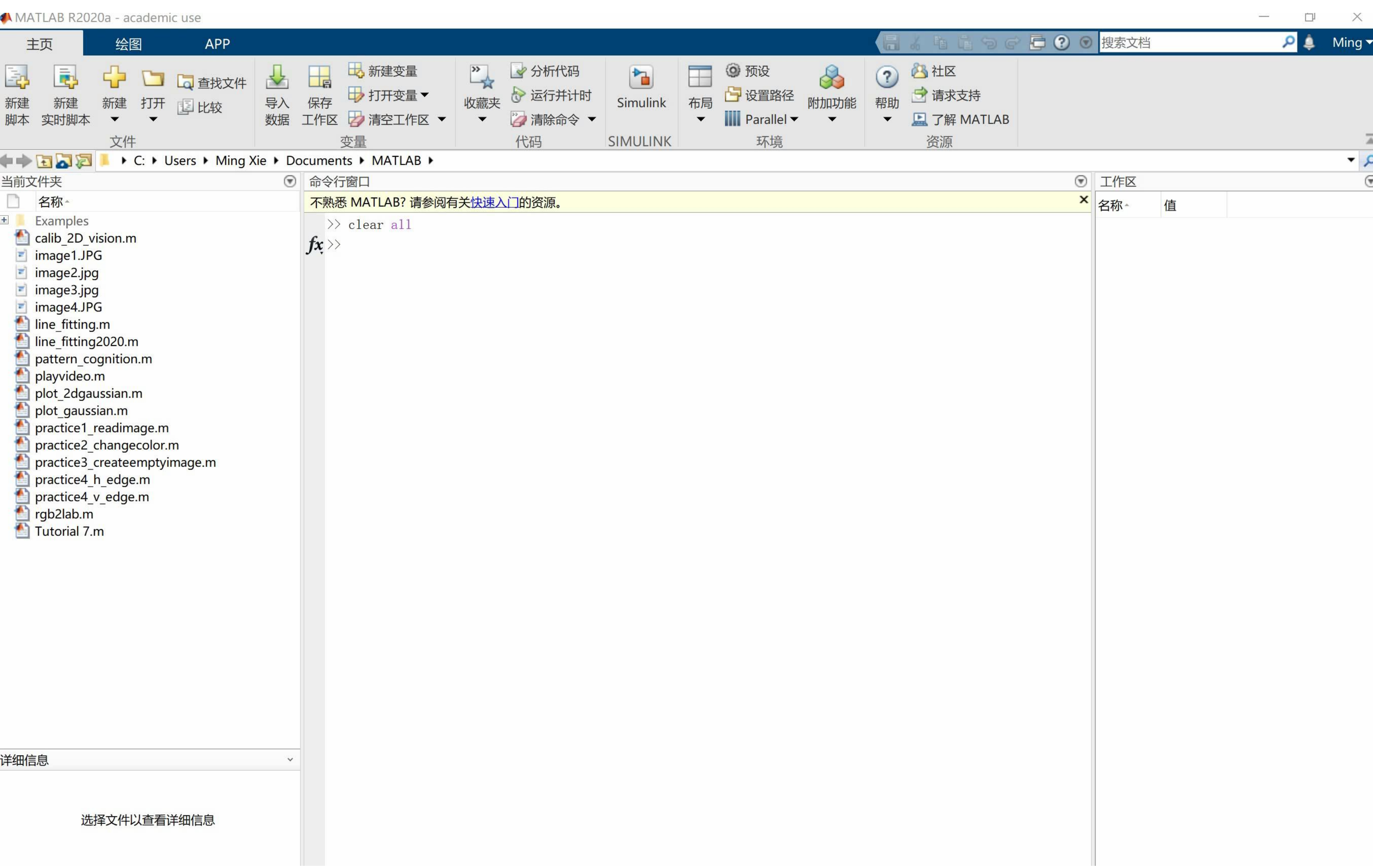
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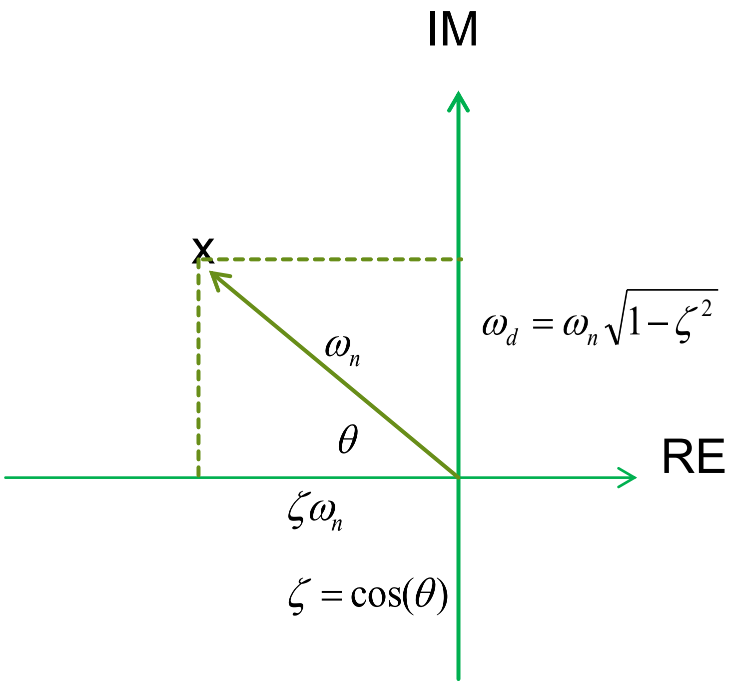
Design Specification in Frequency Domain:

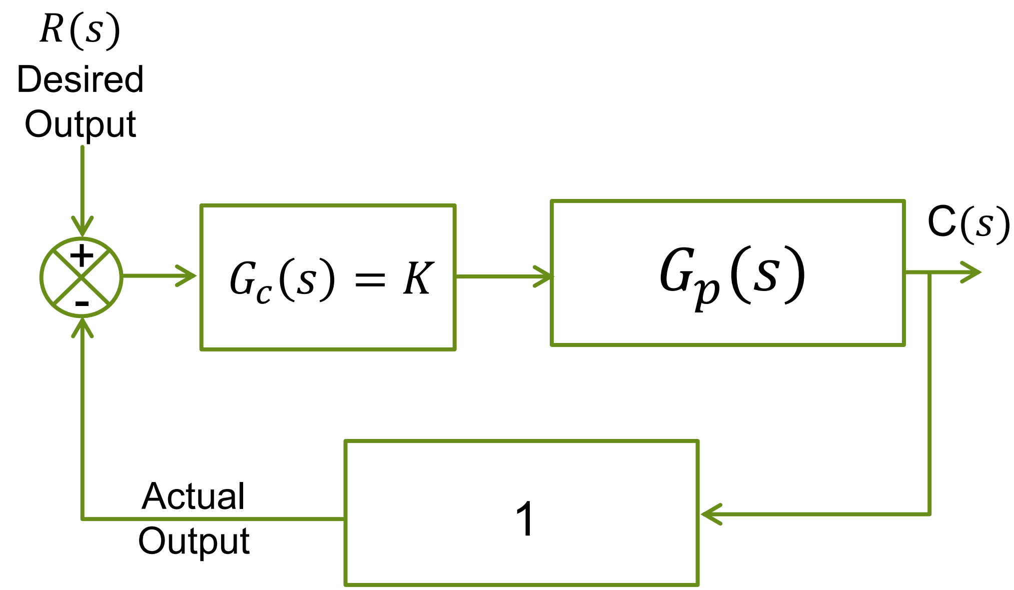
- Locations of desired roots of error control loop systems.



Design Options

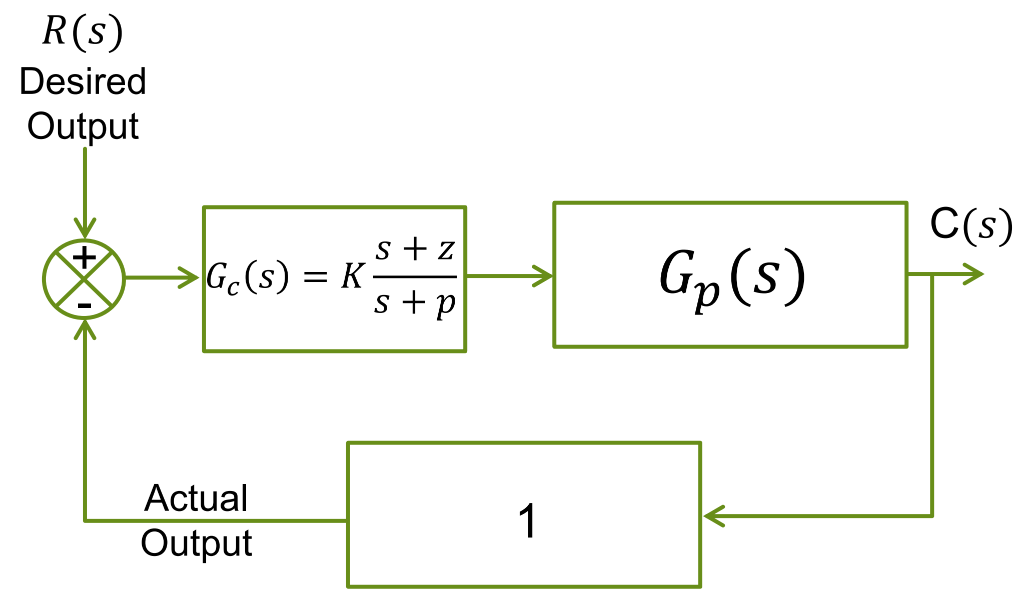
- Option 1: without phase compensator

$$G(s) = G_c(s)G_p(s)$$



- Option 2: with phase compensator:

$$G(s) = G_c(s)G_p(s)$$



Example

Consider the model for an AGV control system shown in Figure 1.

a) Determine the unit step time response of the feedback system, for $G_c(s) = 1$ and $K = 1$.

b) Design the following types of $G_c(s)$ to give a pair of dominant closed loop poles at $s = -1 \pm j1$

(i) Proportional-Derivative controller

(ii) Lead compensator

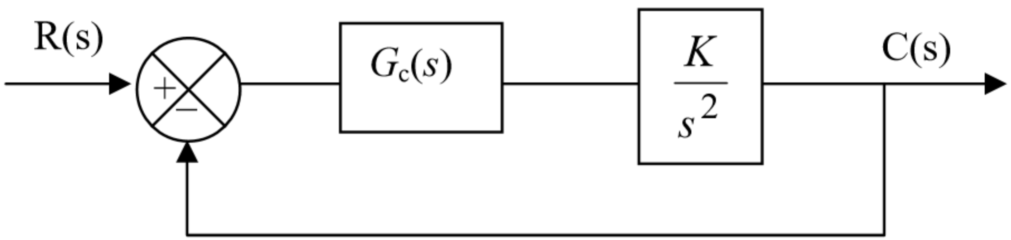
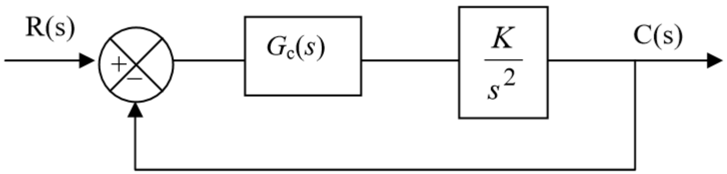


Figure 1

c) What improvement will the compensator bring to the system unit-step response?

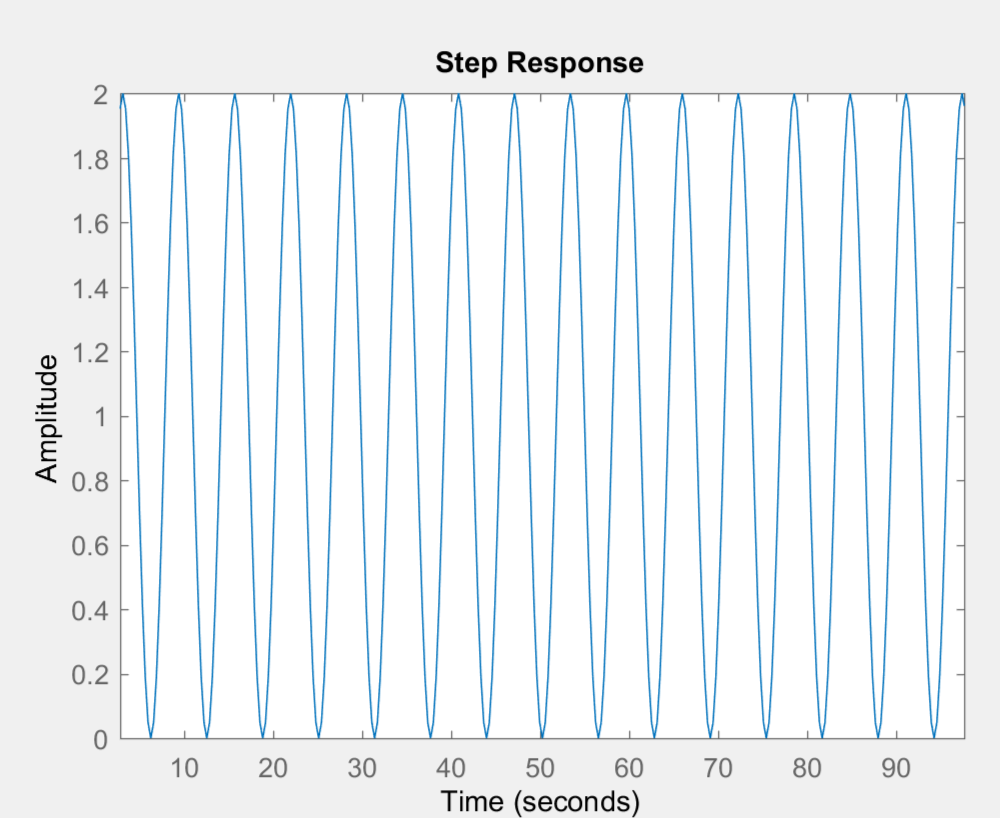
Solution



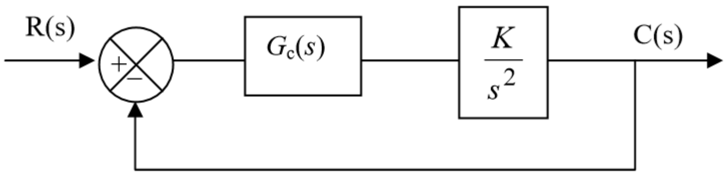
- a) Unit-step Response when Gc(s) = 1 and K = 1:

$$\frac{C(s)}{R(s)} = \frac{G_c(s)\frac{K}{s^2}}{1+G_c(s)\frac{K}{s^2}} = \frac{K \bullet G_c(s)}{s^2 + K \bullet G_c(s)}$$

$$\frac{C(s)}{R(s)} = \frac{1}{s^2+1} = \frac{1}{(s+j)(s-j)}$$

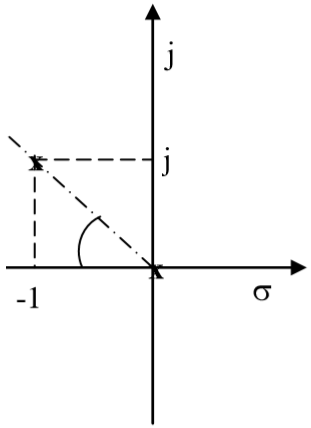


Solution



- b(i) Design of PD Controller: $G_c(s) = k_1 + k_2 s$

$$\frac{C(s)}{R(s)} = \frac{G_c(s)\frac{K}{s^2}}{1+G_c(s)\frac{K}{s^2}} = \frac{K \bullet (k_1 + k_2 s)}{s^2 + K \bullet (k_1 + k_2 s)} = \frac{k_p + k_d s}{s^2 + k_d s + k_p}$$



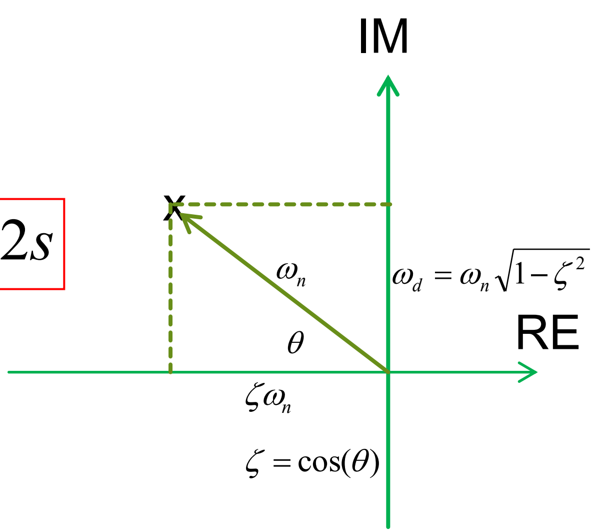
$$\zeta = \cos(45^0) = 0.707$$

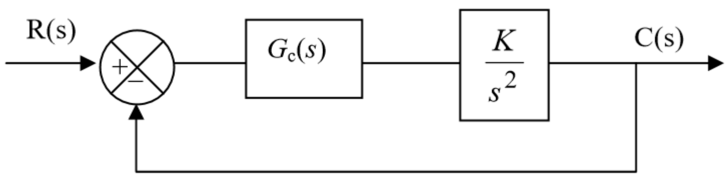
$$\omega_n = \sqrt{1^2 + 1^2} = 1.4142$$

$$k_p = \omega_n^2 = 2$$

$$k_d = 2\zeta\omega_n = 2 \times 0.707 \times 1.4142 = 2$$

$$G_c(s) = 2 + 2s$$



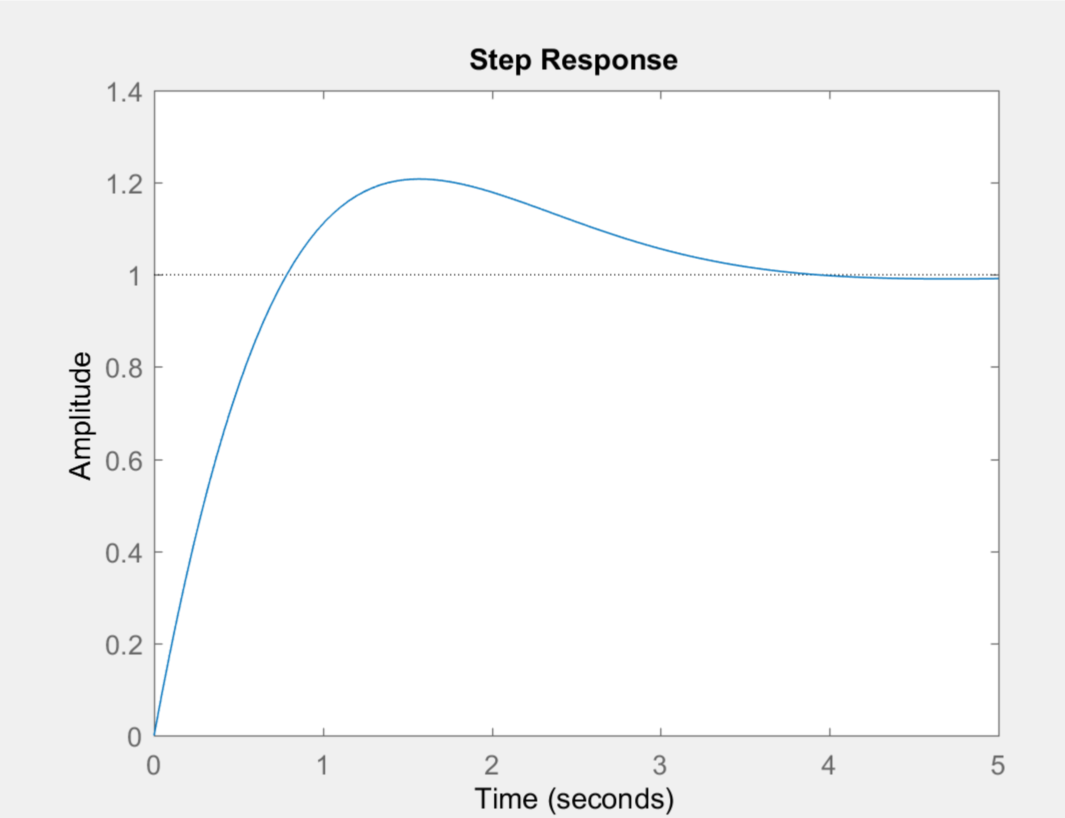


$$\frac{C(s)}{R(s)} = \frac{(2+2s)\frac{1}{s^2}}{1+(2+2s)\frac{1}{s^2}} = \frac{2s+2}{s^2+2s+2}$$

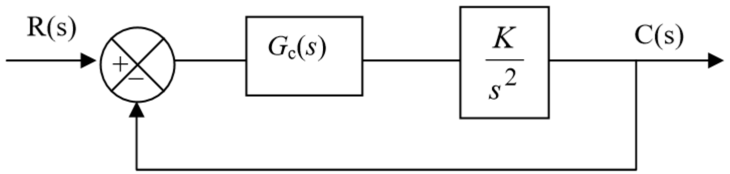
K = 1

$$G_c(s) = 2+2s$$

```
>> TF=(2*s+2)/(s^2+2*s+2)

TF =

     2 s + 2
  -------------
  s^2 + 2 s + 2

Continuous-time transfer function.

>> step(TF)
```

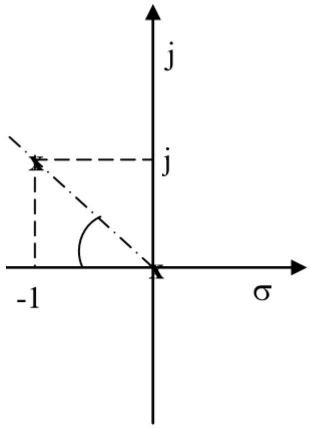


Solution



- b(ii) Design of Phase Compensator:

$$G_c(s) = \frac{s+z}{s+p}$$

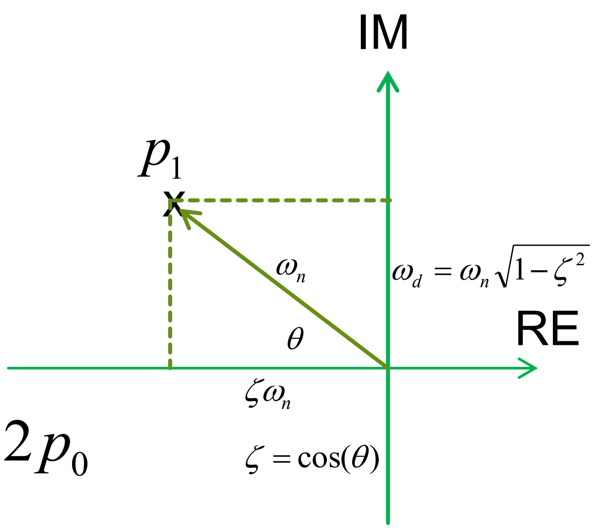


$$1 + G_c(s)\frac{K}{s^2} = 0$$

$$s^2 + K\frac{s+z}{s+p} = 0 \quad \Rightarrow \quad s^3 + ps^2 + Ks + Kz = 0$$

$$s^3 + ps^2 + Ks + Kz = (s+p_0)(s+1+j)(s+1-j)$$

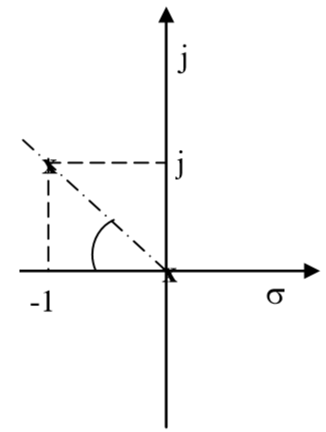
$$s^3 + ps^2 + Ks + Kz = s^3 + (2+p_0)s^2 + (2+2p_0)s + 2p_0$$



$$s^3 + ps^2 + Ks + Kz = s^3 + (2 + p_0)s^2 + (2 + 2p_0)s + 2p_0$$

$$K = 2 + 2p_0$$

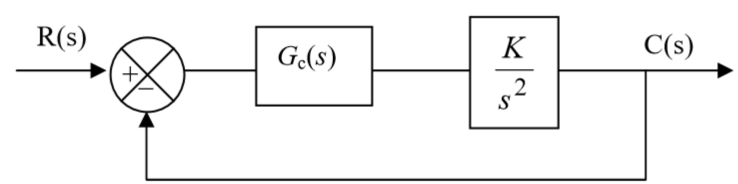
$$p_0 = 0.5K - 1$$



$$p = 2 + p_0 = 0.5K + 1$$

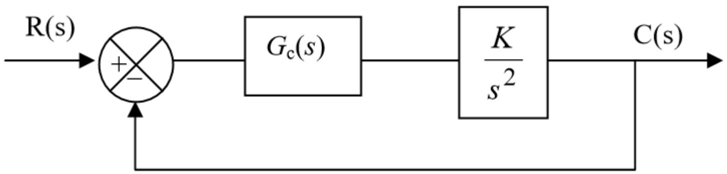
$$Kz = 2p_0 = K - 2$$

$$z = 1 - 2/K$$



$$G_c(s) = \frac{s+z}{s+p} = \frac{s + 1 - 2/K}{s + 0.5K + 1}$$

Solution

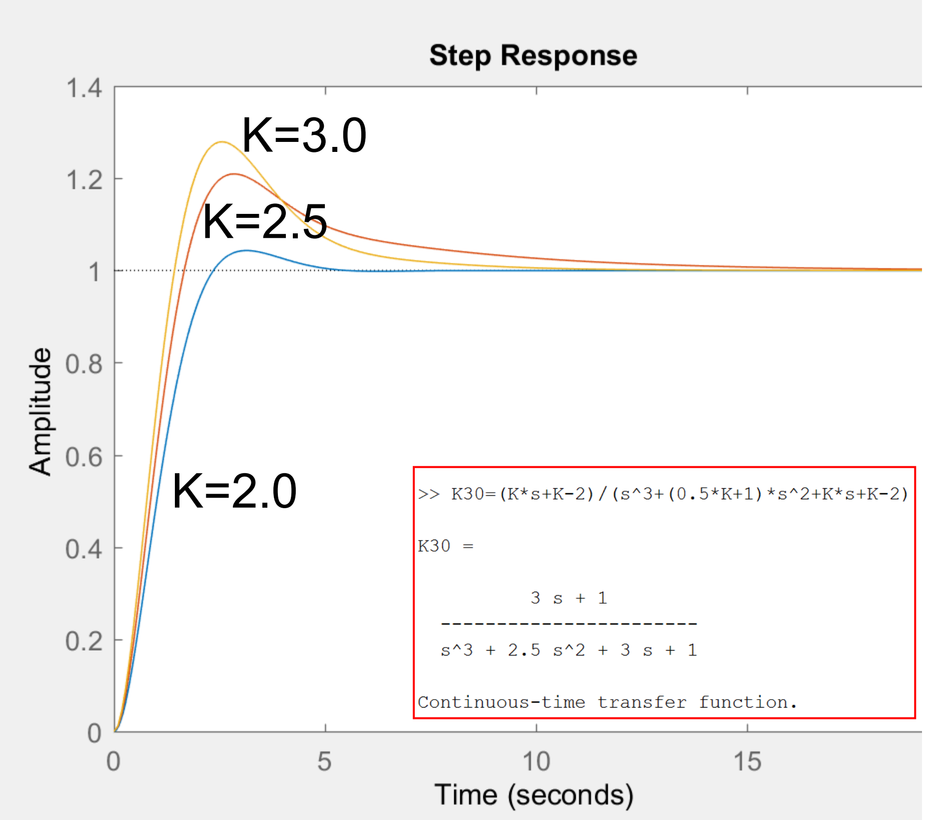


- c) Improvement by Phase Compensator:

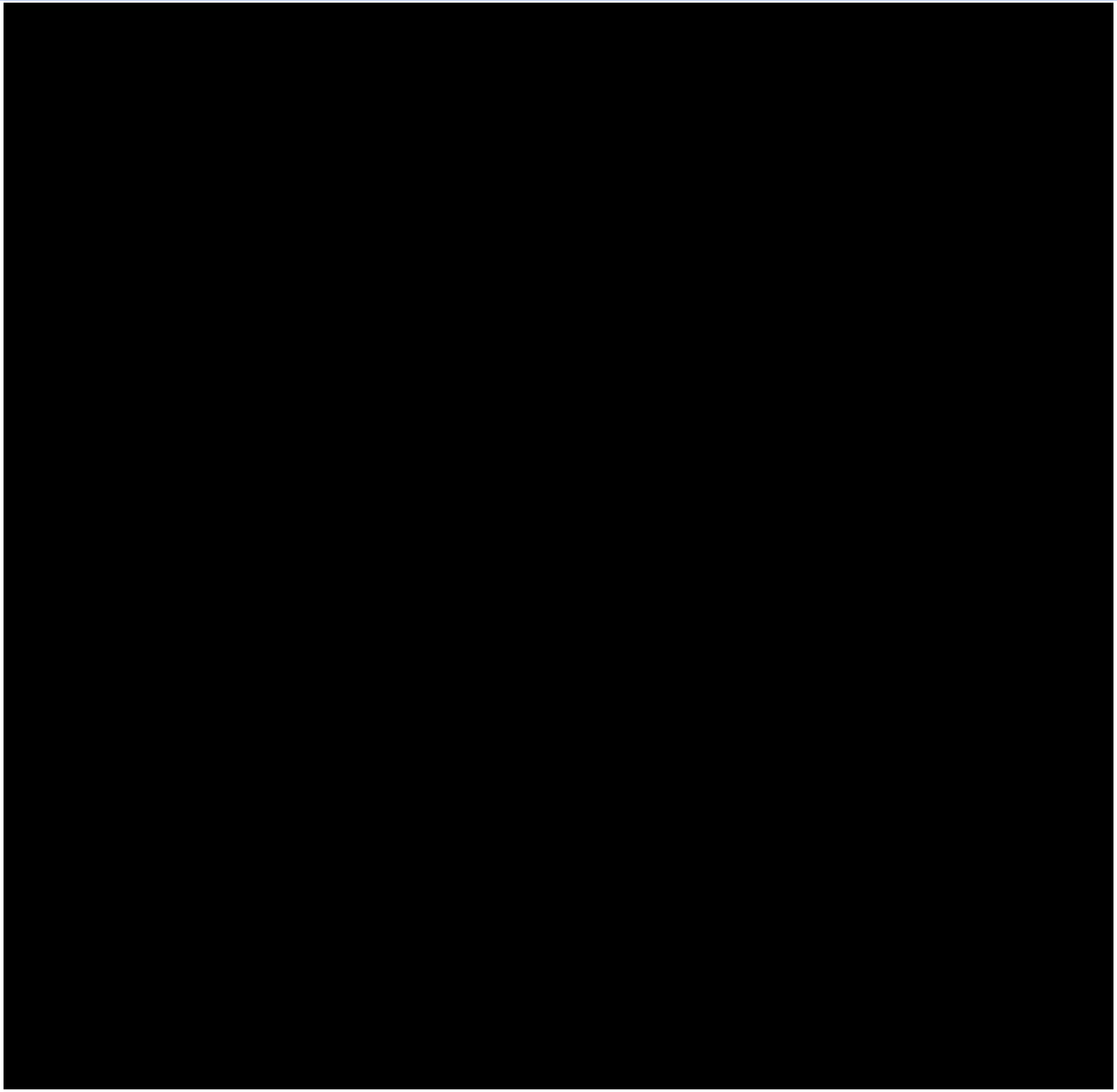
$$G_c(s) = \frac{s+1-2/K}{s+0.5K+1}$$

$$\frac{C(s)}{R(s)} = \frac{\dfrac{K(s+1-2/K)}{s^2(s+0.5K+1)}}{1+\dfrac{K(s+1-2/K)}{s^2(s+0.5K+1)}}$$

$$\frac{C(s)}{R(s)} = \frac{K(s+1-2/K)}{s^3+(0.5K+1)s^2+Ks+(K-2)}$$



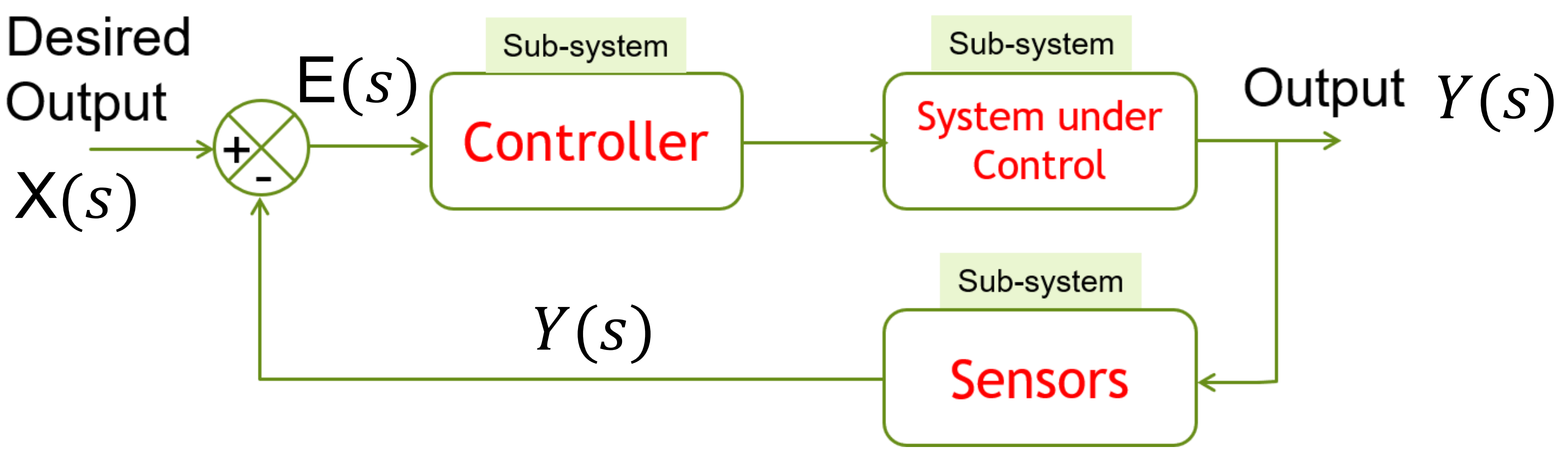
MATLAB Demo



Another Useful Tool: Final Value Theorem

Steady-state Output $$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s \bullet Y(s)$$

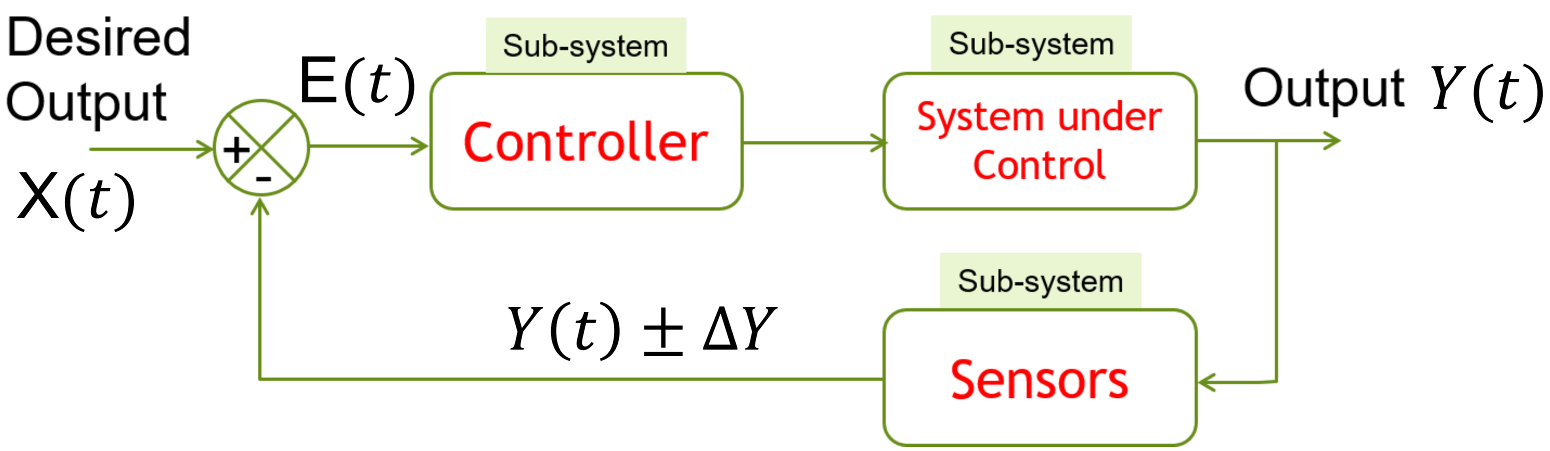
Steady-state Error $$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s \bullet E(s)$$



Caution: Error from Sensor Cannot Be Compensated by Error Control Loop!

When $E(t) = X(t) - Y(t) \pm \Delta Y = 0$

we have $\boxed{Y(t) = X(t) \pm \Delta Y}$

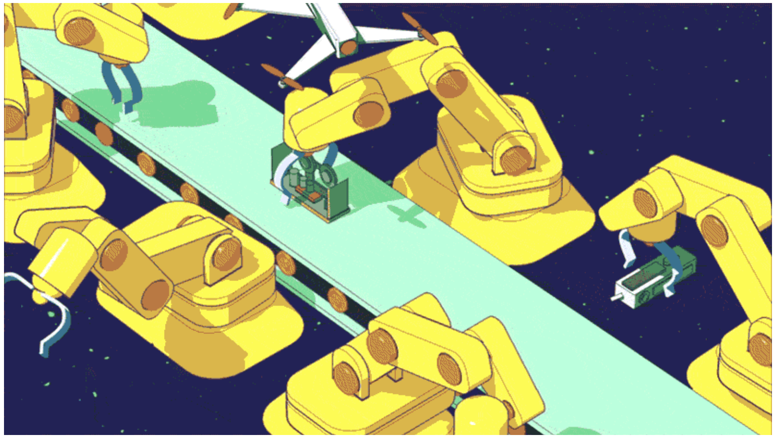


Remaining Questions …

- How is robot brain’s control signal related to desired motion?
- Under what condition will a robotic arm behave like an ideal dynamic system?

MIMO = Sum of SISOs

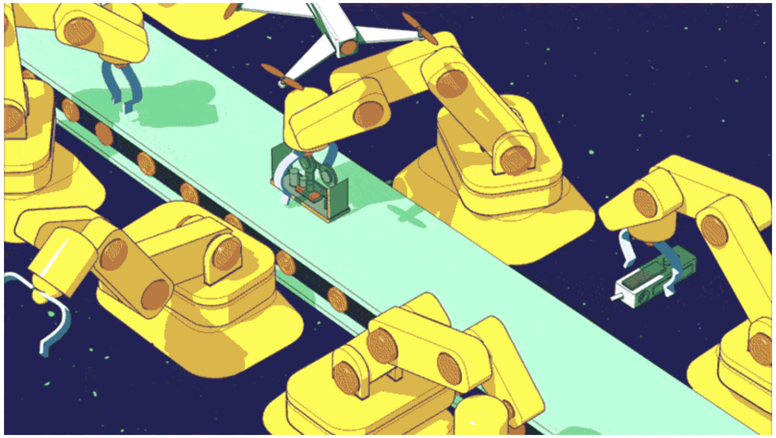
Summary of Lecture 3



- Basics of Systems
- Basics of Control Systems
- Design Solutions

Outline of Module 4

- Dynamics under Control
- Signal Flow Diagram
- Design of Control Systems
- Control in Joint-Space
- Control in Task-Space



School of Mechanical & Aerospace Engineering

Design, Machine, Control, Intelligence

Module 4

MA4825 Robotics

Lecture 4

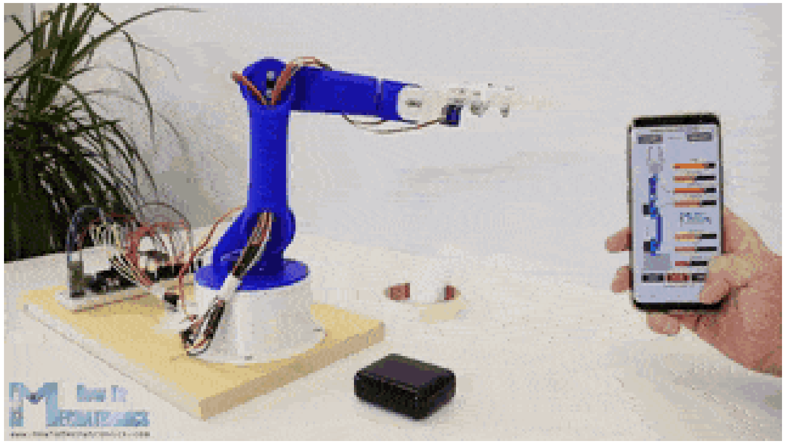
Control in Joint-Space

Xie Ming, PhD (France)

http://personal.ntu.edu.sg/mmxie

Outline of Lecture 4

- Joint Space
- Control Input
- Control Feedback
- Control with Known Dynamics
- Control with Unknown Dynamics



Outline of Lecture 4

- Joint Space
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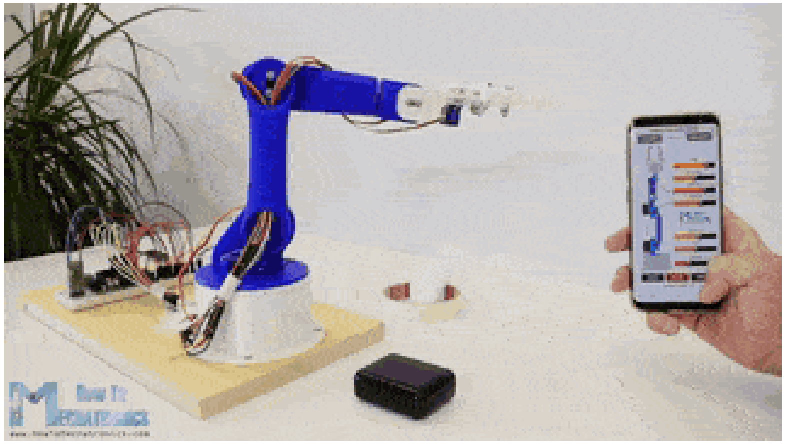
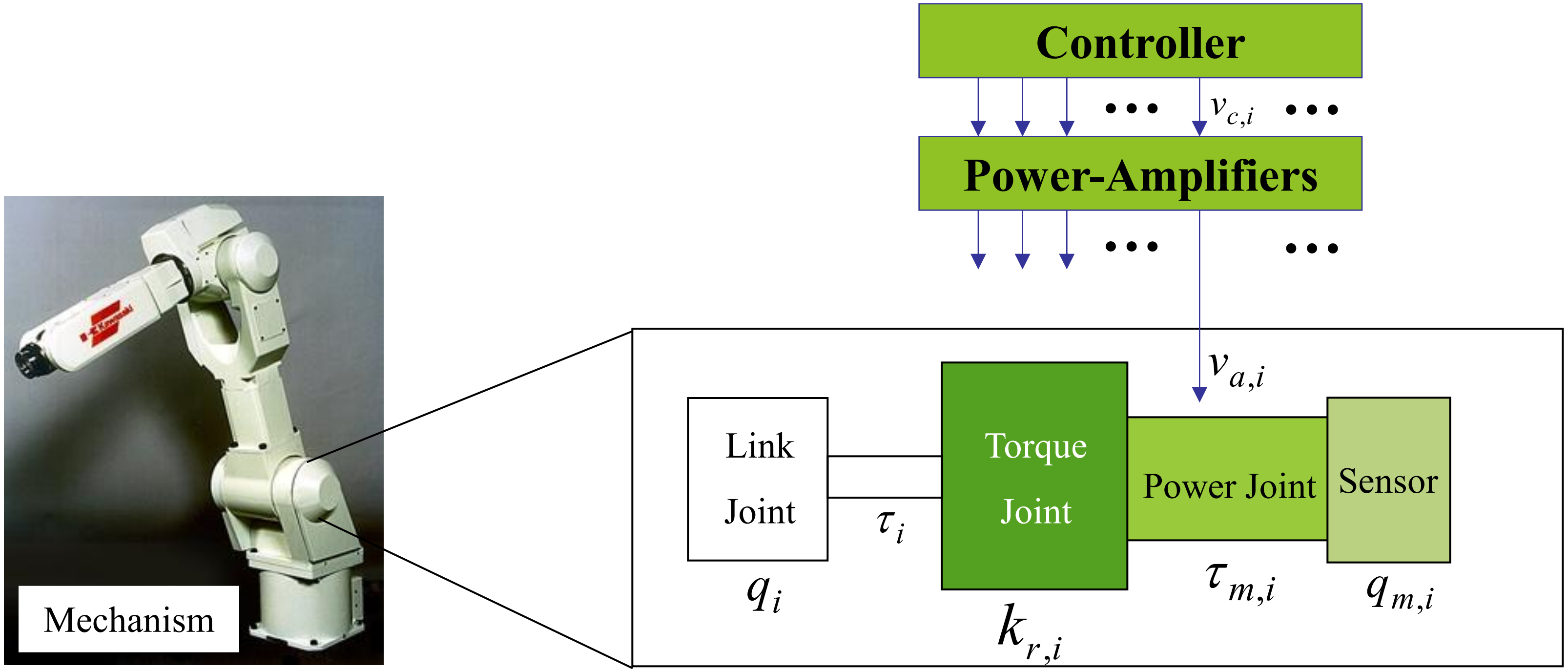


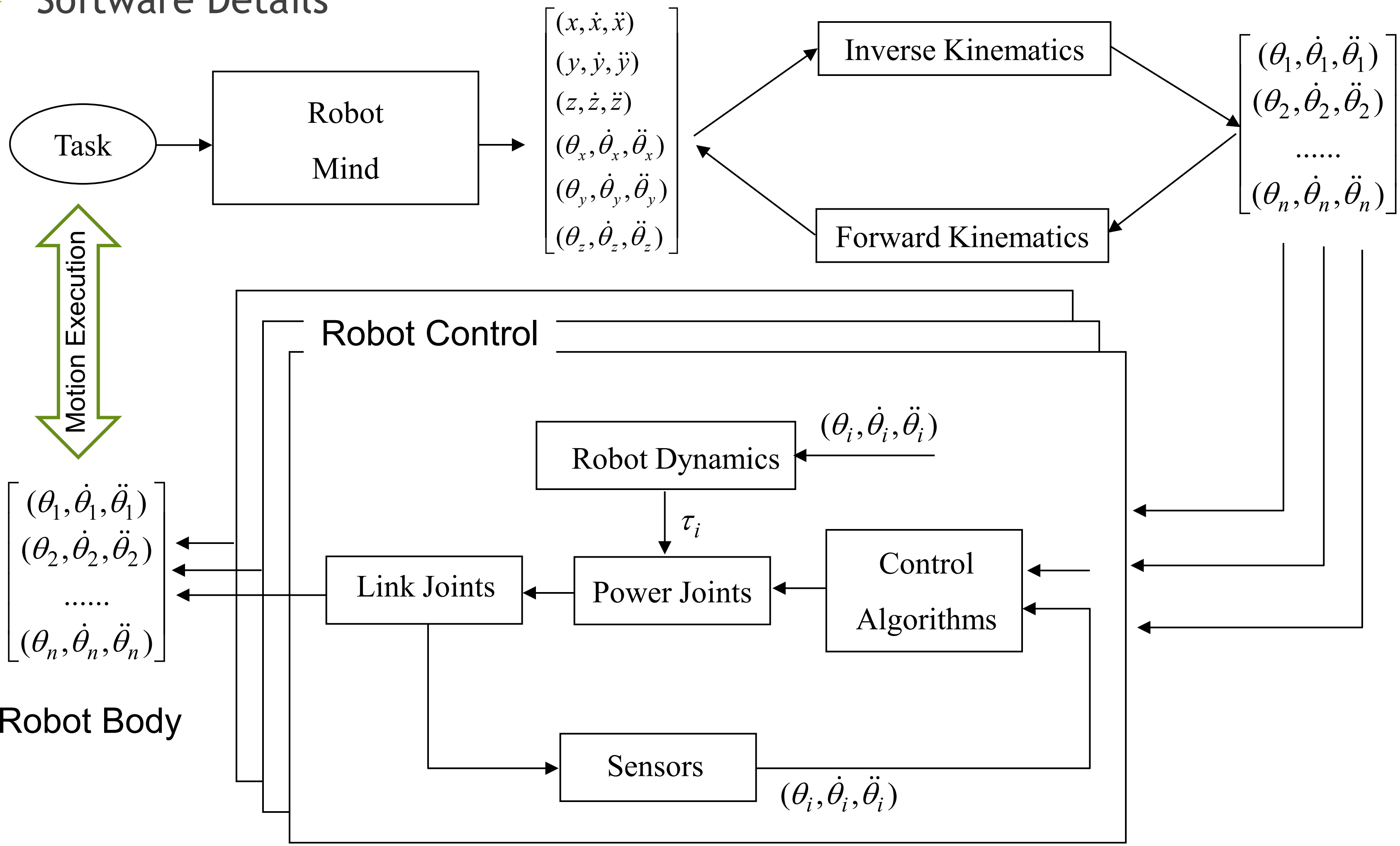
Robot's Control Systems

- Hardware Details



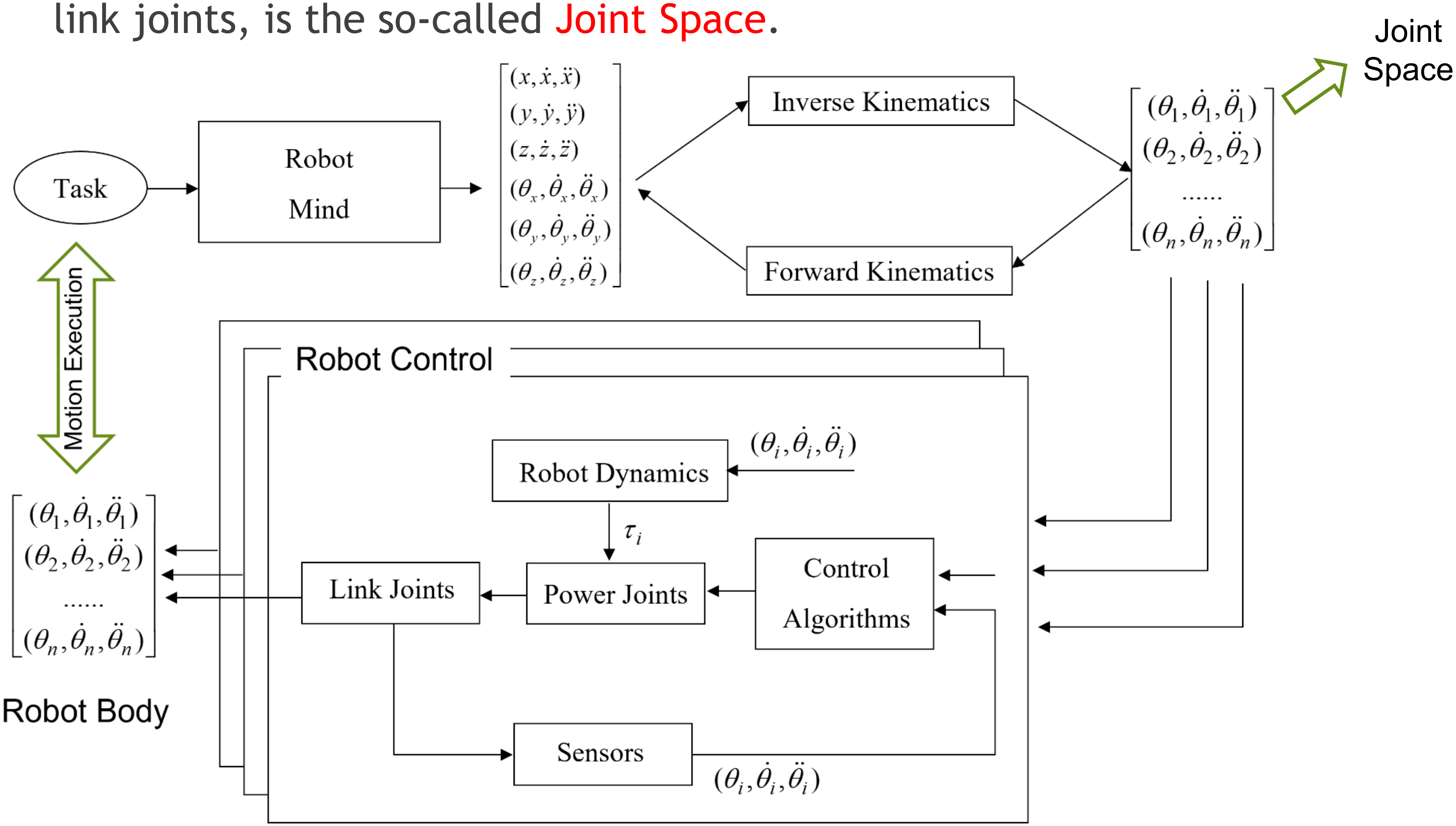
Robot's Control Systems

- Software Details



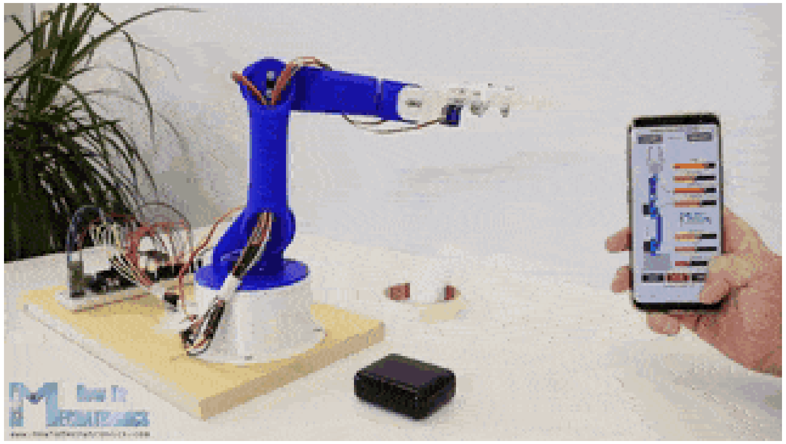
Definition of Joint Space

- The space, which is defined by the linear or angular positions of link joints, is the so-called Joint Space.

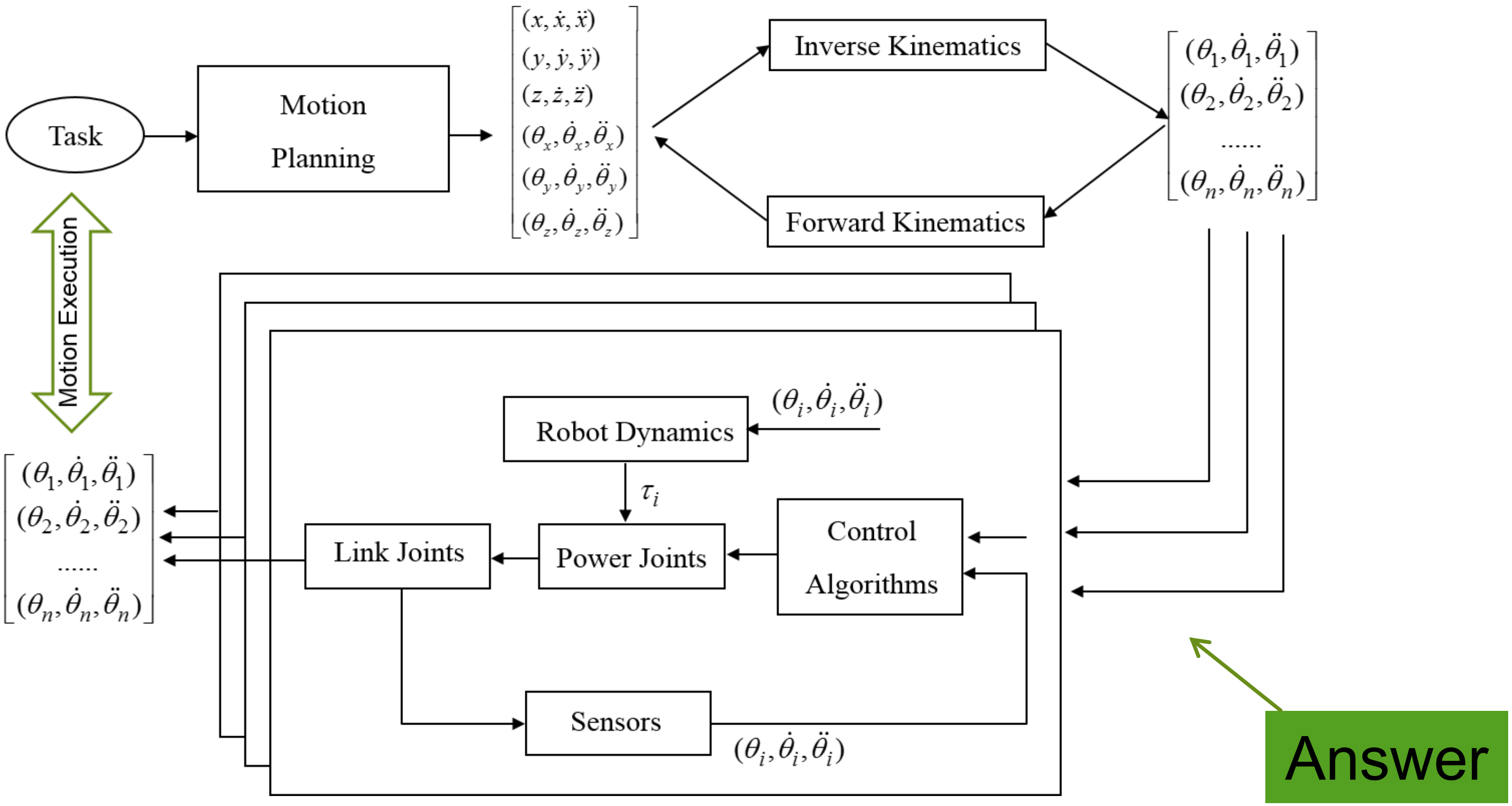


Outline of Lecture 4

- Joint Space
- Control Input
- Control Feedback
- Control with Known Dynamics
- Control with Unknown Dynamics

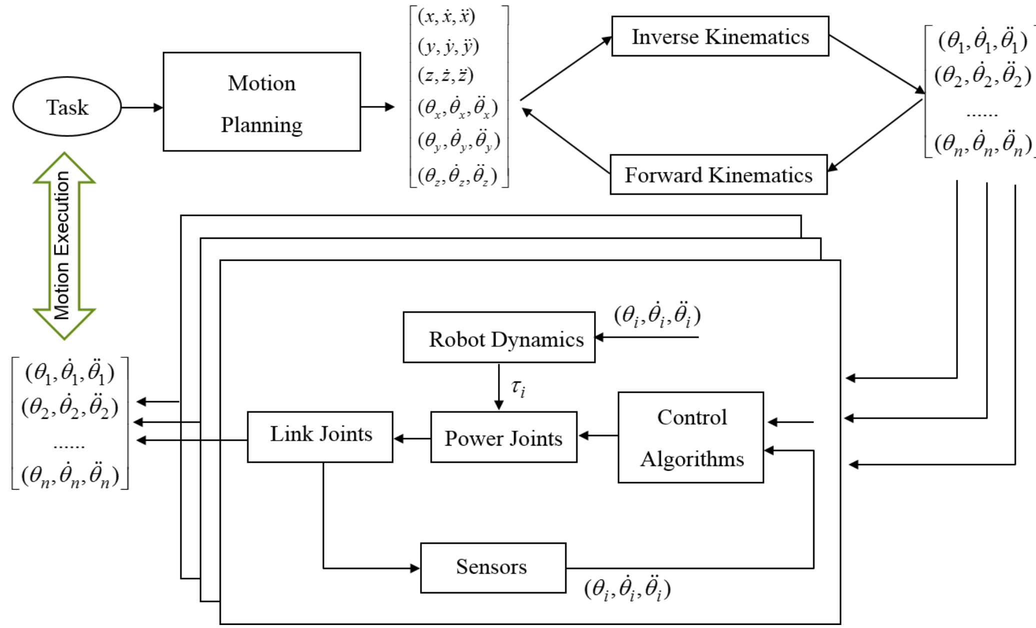


What are the controllable variables in joint space?



Controllable Variables in Joint Space

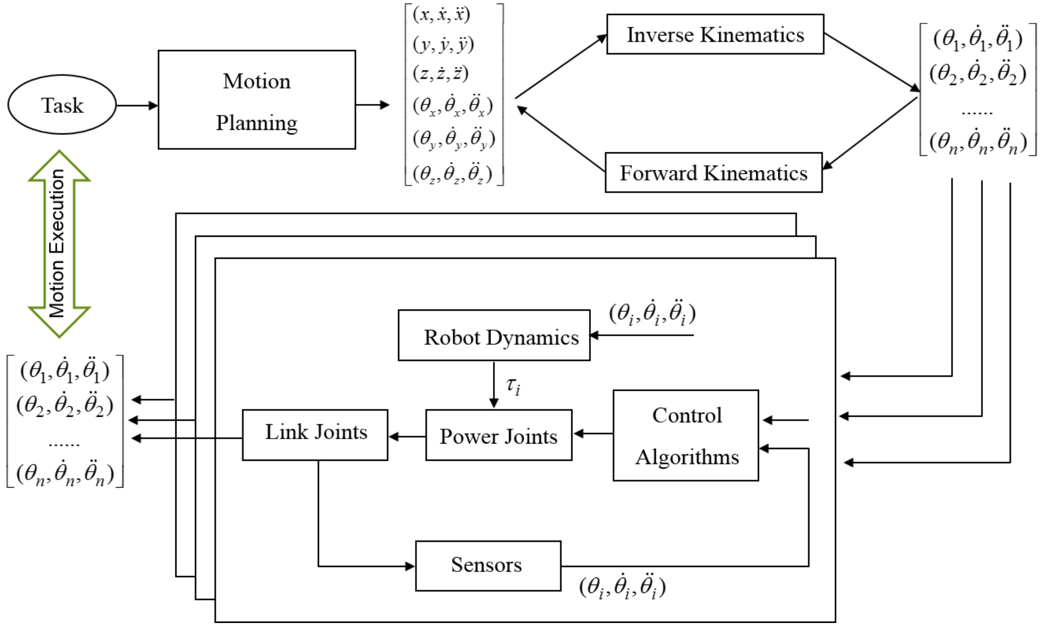
- Positions
- Velocities
- Forces/Torques (related to accelerations)



Next Question: How to determine desired output of Controllable Variables in Joint Space?

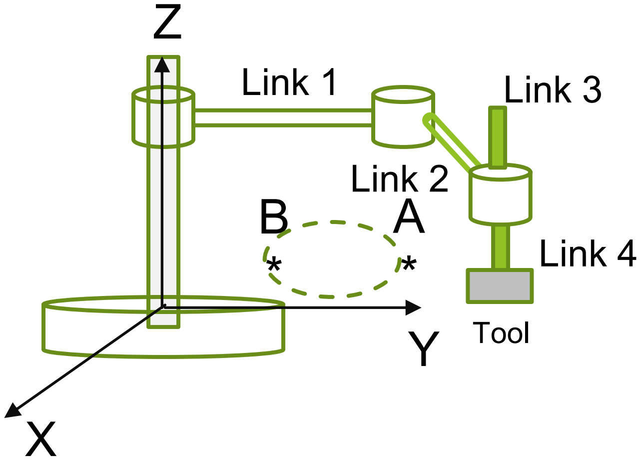
Answer:

- For Robot Manipulators:
  - Use of Inverse Kinematics
- For Robot Mobile Base:
  - Use of Inverse Kinematics



Example 1: Inverse Kinematics of Robot Manipulator with Two Links …

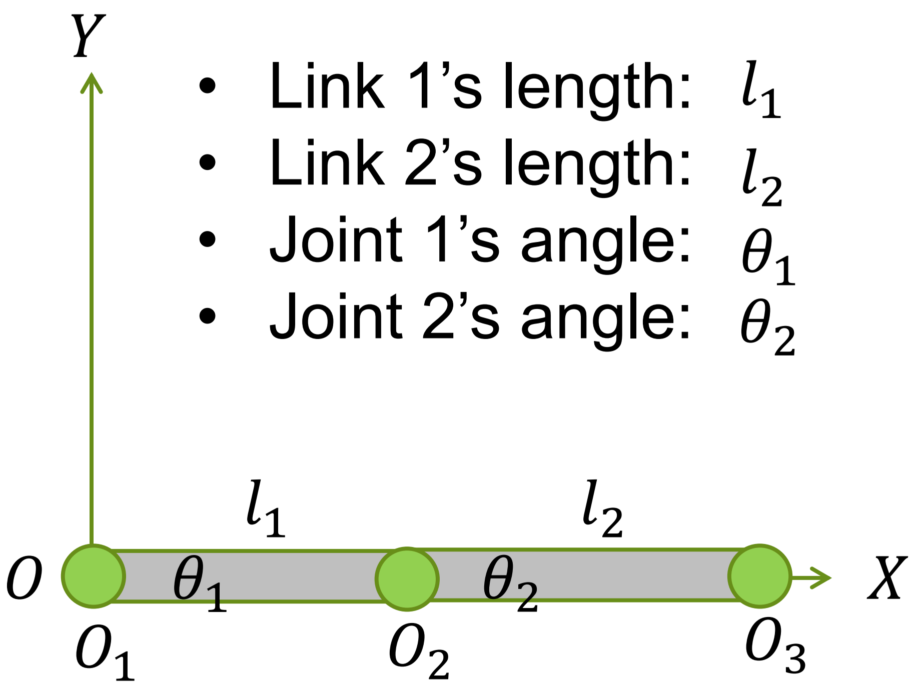
- A SCARA robot has four links. The first two links are revolute links. What should be the angles of joint 1 and joint 2 if the tool tip's coordinates are at (x(t), y(t), z(t))?

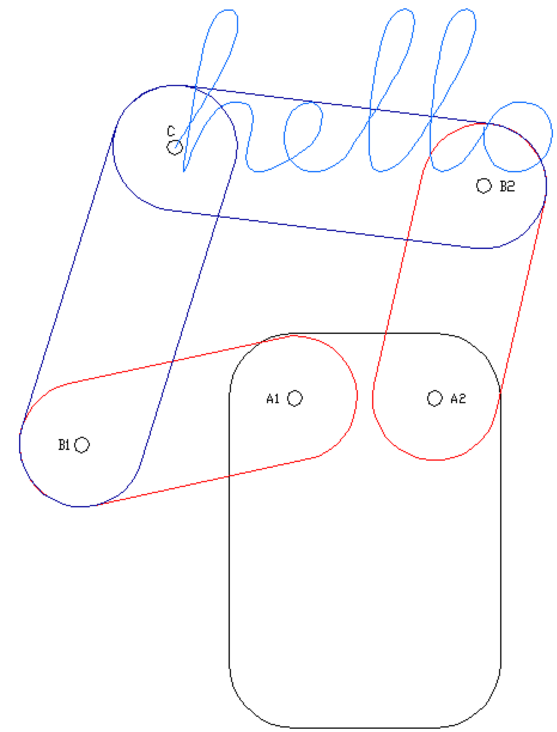


Solution

- Home positions of the first two links:

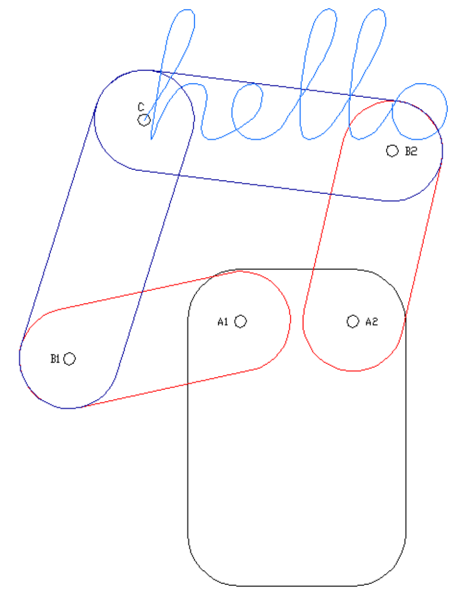
  - Link 1's length: l_1
  - Link 2's length: l_2
  - Joint 1's angle: θ_1
  - Joint 2's angle: θ_2

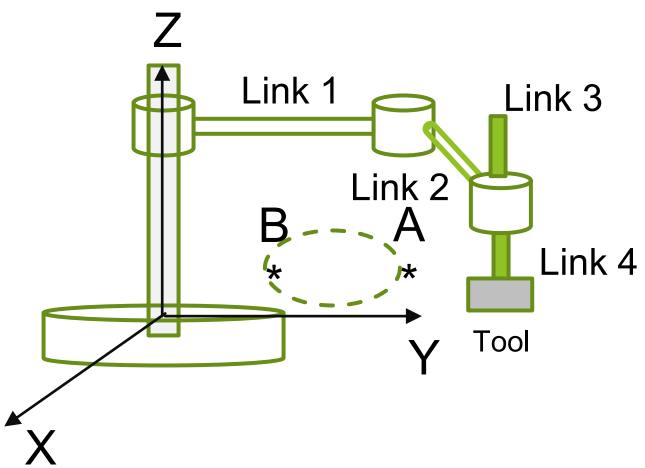


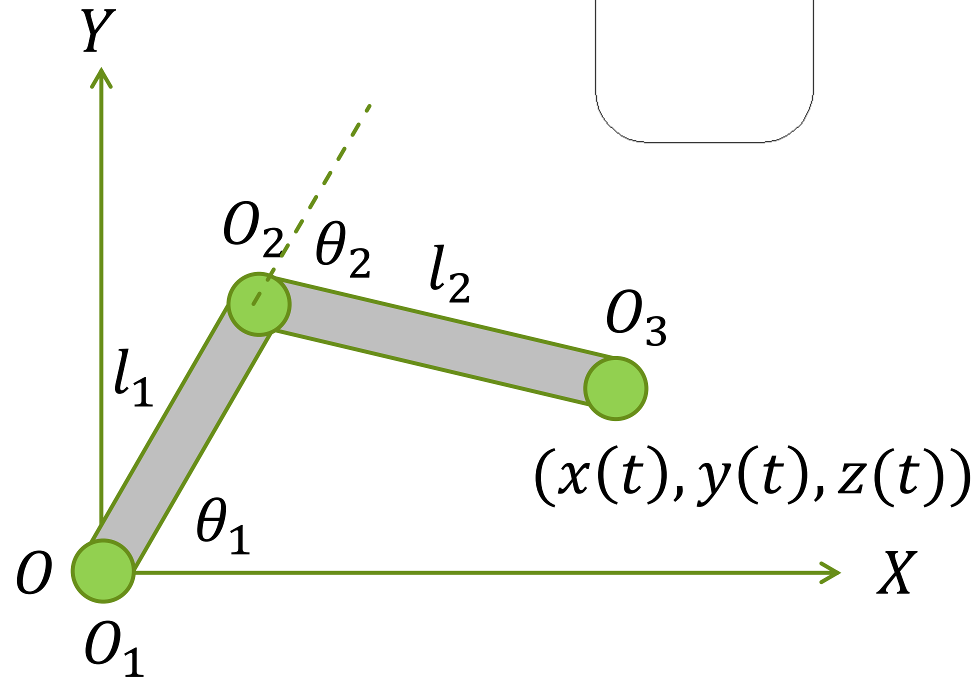


Solution (continued)

- Input of coordinates (x(t), y(t), z(t)) at O3:





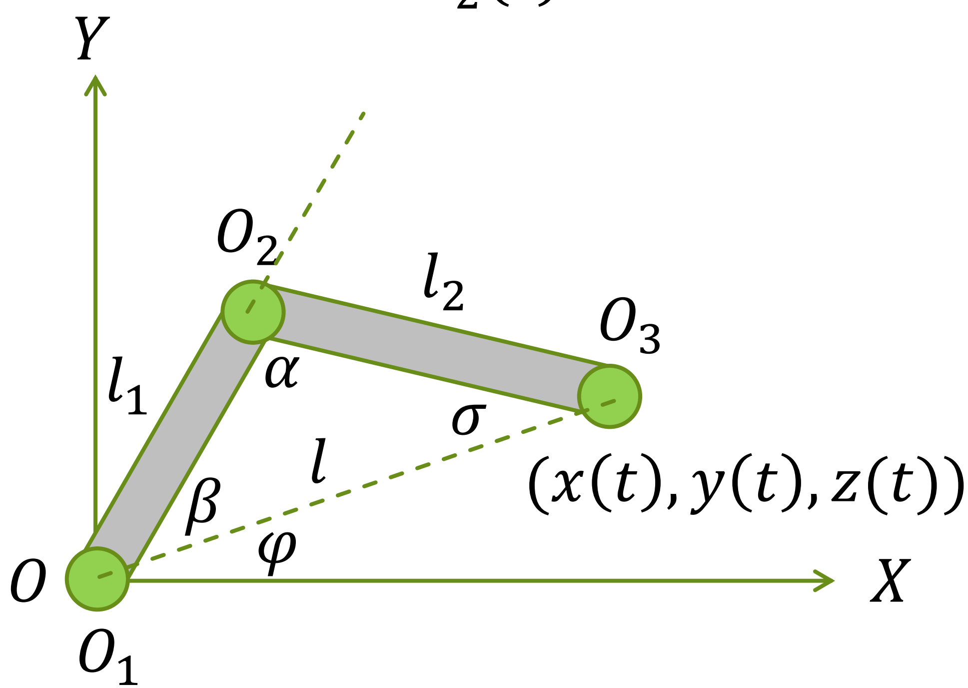


Solution (continued)

- Output of angles of link 1 and link 2:

$$\theta_1(t) = \beta + \varphi$$

$$\theta_2(t) = -\pi + \alpha$$



$$\alpha = \arccos\left(\frac{l_1^2 + l_2^2 - l^2}{2l_1 l_2}\right)$$

$$\beta = \arccos\left(\frac{l_1^2 + l^2 - l_2^2}{2l_1 l}\right)$$

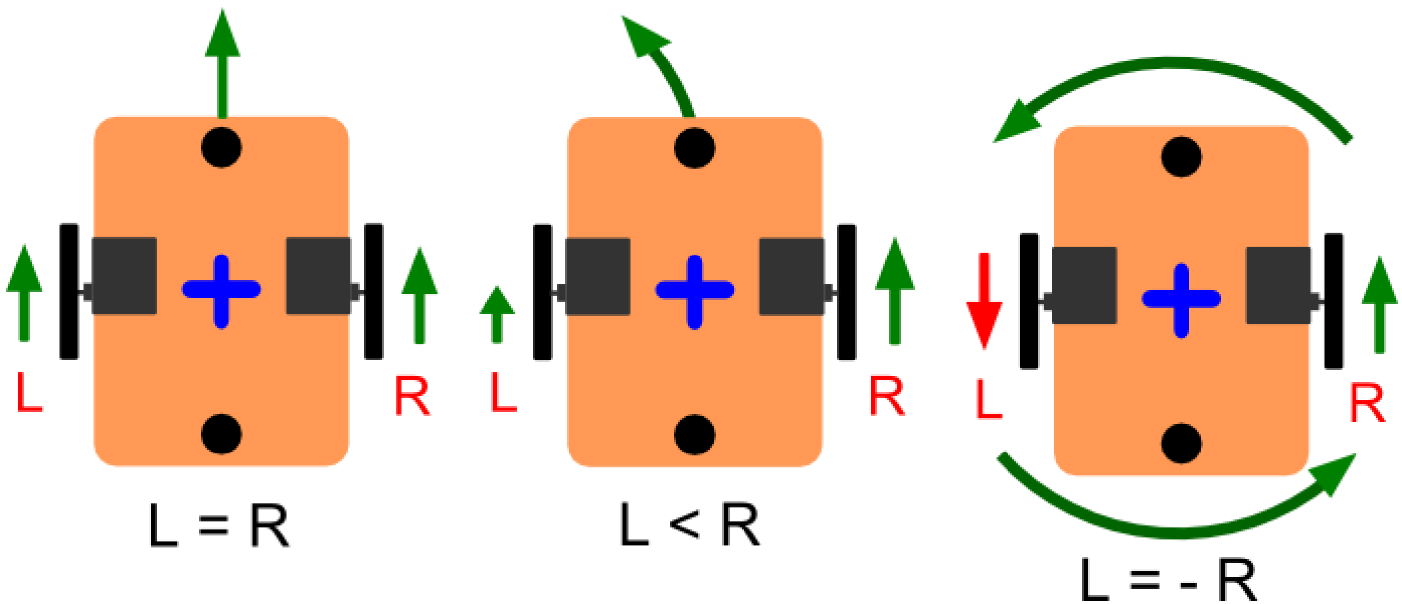
$$\sigma = \arccos\left(\frac{l_2^2 + l^2 - l_1^2}{2l_2 l}\right)$$

$$l = \sqrt{x(t)^2 + y(t)^2}$$

$$\varphi = \arctan\left(\frac{y(t)}{x(t)}\right)$$

Example 2: Inverse Kinematics of Mobile Robot with Two Differential-Drive Wheels

- How to transform desired motion in task space into desired motion in joint space? (Forward Kinematics + Inverse Kinematics)

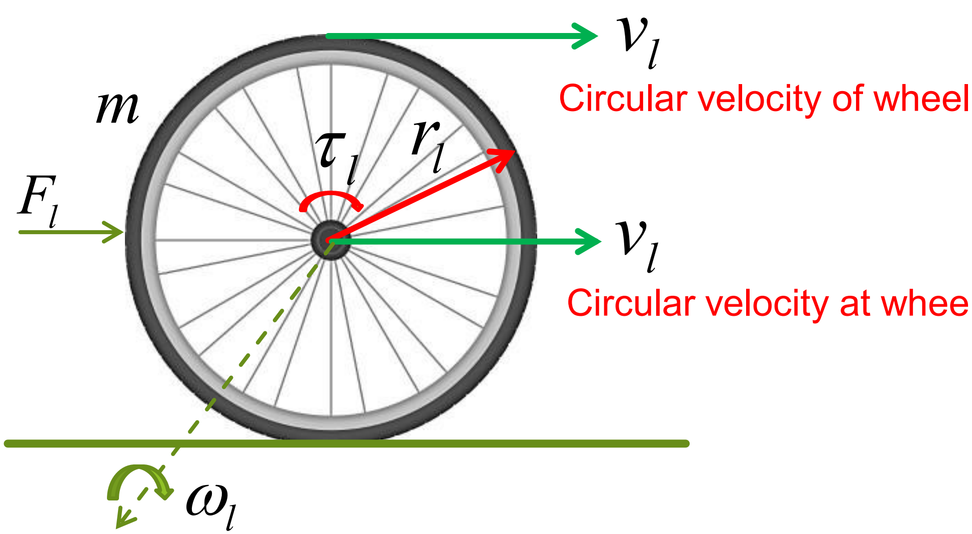


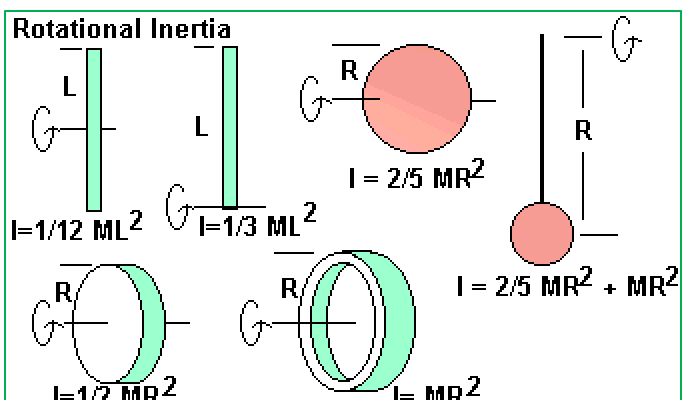
Pay attention to these variables and their meanings:

- Angular velocities of two wheels.
- Angular velocity of mobile robot.
- Circular velocities of two wheels due to wheels' angular velocities.
- Circular velocities at two wheels, which create robot's angular velocity.

Solution

- Equations of Wheels:





$$\tau_l = I\dot{\omega}_l = mr_l^2\dot{\omega}_l$$

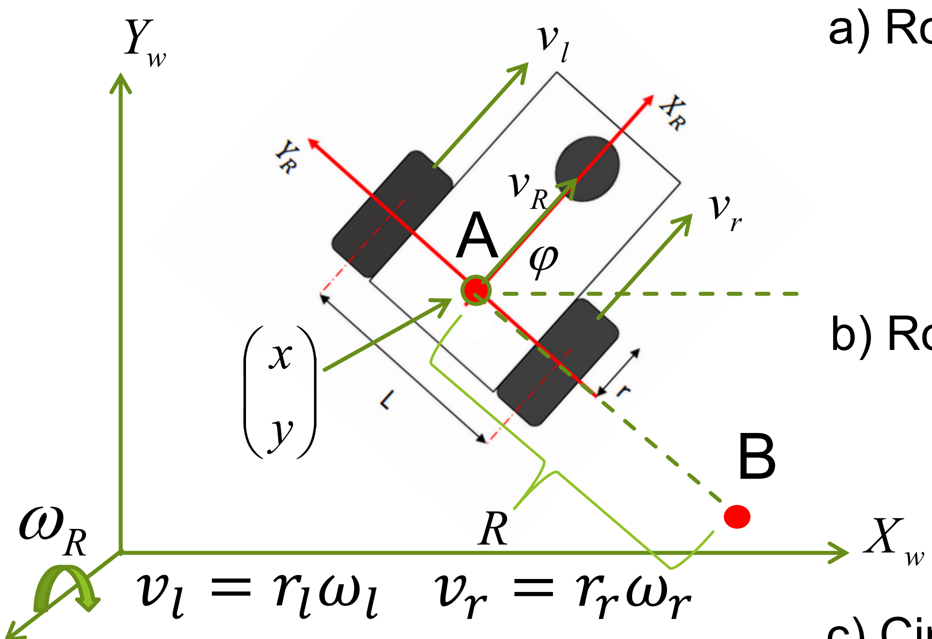
$$F_l = \frac{\tau_l}{r_l}$$

$$v_l = r_l\omega_l$$

$$\omega_l = \frac{v_l}{r_l}$$

Solution (continued)

- Equations of Mobile Robot:



Forward Kinematics

a) Robot rotates about Z axis passing through A:

$$\omega_R = \frac{1}{L} v_l - \frac{1}{L} v_r$$

b) Robot rotates about Z axis passing through B:

$$v_\mathrm{R} = \frac{1}{2}(v_l + v_r)$$

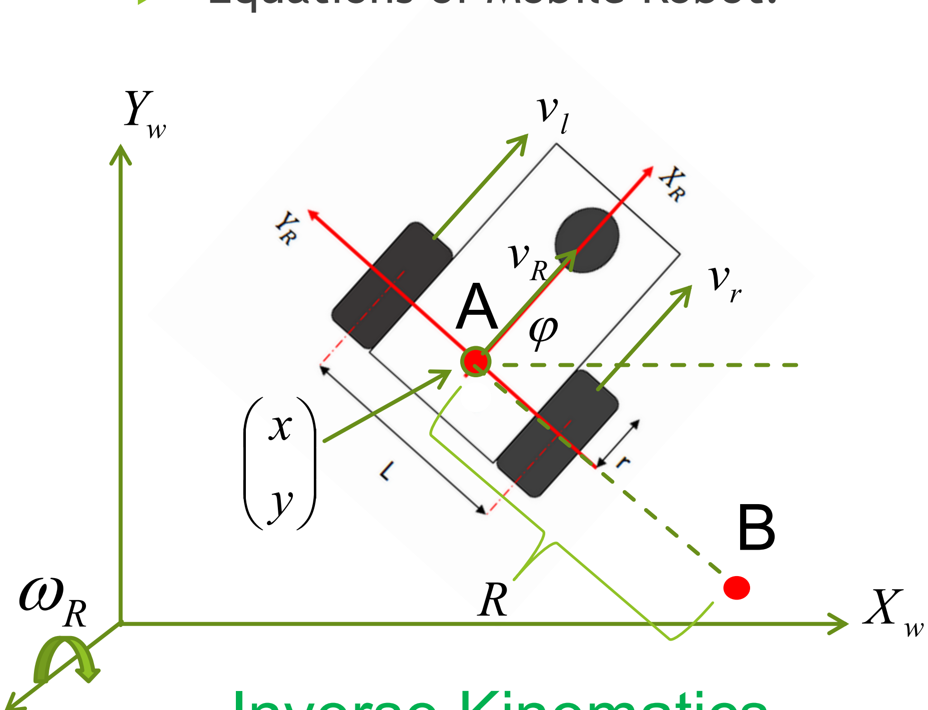
c) Circular velocities at wheels, will create the same circular velocity of robot:

$$\omega_R = \frac{v_l}{R + L/2} = \frac{v_\mathrm{R}}{R}$$

$$\omega_R = \frac{v_r}{R - L/2} = \frac{v_\mathrm{R}}{R}$$

Solution (continued)

- Equations of Mobile Robot:



$$\omega_R = \frac{v_l}{R + L/2} = \frac{v_R}{R}$$

$$\omega_R = \frac{v_r}{R - L/2} = \frac{v_R}{R}$$

$$\begin{cases} v_l = \frac{1}{2}(2v_R + L \times \omega_R) \\ v_r = \frac{1}{2}(2v_R - L \times \omega_R) \end{cases}$$

Inverse Kinematics

$$\omega_l = \frac{v_l}{r_l} \qquad \omega_r = \frac{v_r}{r_r}$$

Solution (continued)

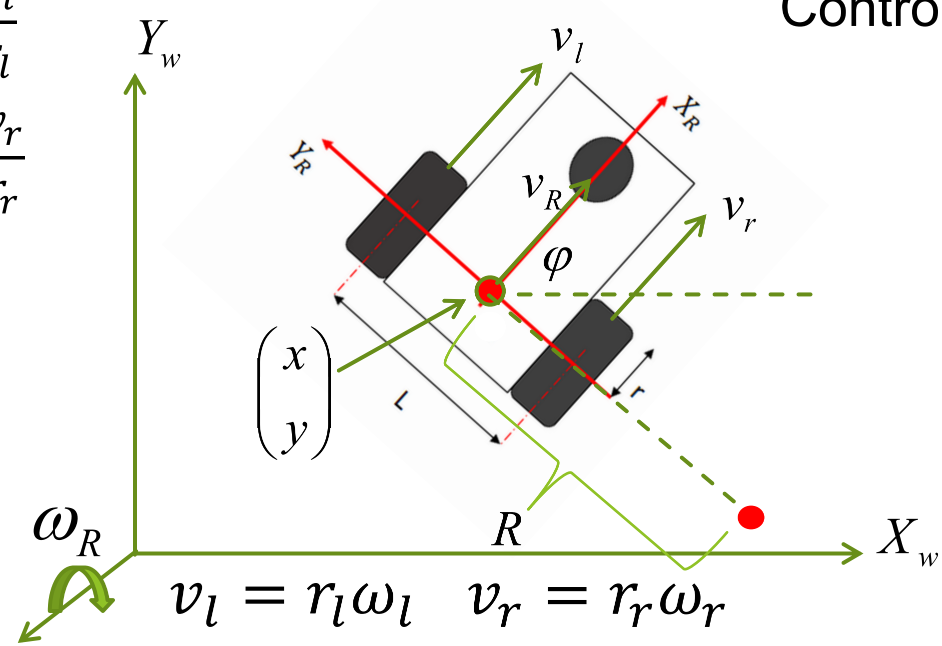
- Desired Output of Robot's Error Control Systems:

Input to Control Loops

$$\begin{cases} v_l = \frac{1}{2}(2v_R + L \times \omega_R) \\ v_r = \frac{1}{2}(2v_R - L \times \omega_R) \end{cases}$$

$$\omega_l = \frac{v_l}{r_l} \qquad \omega_r = \frac{v_r}{r_r}$$

Control



$$v_l = r_l \omega_l \qquad v_r = r_r \omega_r$$

$$\begin{pmatrix} x \\ y \\ \varphi \end{pmatrix}$$

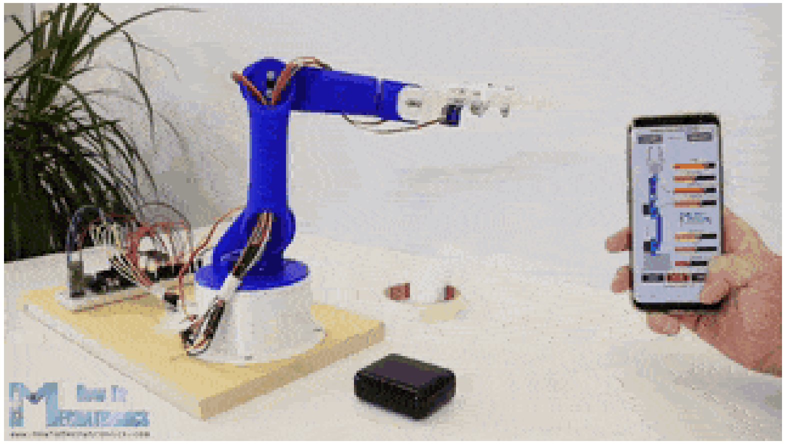
Results from Motion Planning

$$\begin{cases} \dot{x} = v_\mathrm{R} \cos(\phi) \\ \dot{y} = v_\mathrm{R} \sin(\phi) \\ \dot{\phi} = \omega_R = \frac{v_\mathrm{R}}{R} \end{cases}$$

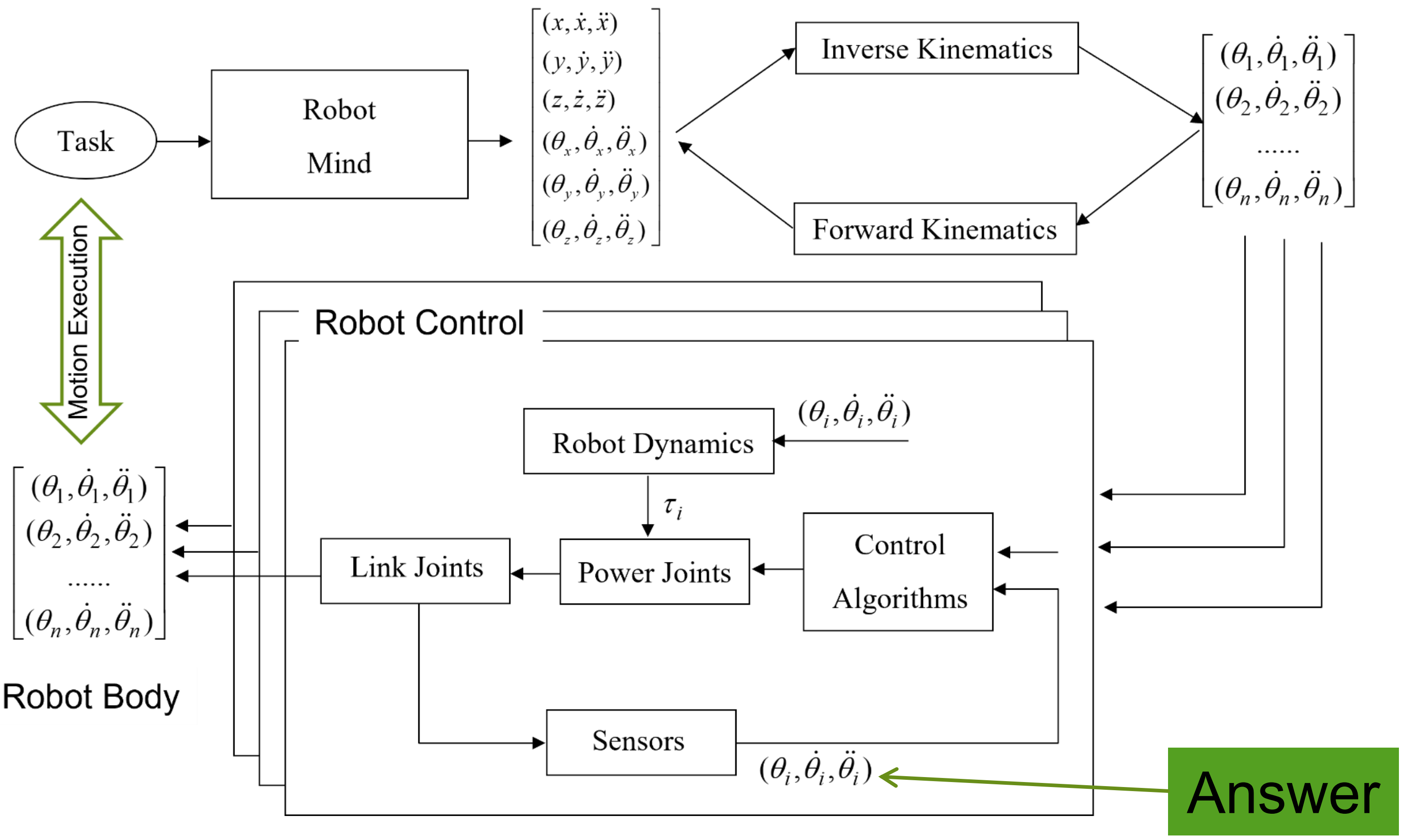
$$\begin{pmatrix} x^d \\ y^d \\ \varphi^d \end{pmatrix}$$ Desired Trajectories in Task Space

Outline of Lecture 4

- Joint Space
- Control Input
- Control Feedback
- Control with Known Dynamics
- Control with Unknown Dynamics

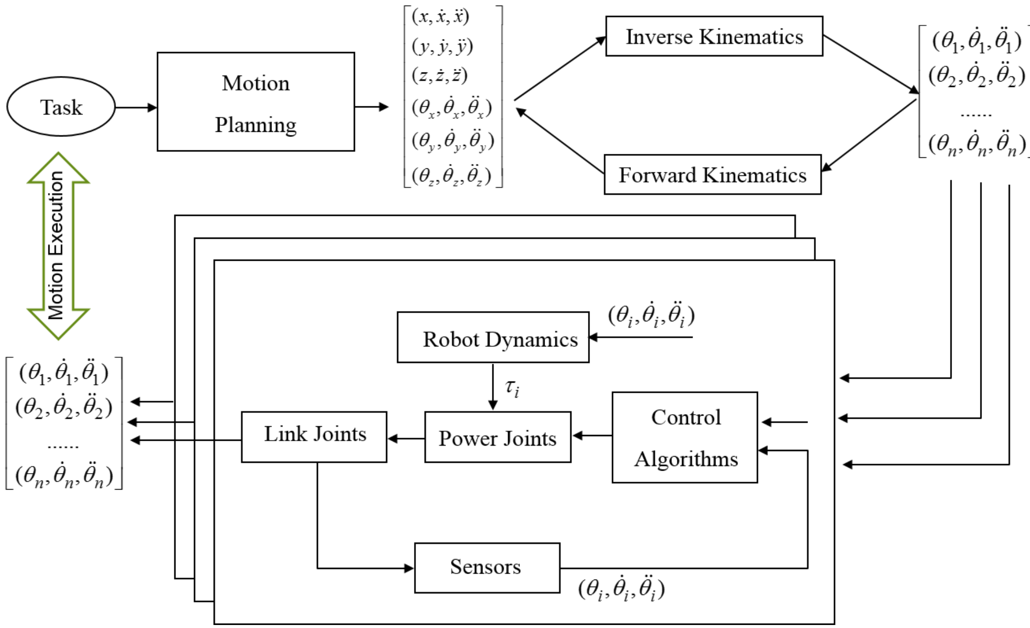


What are the observable variables in joint space?



Observable Variables in Joint Space

- Positions
- Velocities
- Forces/Torques (related to accelerations)



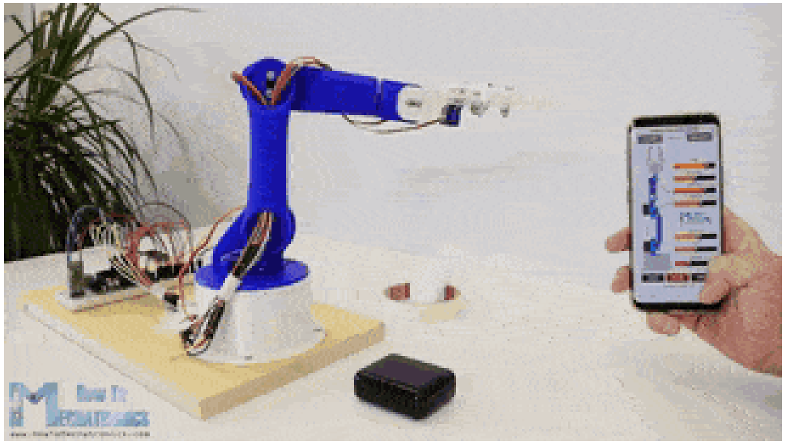
Example of Joint Space Control

How to construct the error control systems in robots?

- There are two control schemes:
  - Control with Known Dynamics
  - Control with Unknown Dynamics

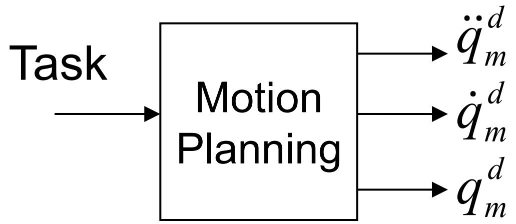
Outline of Lecture 4

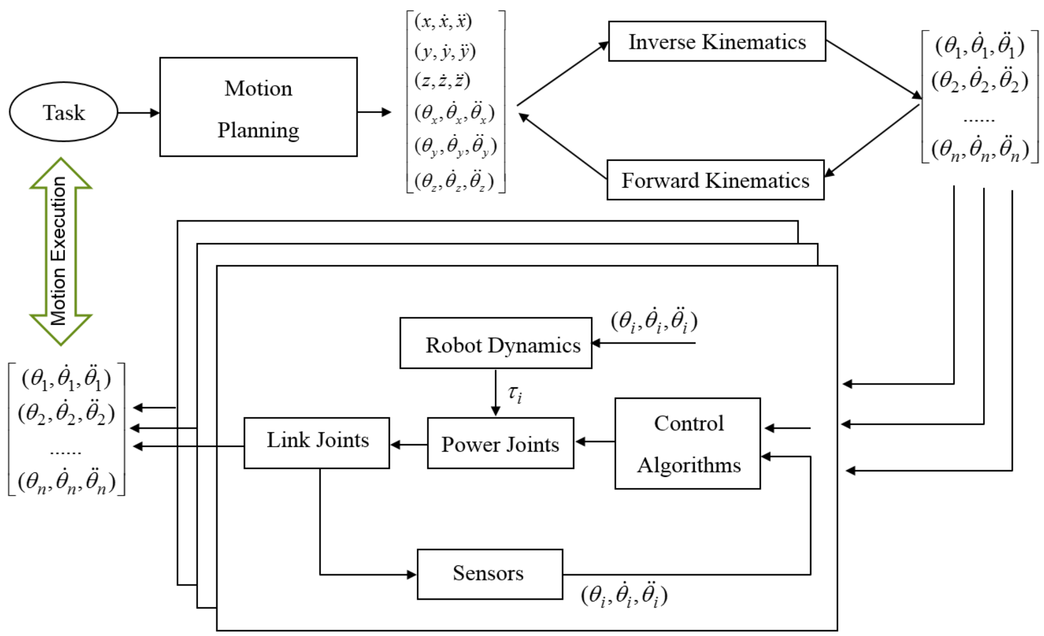
- Joint Space
- Control Input
- Control Feedback
- Control with Known Dynamics
- Control with Unknown Dynamics



Control with Known Dynamics

- Step 1: We can determine desired motions from desired tasks.





Control with Known Dynamics (continued)

- Step 2: We can determine desired torques from desired motions.

$$\tau_m^d = K_r^{-1} B_{diag} K_r^{-1} \ddot{q}_m^d + d^d$$

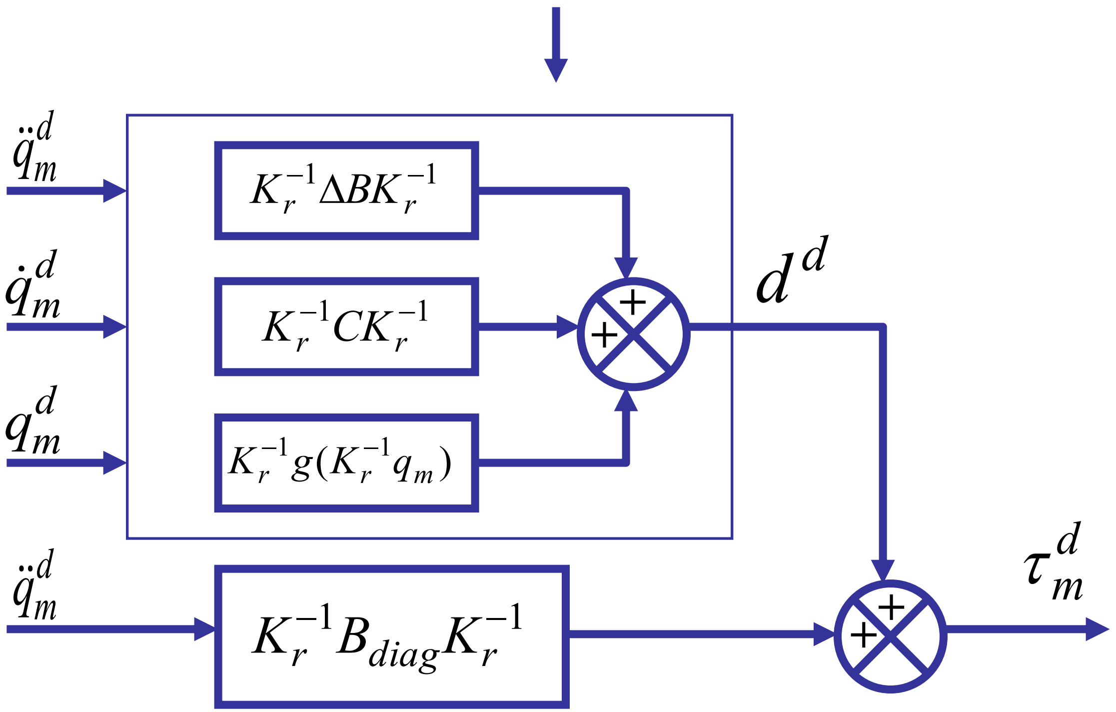
with :

$$d^d = K_r^{-1} \Delta B K_r^{-1} \ddot{q}_m^d + K_r^{-1} C K_r^{-1} \dot{q}_m^d + K_r^{-1} g(K_r^{-1} q_m^d)$$

$$\tau_m^d = K_r^{-1} B_{diag} K_r^{-1} \ddot{q}_m^d + d^d$$

with :

$$d^d = K_r^{-1} \Delta B K_r^{-1} \ddot{q}_m^d + K_r^{-1} C K_r^{-1} \dot{q}_m^d + K_r^{-1} g(K_r^{-1} q_m^d)$$



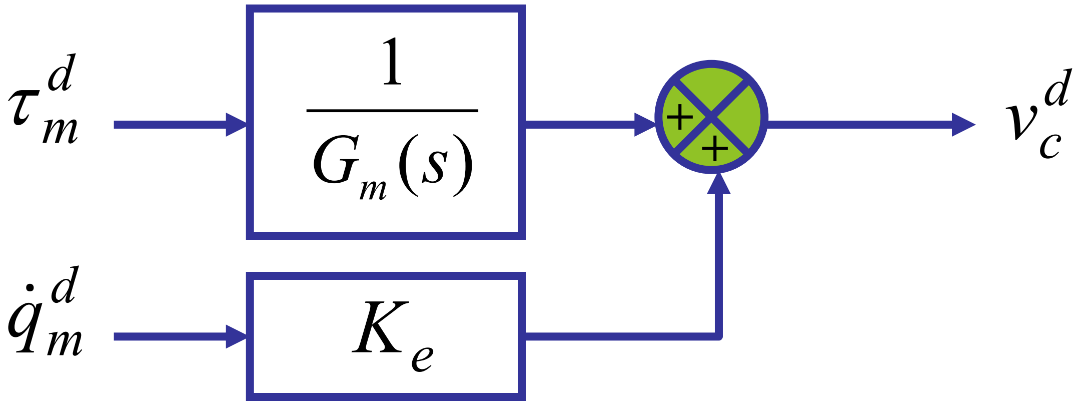
Control with Known Dynamics (continued)

- Step 3: We can determine desired control signals from desired torque.

$$\tau^{d}{}_{m}(s) = G_m(s)\{v^{d}{}_{c}(s) - K_e \dot{q}^{d}{}_{m}(s)\}$$

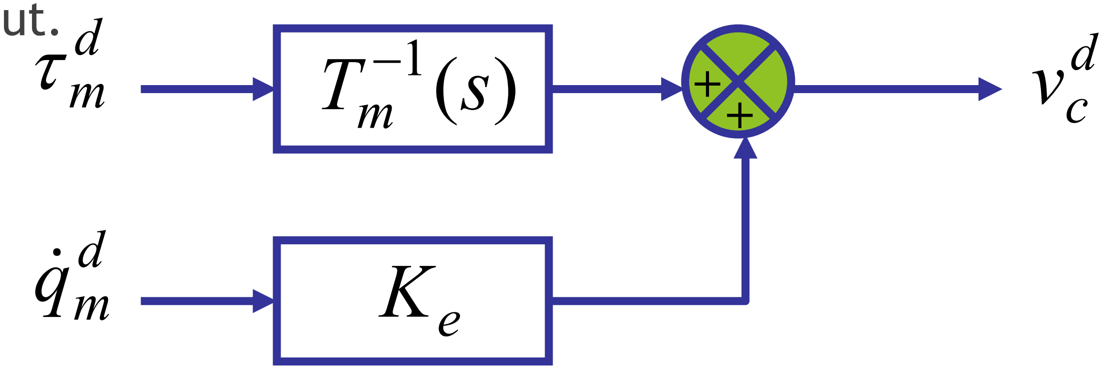
$$v^{d}{}_{c}(s) = \frac{1}{G_m(s)}\tau^{d}{}_{m}(s) + K_e \dot{q}^{d}{}_{m}(s)$$

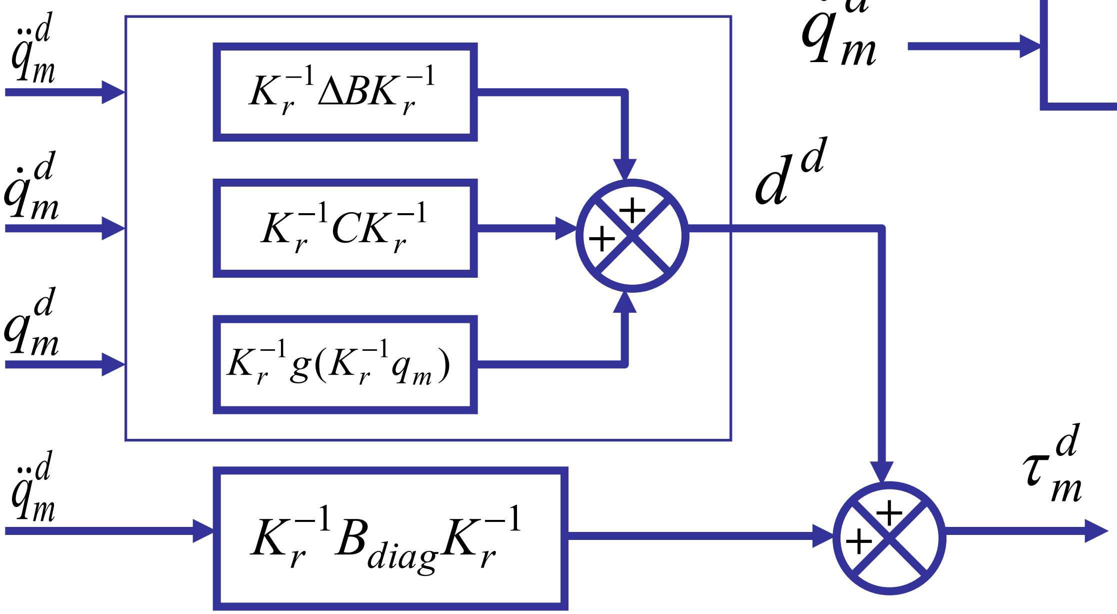
$$v_c^d(s) = \frac{1}{G_m(s)} \tau_m^d(s) + K_e \dot{q}_m^d(s)$$



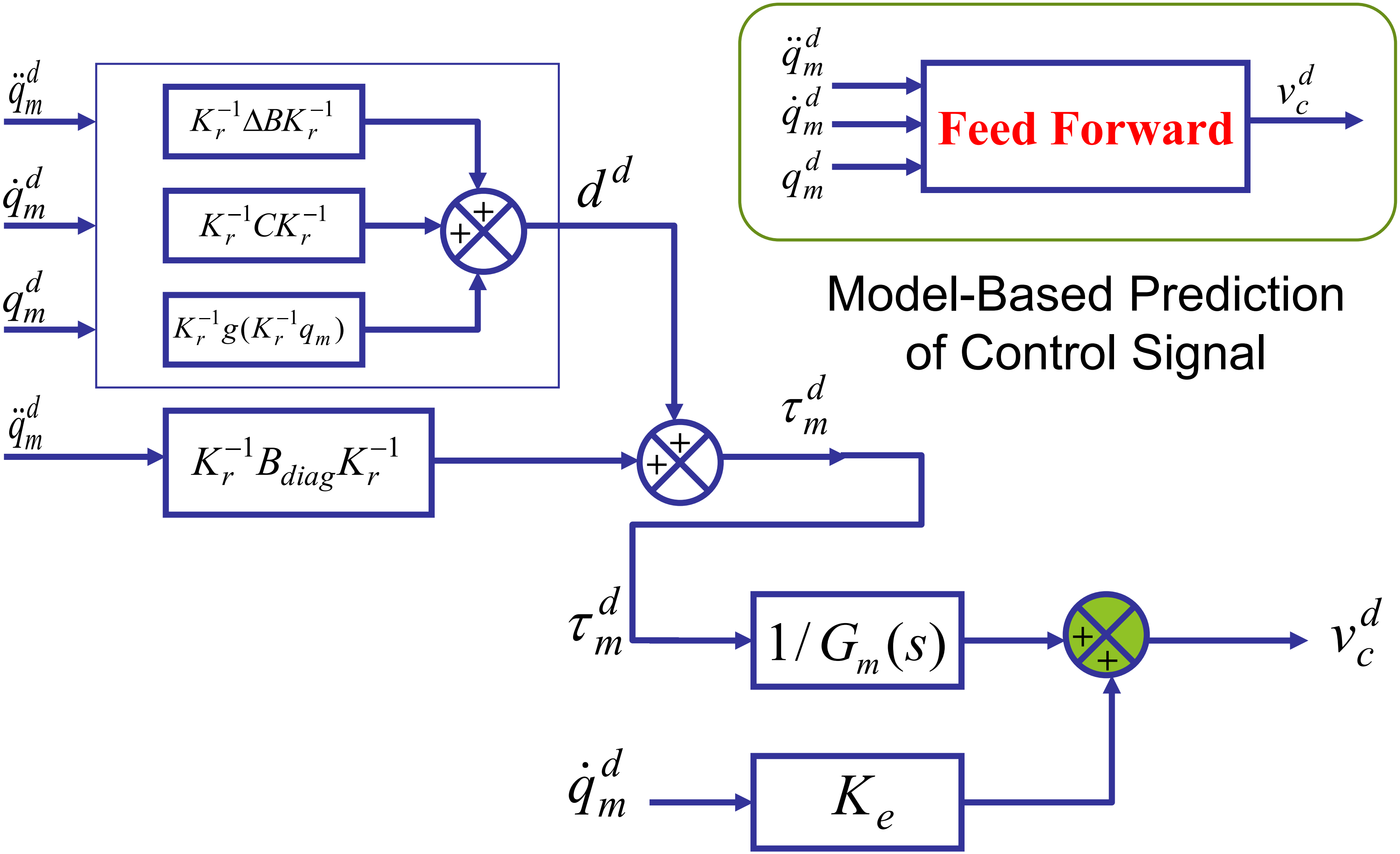
Control with Known Dynamics (continued)

- Step 4: In other words, we can determine desired control signals from desired motions at the input.



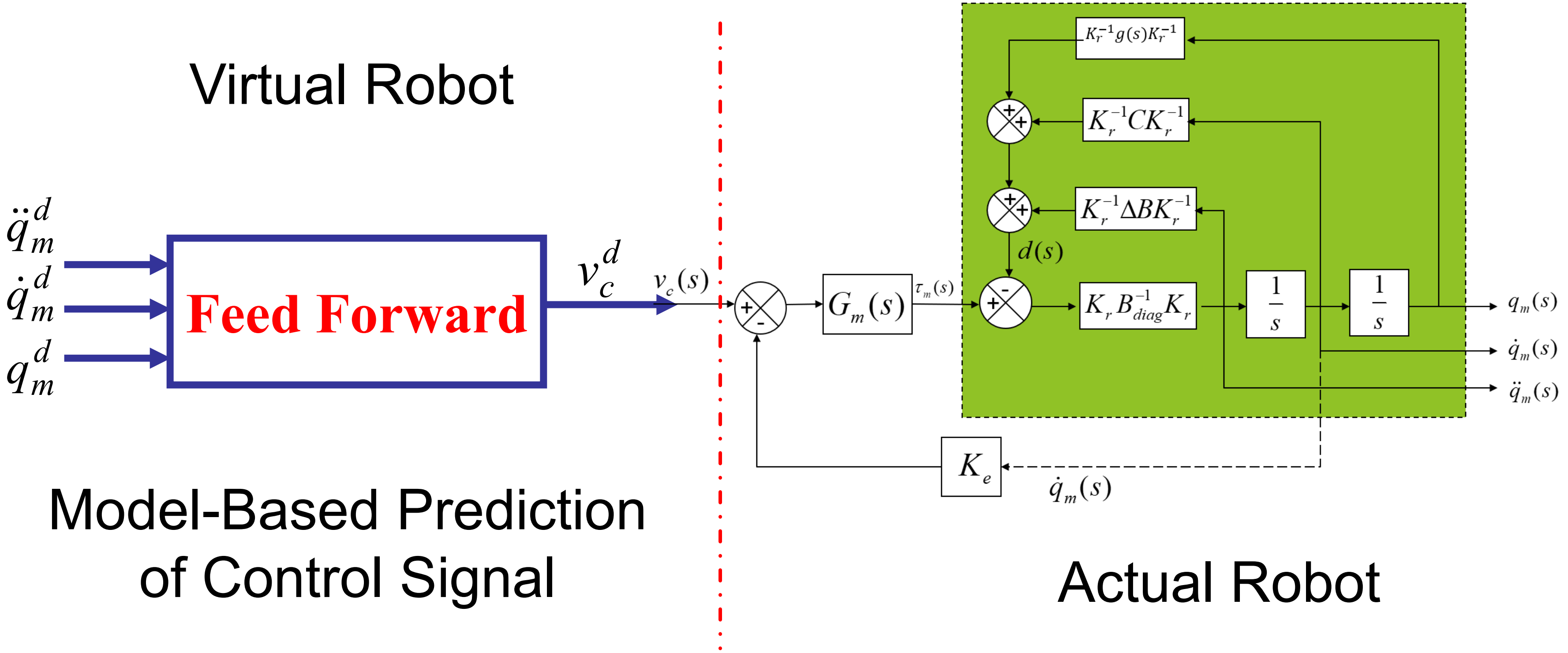


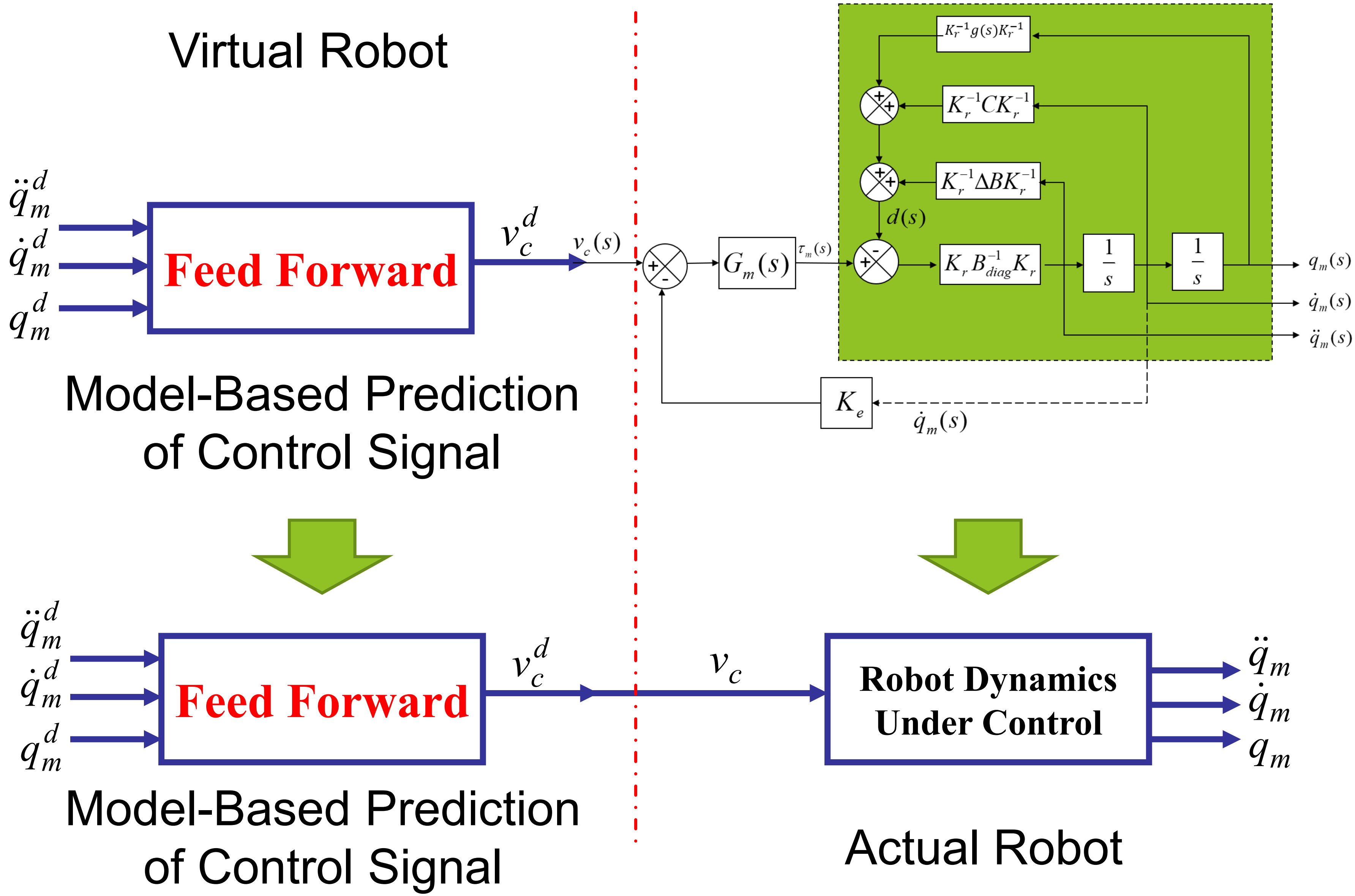
Model-Based Prediction of Control Signal



Control with Known Dynamics (continued)

- Step 5: Then, we can use such desired control signals to produce feedforward compensation.

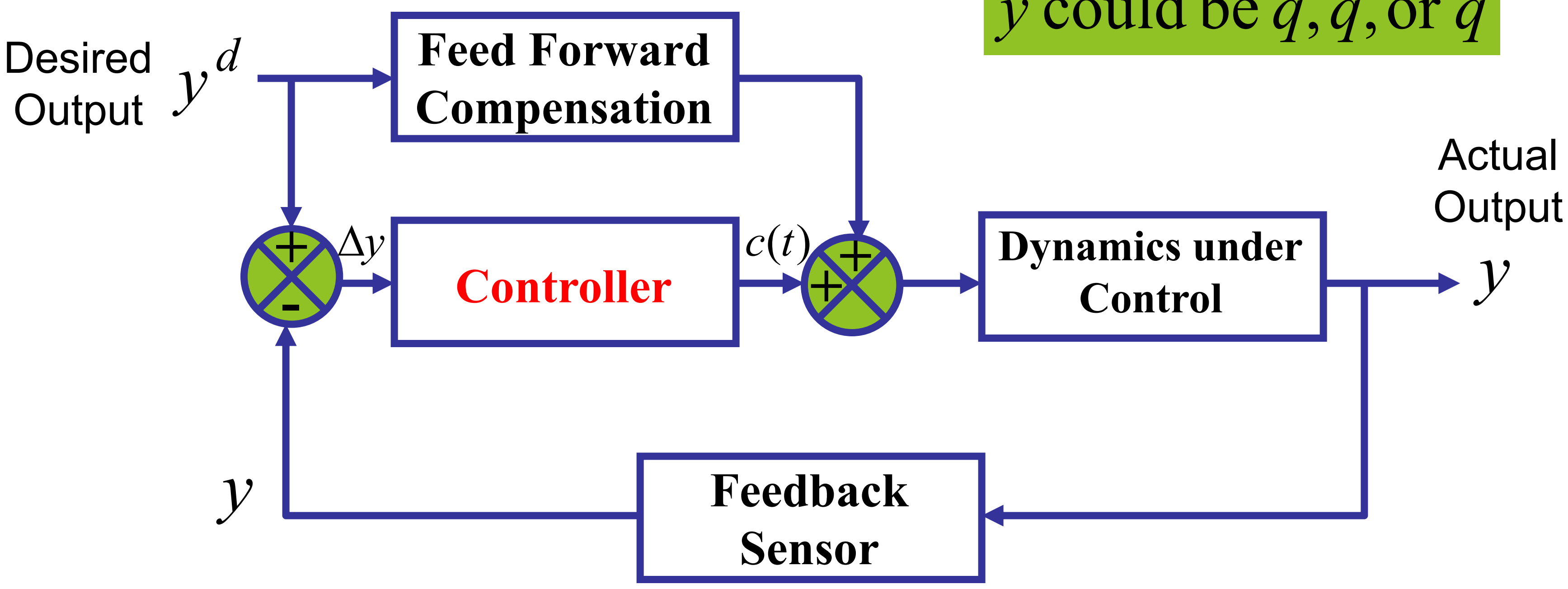



Virtual Robot
Feed Forward
Model-Based Prediction
of Control Signal
Feed Forward
Model-Based Prediction
of Control Signal
Robot Dynamics
Under Control
Actual Robot

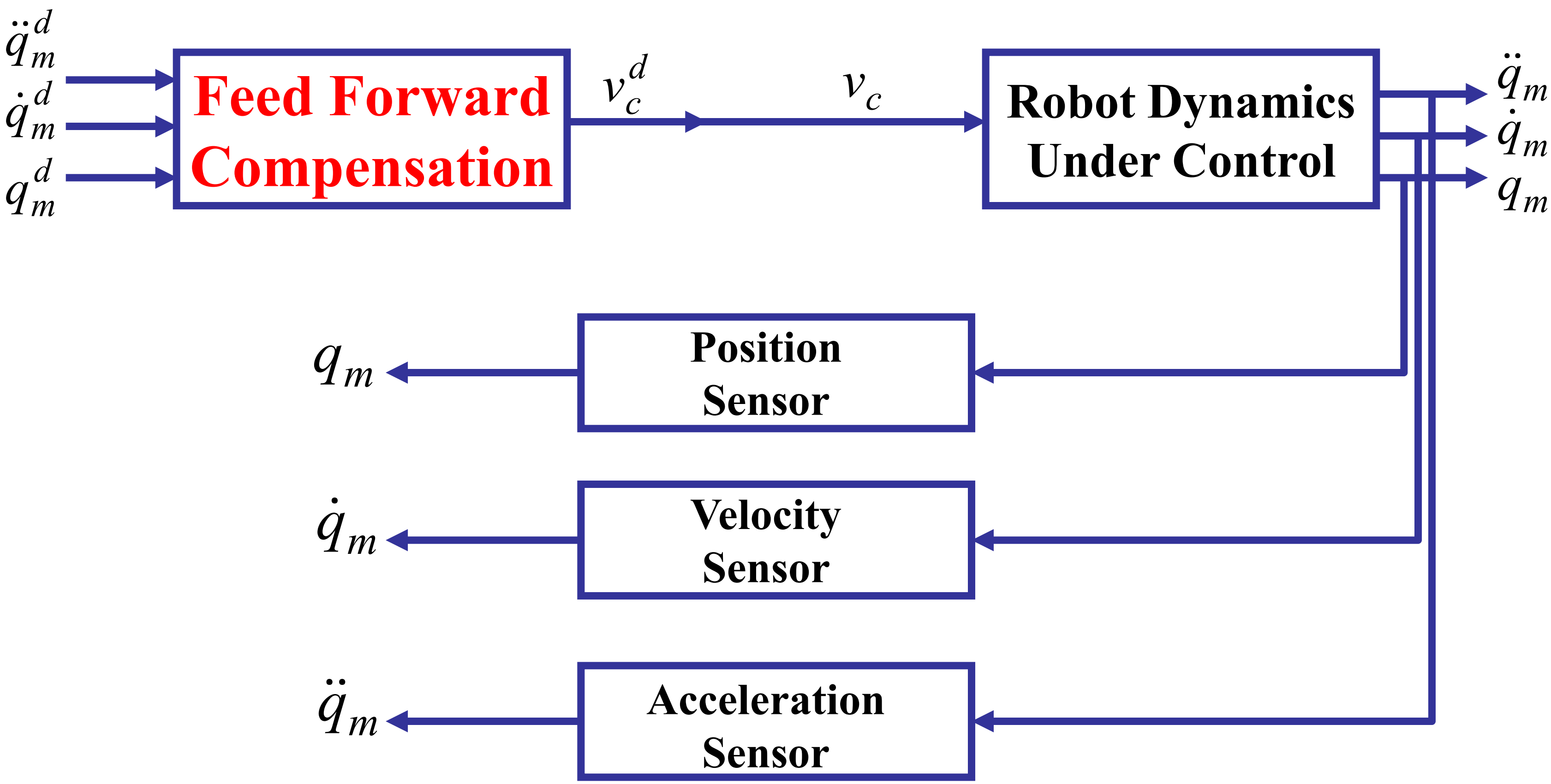
Control with Known Dynamics (continued)

- Step 6: Finally, we can come out the design of the error control system with feedforward compensation.

y could be $q, \dot{q}$, or $\ddot{q}$



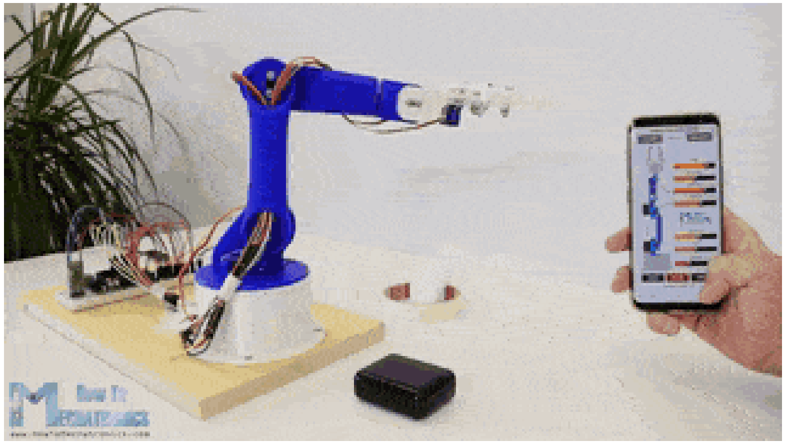
Types of Sensory Feedback



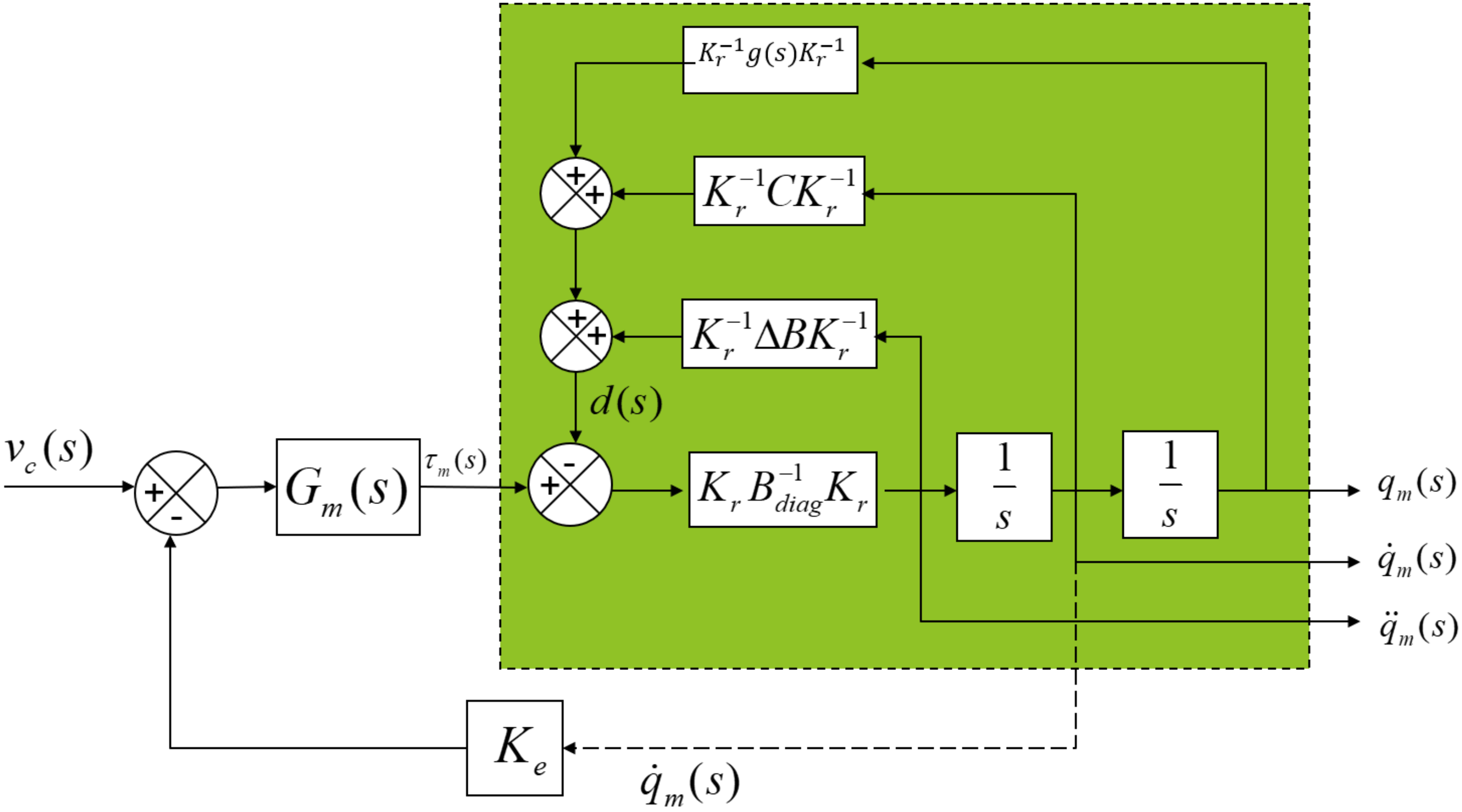
Example of Joint Space Control

Outline of Lecture 4

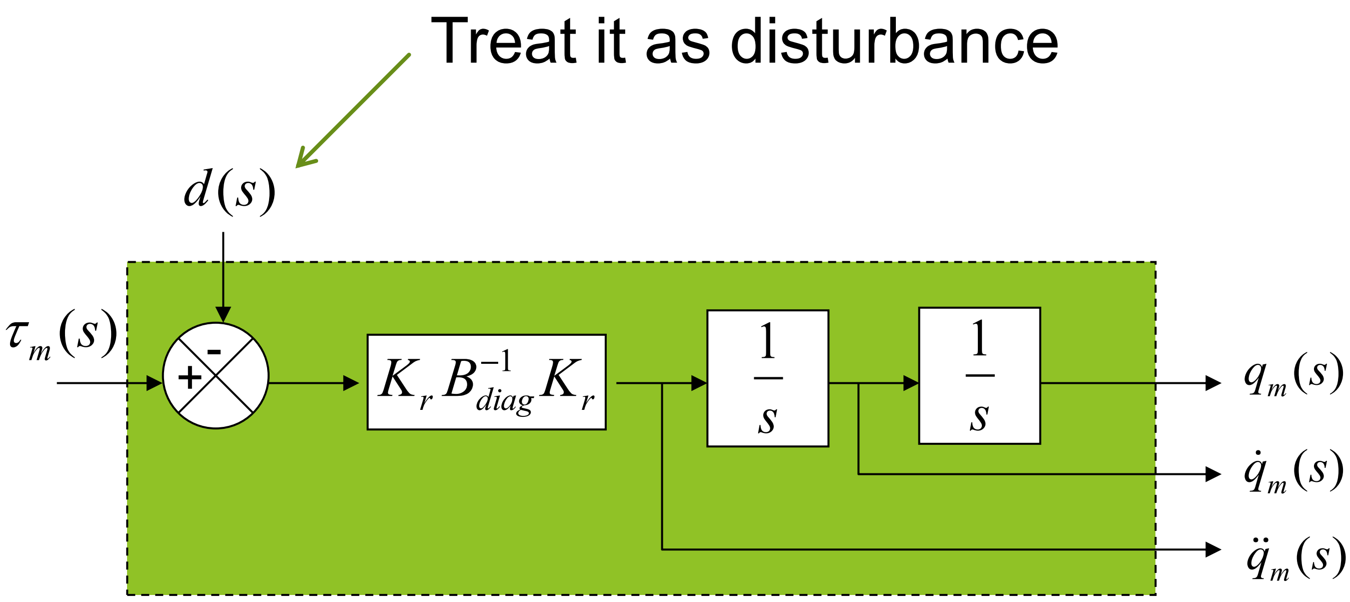
- Joint Space
- Control Input
- Control Feedback
- Control with Known Dynamics
- Control with Unknown Dynamics



Robot's Dynamics under Control

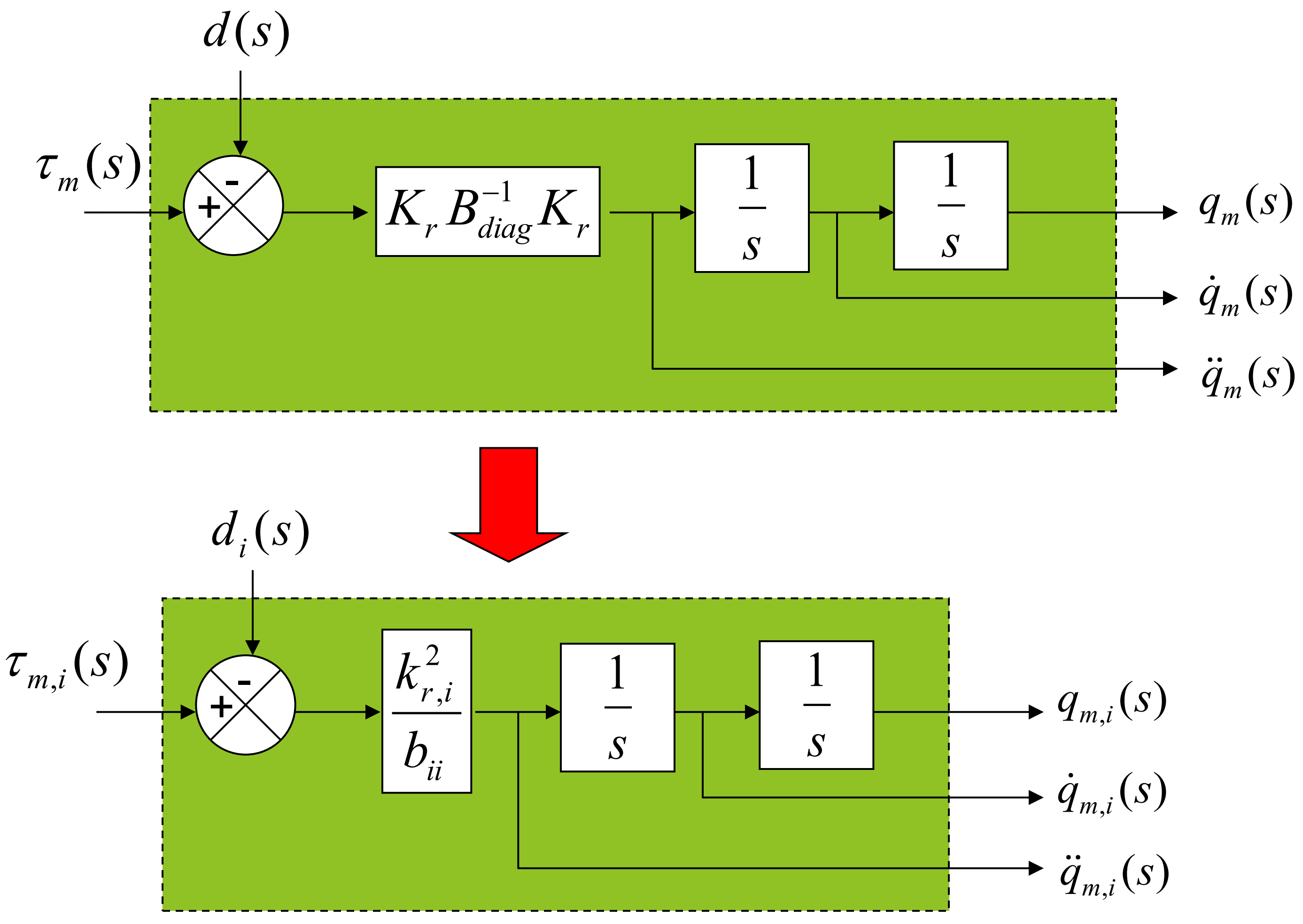


If reduction ratios are big enough,



MIMO = Sum of SISOs

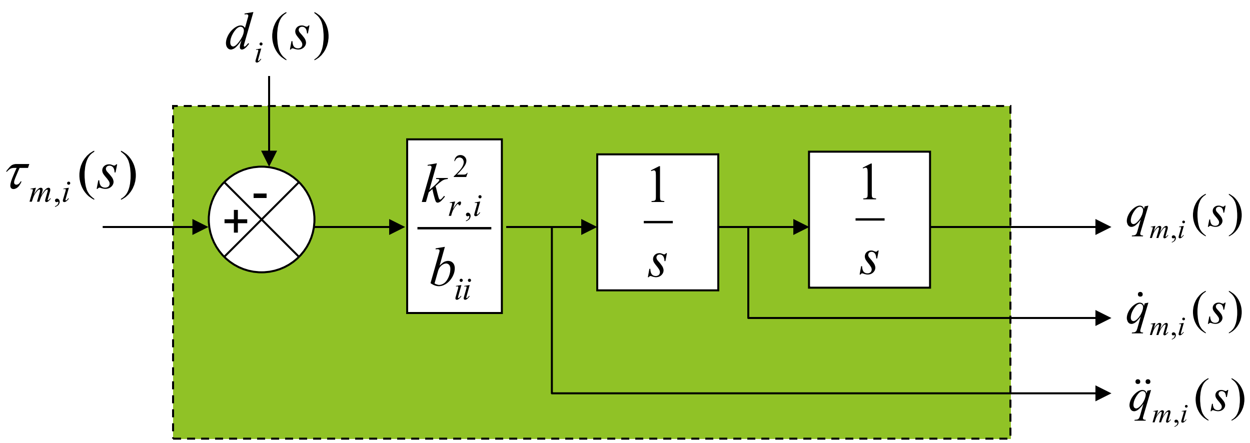
Decoupling Dynamics at Torque Joints



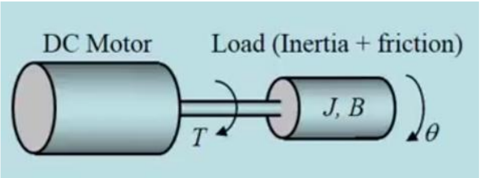
$i = 1,2,3,\ldots,n$

How to Control a Robot Arm with Unknown Dynamics?

- Step 1: We start with the independent unknown dynamics at torque joints.

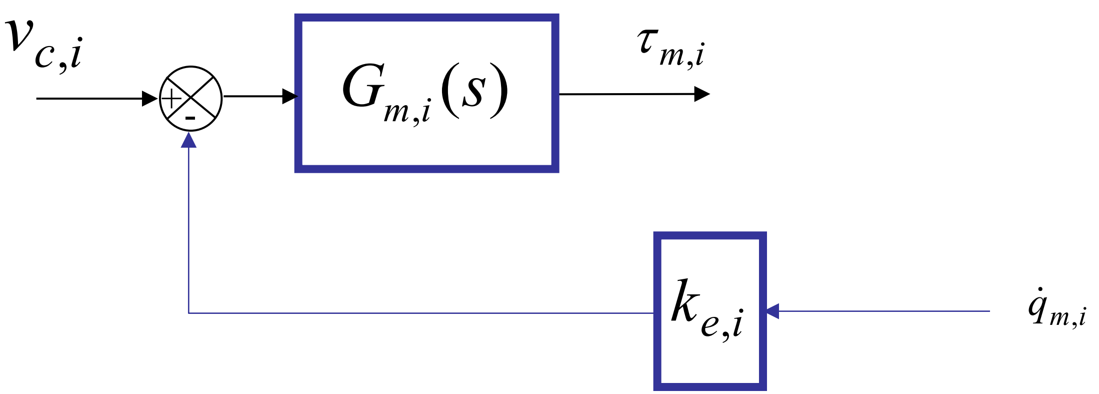


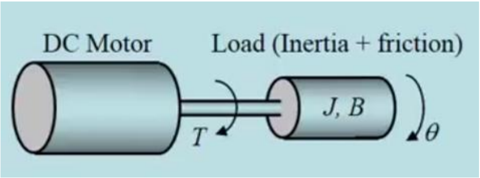
$i = 1,2,3,\ldots,n$



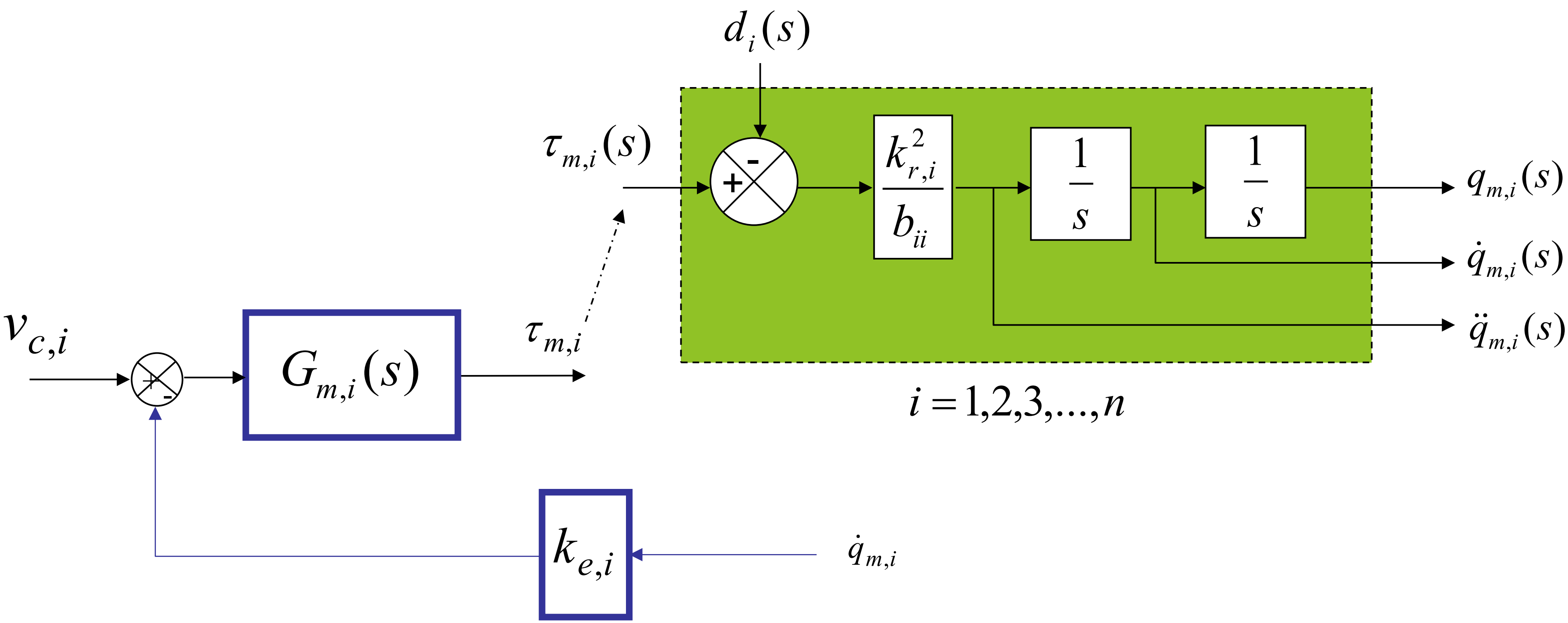
How to Control a Robot Arm with Unknown Dynamics? (continued)

- Step 2: Then, we get the independent unknown dynamics at power joints.

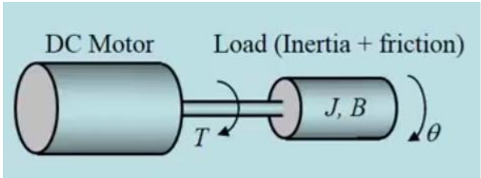




$$i = 1,2,\ldots n$$

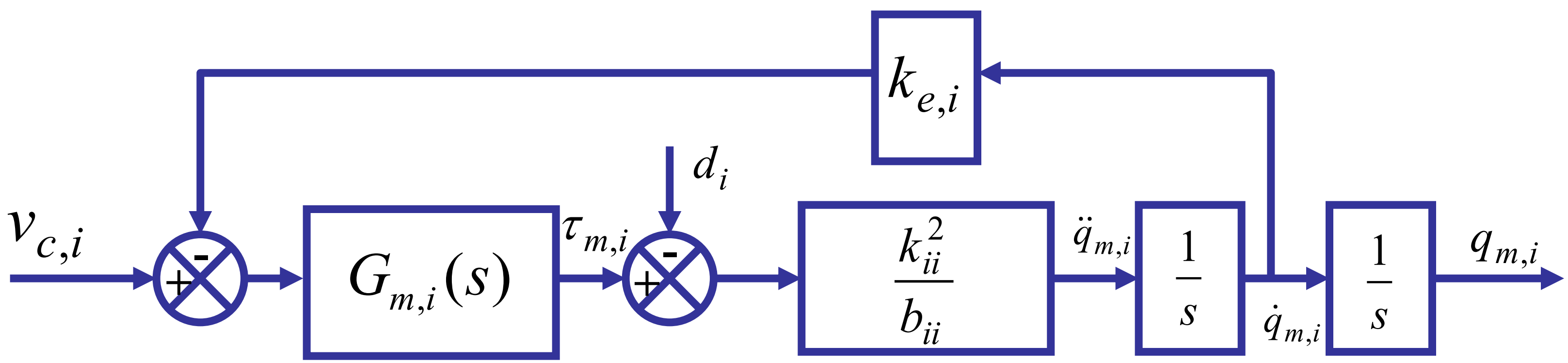

$d_i(s)$
$\tau_{m,i}(s)$
+
-
$\frac{k_{r,i}^2}{b_{ii}}$
$\frac{1}{s}$
$\frac{1}{s}$
$q_{m,i}(s)$
$\dot{q}_{m,i}(s)$
$\ddot{q}_{m,i}(s)$
$i = 1,2,3,\ldots,n$
$v_{c,i}$
+
-
$G_{m,i}(s)$
$\tau_{m,i}$
$k_{e,i}$
$\dot{q}_{m,i}$

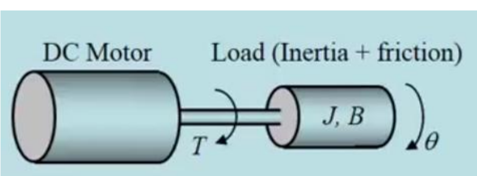
$$i = 1,2,\ldots n$$


DC Motor
Load (Inertia + friction)
J, B
T
θ

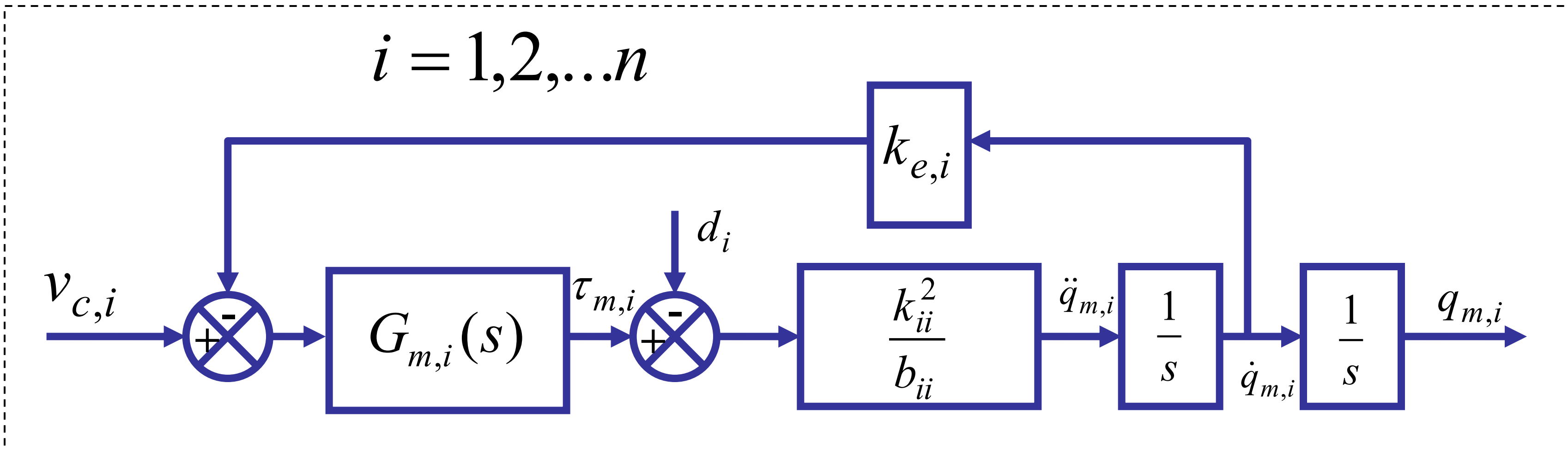
How to Control a Robot Arm with Unknown Dynamics? (continued)

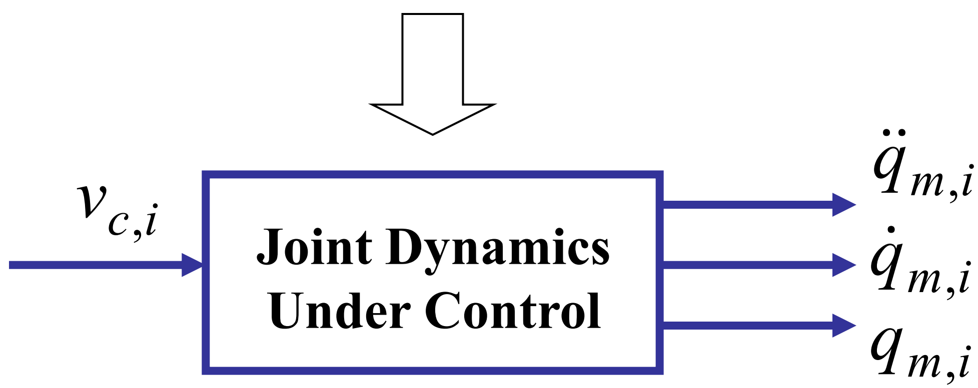
- Step 3: Subsequently, we have the independent unknown dynamics under control at power joints.





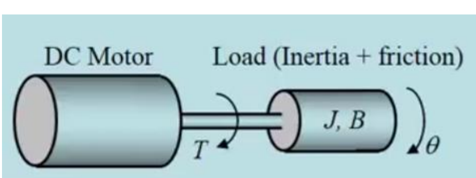
$$i = 1,2,\ldots n$$





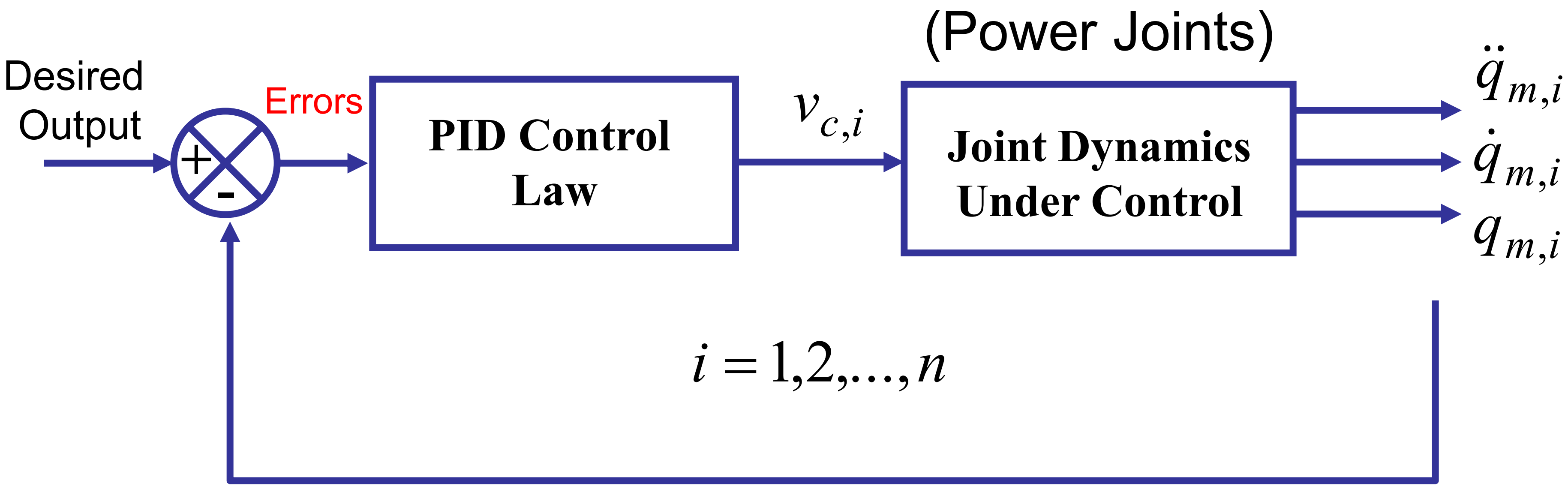
$i = 1,2,\ldots,n$

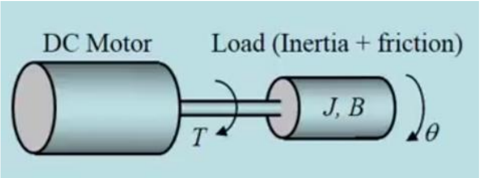
(Power Joints)



Control with Unknown Dynamics (continued)

- Step 4: Finally, we construct the independent error control systems at power joints.





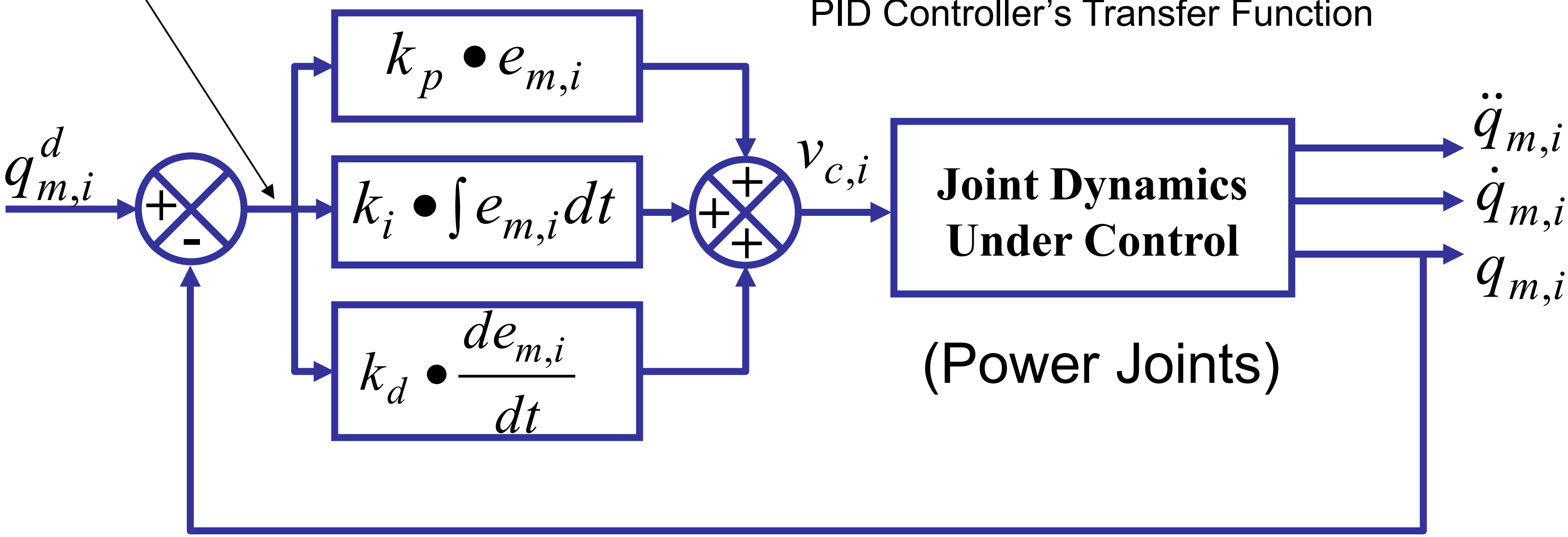
MIMO = Sum of SISOs

Example: Position Error Control

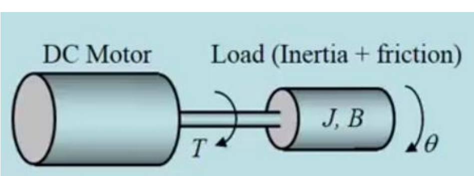
$$e_{m,i} = q^{d}_{m,i} - q_{m,i}$$

$$G_c(s) = k_p + k_i \frac{1}{s} + k_d s$$

PID Controller's Transfer Function



$$i = 1,2,\ldots,n$$

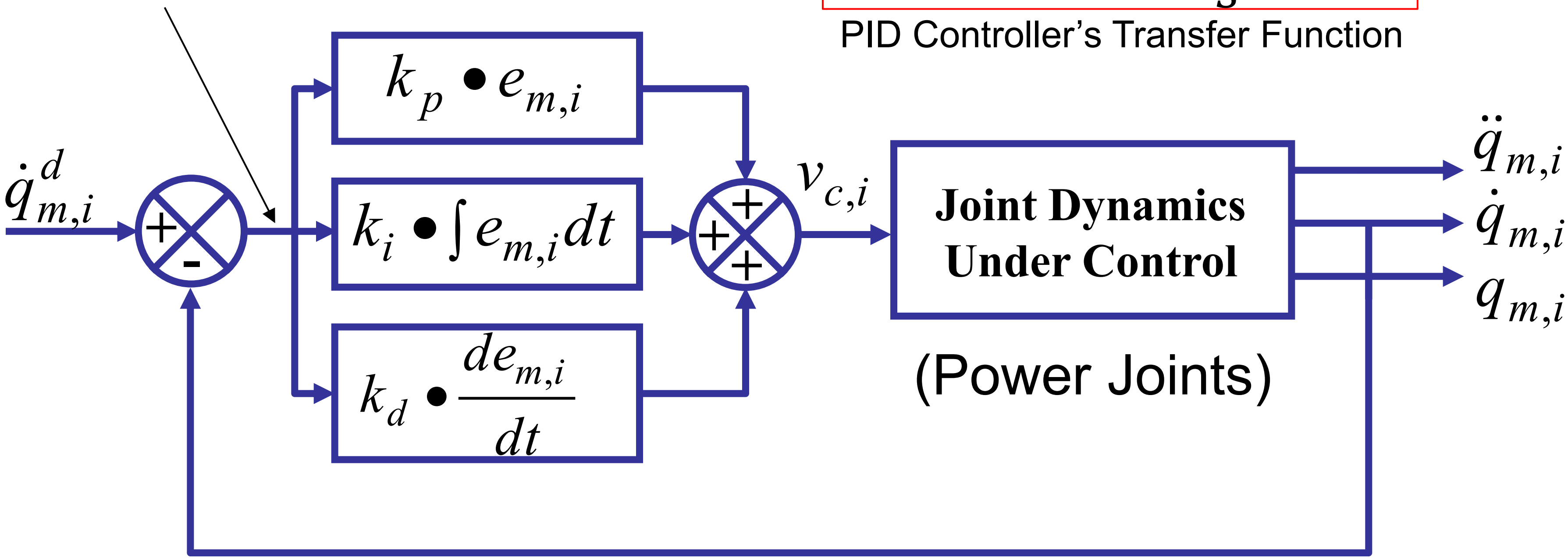


Example: Velocity Error Control

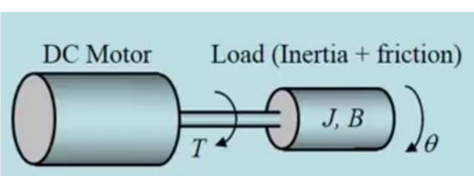
$$e_{m,i} = \dot{q}^{d}_{m,i} - \dot{q}_{m,i}$$

$$G_c(s) = k_p + k_i \frac{1}{s} + k_d s$$

PID Controller’s Transfer Function

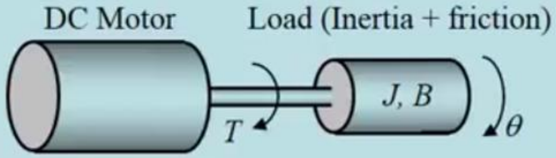


(Power Joints)



$$i = 1,2,\ldots,n$$

Example of Position Error Control without Inner Velocity Error Control Loop …

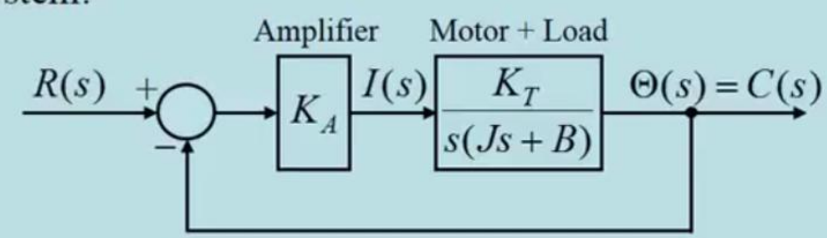


$$T \propto i \quad \Rightarrow \quad T = K_T i$$

Torque-balance equation: $T = J\ddot{\theta} + B\dot{\theta} = K_T i \quad \Rightarrow \quad Js^2\Theta(s) + Bs\Theta(s) = K_T I(s)$

$$\frac{\Theta(s)}{I(s)} = \frac{K_T}{Js^2 + Bs} = \frac{K_T}{(Js+B)s}$$

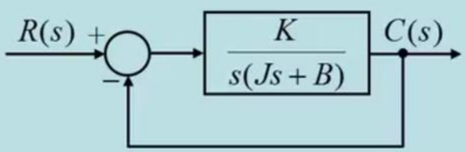
Closed-loop system:



$$\frac{C(s)}{R(s)} = \frac{\dfrac{K}{Js^2 + Bs}}{1 + \dfrac{K}{Js^2 + Bs}} = \frac{K}{Js^2 + Bs + K} \qquad K = K_A K_T$$

Example of Position Error Control without Inner Velocity Error Control Loop (continued) …

Second-Order Systems



$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + Bs + K}$$

$$\frac{C(s)}{R(s)} = \frac{K/J}{s^2 + (B/J)s + K/J} = \frac{\frac{K}{J}}{\left(s + \frac{B}{2J} + \sqrt{\left(\frac{B}{2J}\right)^2 - \frac{K}{J}}\right)\left(s + \frac{B}{2J} - \sqrt{\left(\frac{B}{2J}\right)^2 - \frac{K}{J}}\right)}$$

$B^2 - 4JK < 0 \Rightarrow$ Complex conjugate poles

$B^2 - 4JK \geq 0 \Rightarrow$ Real poles

Let $\frac{K}{J} = \omega_n^2$ and $\frac{B}{J} = 2\zeta\omega_n = 2\sigma$

Undamped natural frequency ← ω_n

Damping ratio ← ζ

Attenuation ← σ

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

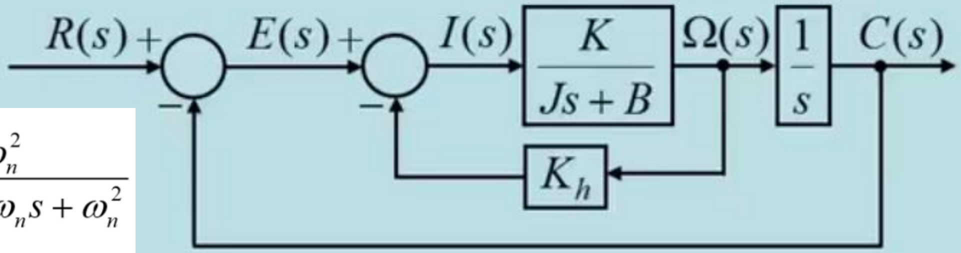
$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$$

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

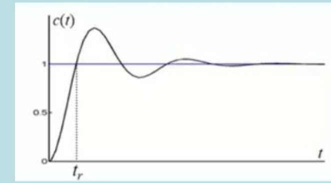
$$t_s = \frac{4}{\zeta\omega_n}$$

$$t_s = \frac{3}{\zeta\omega_n}$$

Example of Position Error Control with Inner Velocity Error Control Loop



$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

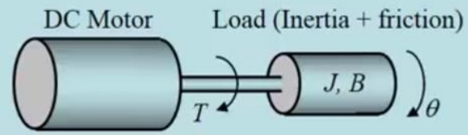


Assume that $J = 1$ kg·m² and $B = 1$ N·m/(rad/sec). The desired performance is specified as $M_p = 0.2$ and $t_p = 1$ sec for the unit-step response.

Determine K and K_h. Also, find the rise time and settling time.

$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 0.2 \;\Rightarrow\; \frac{\zeta}{\sqrt{1-\zeta^2}}\pi = 1.61 \;\Rightarrow\; \zeta^2 = 0.2625(1-\zeta^2) \;\Rightarrow\; \zeta = 0.456$$

$$t_p = \frac{\pi}{\omega_d} = 1 \;\Rightarrow\; \omega_d = 3.14 \;\Rightarrow\; \omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = 3.53 \text{ rad/sec}$$

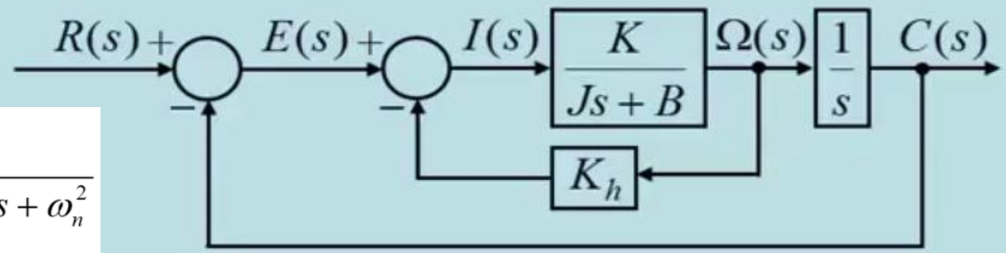


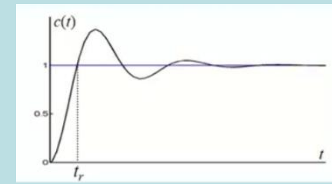
$$\omega_n = \sqrt{\frac{K}{J}} = \sqrt{K} \;\Rightarrow\; K = \omega_n^2 = 12.5 \text{ Nm}$$

$$2\zeta\omega_n = \frac{B + KK_h}{J} \;\Rightarrow\; K_h = \frac{2\sqrt{JK}\zeta - B}{K} = \frac{2\sqrt{12.5}\times 0.456 - 1}{12.5} = 0.178 \text{ sec}$$

Example of Position Error Control with Inner Velocity Error Control Loop

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$



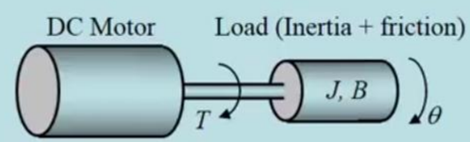


$\omega_n = 3.53$ rad/sec $\quad \zeta = 0.456 \quad K = 12.5$ Nm $\quad K_h = 0.178$ sec

Rise time:

$$t_r = \frac{\pi - \beta}{\omega_d} \qquad \beta = \tan^{-1}\frac{\omega_d}{\sigma} = \tan^{-1}\frac{\pi}{0.456 \times 3.53} = 1.10 \text{ rad}$$

$$\sigma = \zeta\omega_n \qquad t_r = \frac{\pi - 1.10}{\pi} = 0.65 \text{ sec}$$



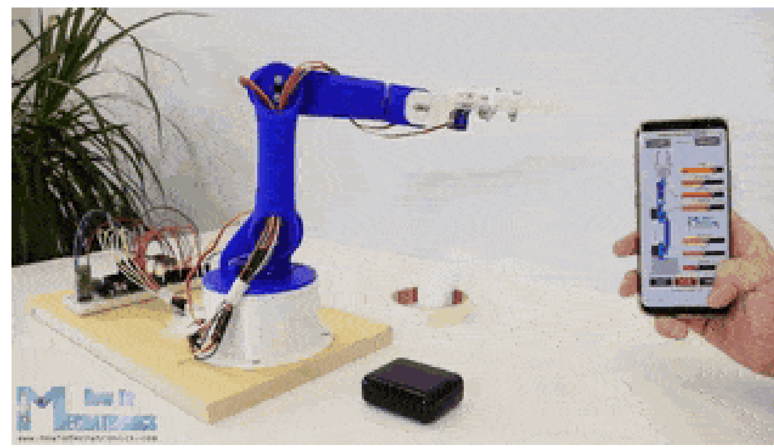
Settling time:

$$t_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.456 \times 3.53} = 2.48 \text{ sec} \quad \text{(2\% criterion)}$$

$$t_s = \frac{3}{\zeta\omega_n} = \frac{3}{0.456 \times 3.53} = 1.86 \text{ sec} \quad \text{(5\% criterion)}$$

Examples

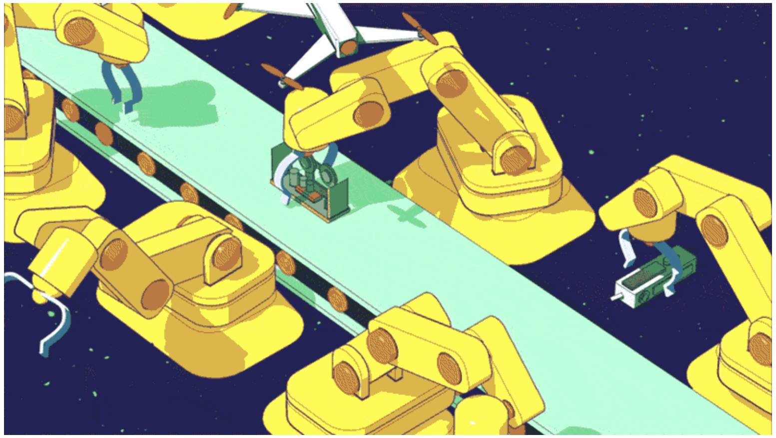
Summary of Lecture 4



- Joint Space
- Control Input
- Control Feedback
- Control with Known Dynamics
- Control with Unknown Dynamics

Outline of Module 4

- Dynamics under Control
- Signal Flow Diagram
- Design of Control Systems
- Control in Joint-Space
- Control in Task-Space



School of Mechanical & Aerospace Engineering

Design, Machine, Control, Intelligence

Module 4

MA4825 Robotics

Lecture 5

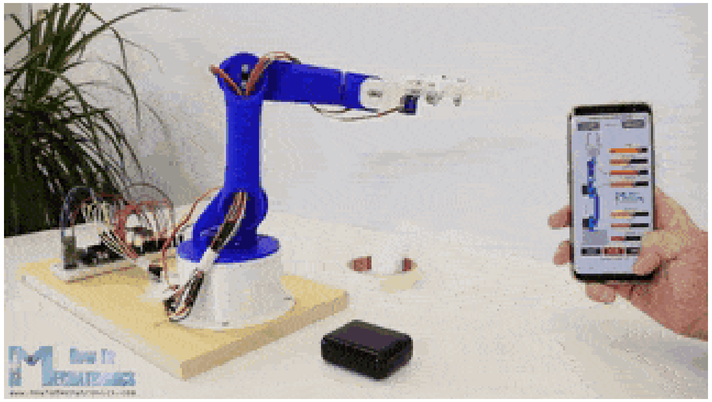
Control in Task-Space

Xie Ming, PhD (France)

http://personal.ntu.edu.sg/mmxie

Outline of Lecture 5

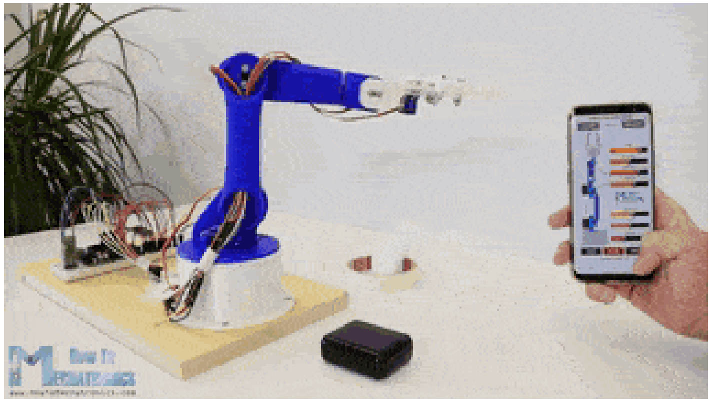
- Task Space



- Control Input
- Control Feedback
- Control of Unconstrained Motion
- Control of Constrained Motion

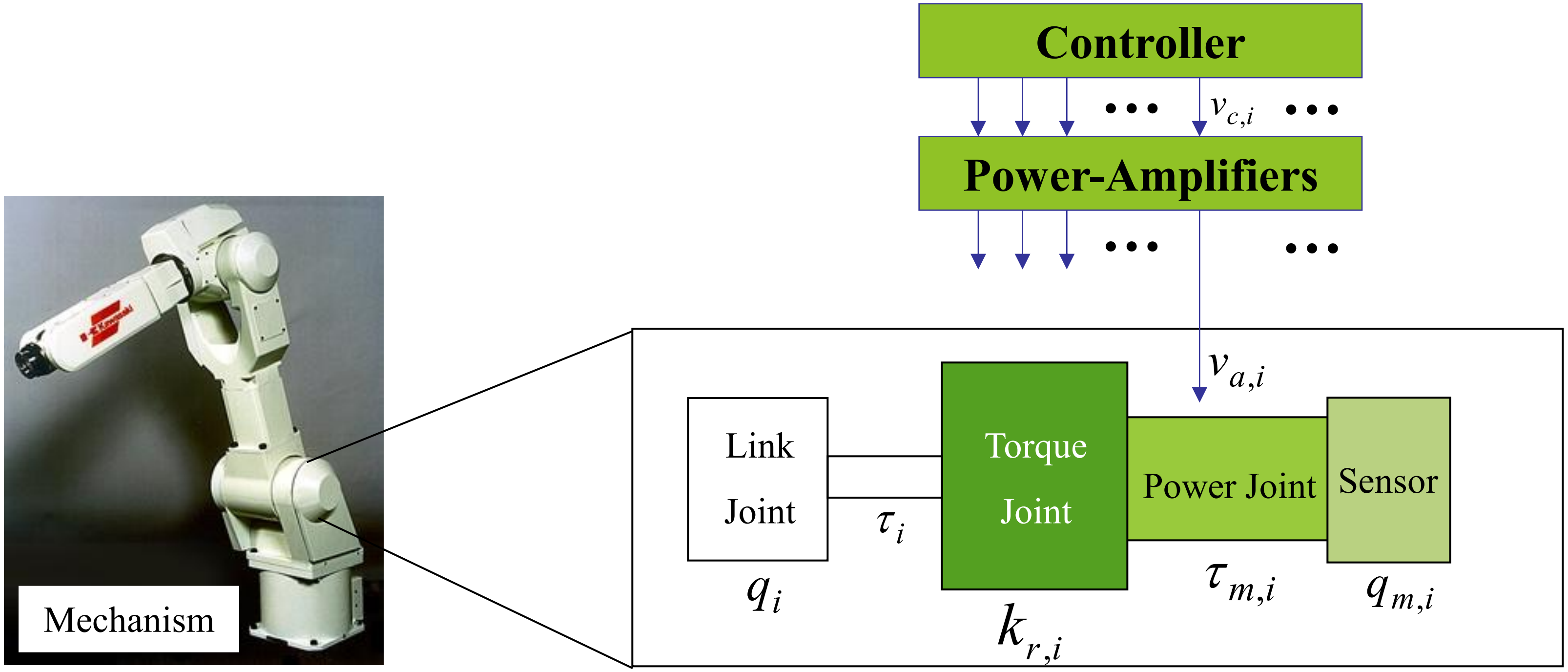
Outline of Lecture 5

- Task Space
- Control Input
- Control Feedback
- Control of Unconstrained Motion
- Control of Constrained Motion



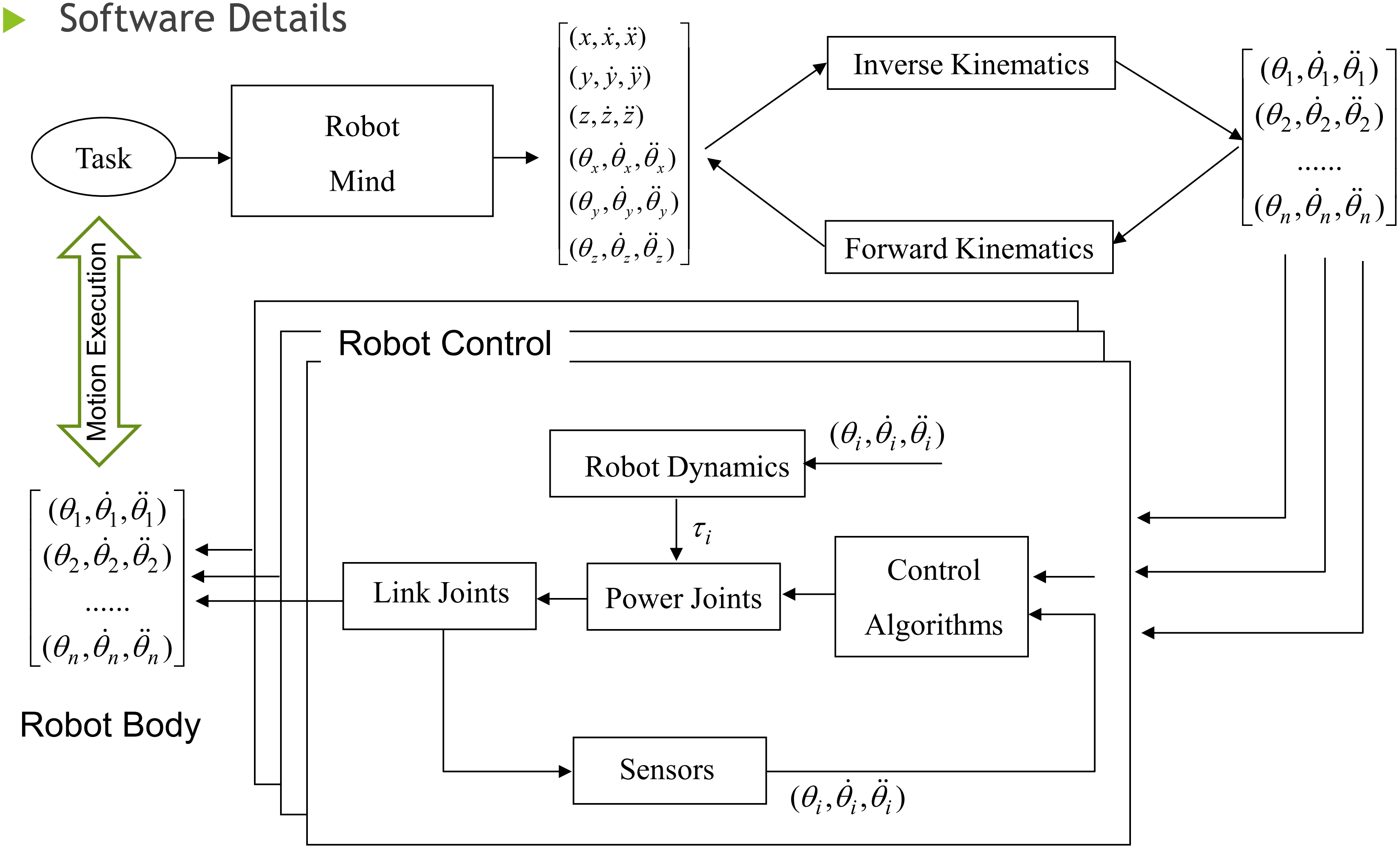
Robot's Control Systems

- Hardware Details



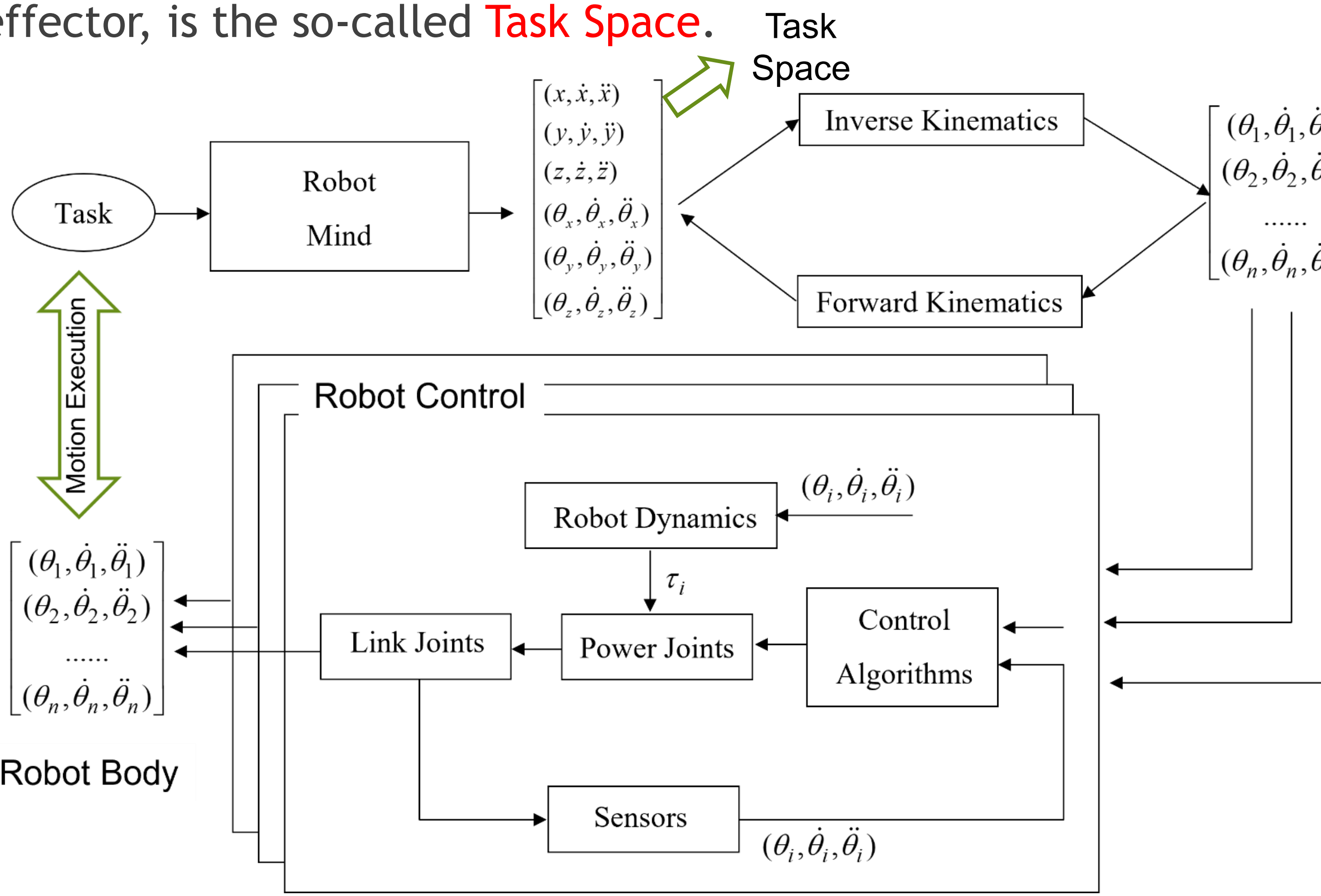
Robot's Control Systems

- Software Details



Definition of Task Space

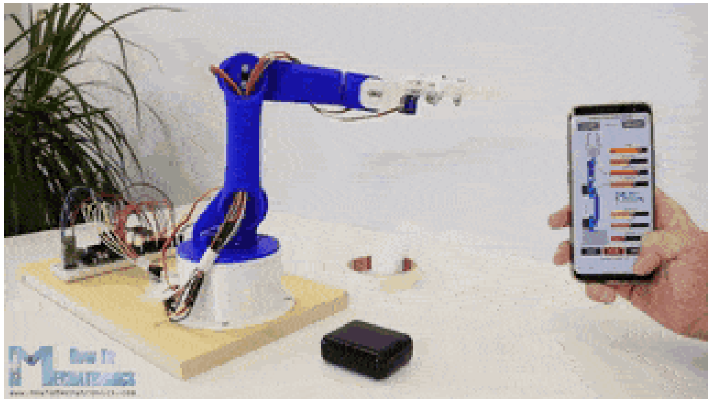
- The space, which is defined by the linear and angular positions of end-effector, is the so-called Task Space.



Example of Controlling Human-Robot Interaction in Task Space

Outline of Lecture 5

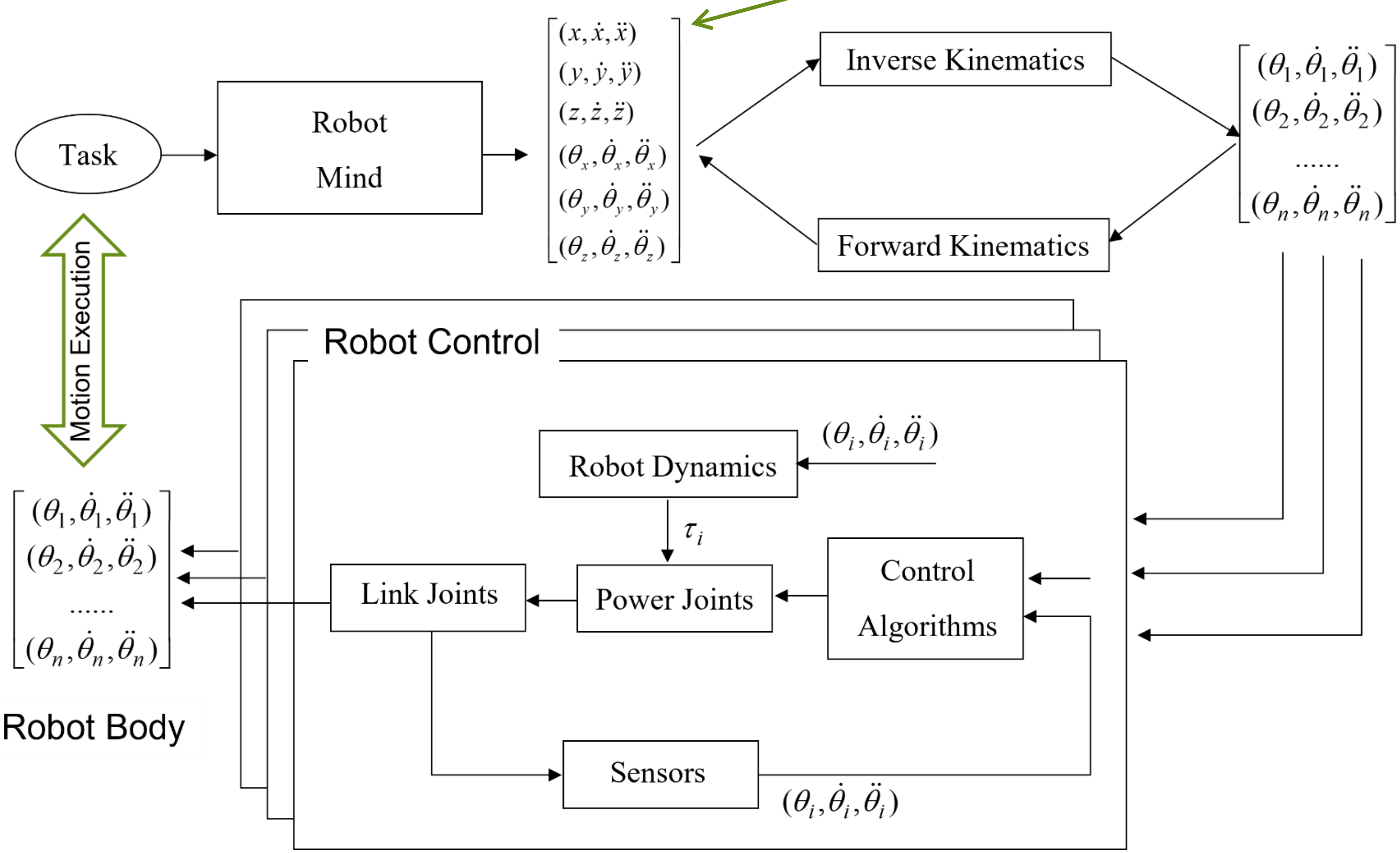
- Task Space
- Control Input



- Control Feedback
- Control of Unconstrained Motion
- Control of Constrained Motion

What are the controllable variables in task space?

Answer



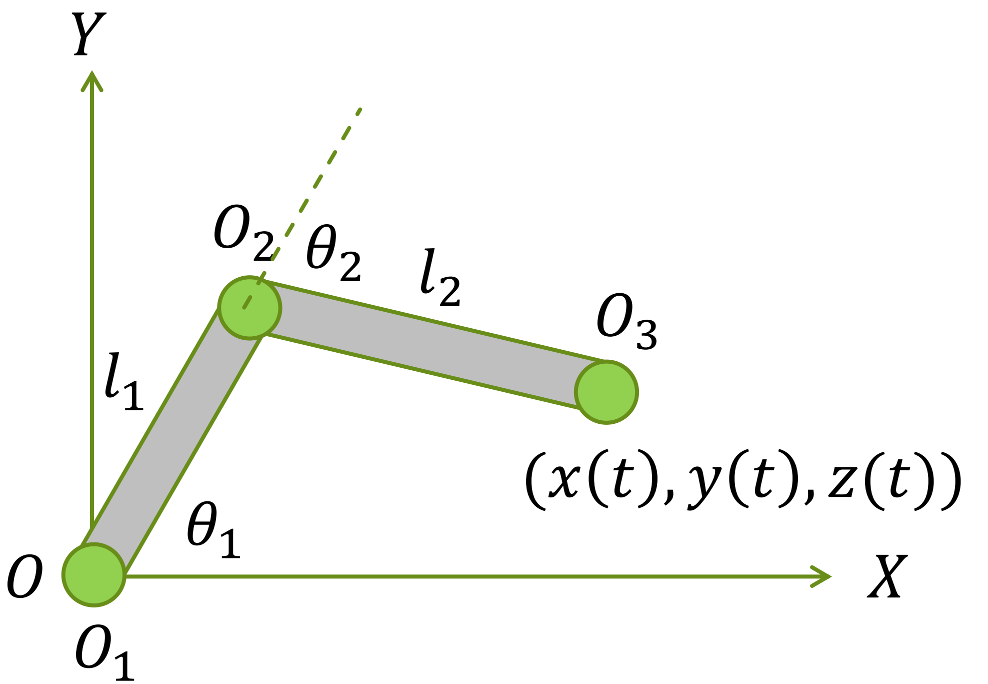
How to Measure Controllable Variables in Task Space Which Include:

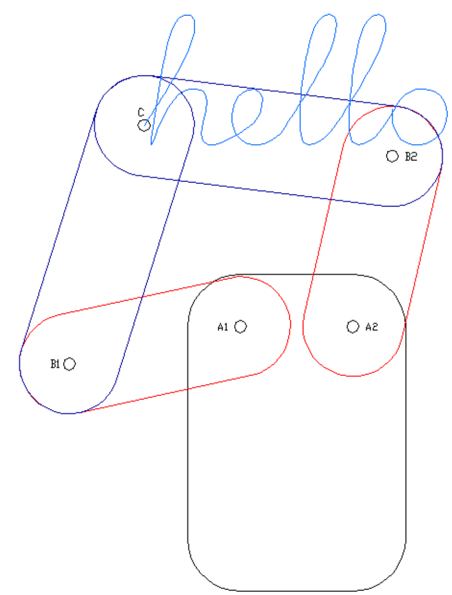
- Positions of End-Effector
- Velocities of End-Effector
- Forces/Torques (related to accelerations)

Use of Relationship between Task Space and Joint Space: Forward Kinematics of Arm

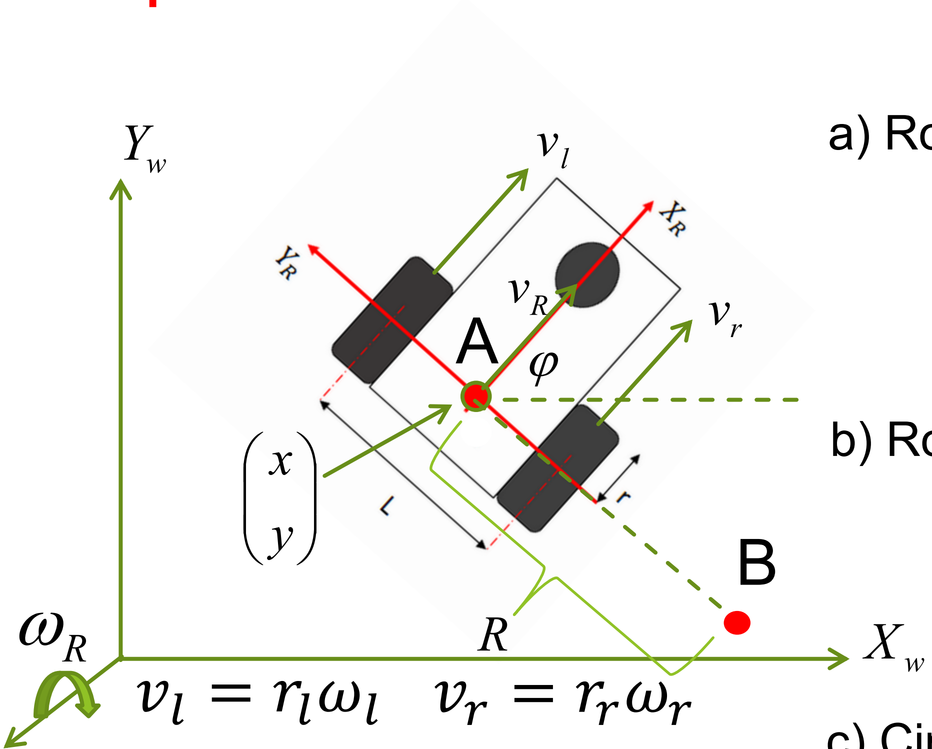
$$x(t) = l_1 \cos\big(\theta_1(t)\big) + l_2 \cos[\theta_1(t) + \theta_2(t)]$$

$$y(t) = l_1 \sin\big(\theta_1(t)\big) + l_2 \sin[\theta_1(t) + \theta_2(t)]$$





Use of Relationship between Task Space and Joint Space: Forward Kinematics of Mobile Base



a) Robot rotates about Z axis passing through A:

$$\omega_R = \frac{1}{L} v_l - \frac{1}{L} v_r$$

b) Robot rotates about Z axis passing through B:

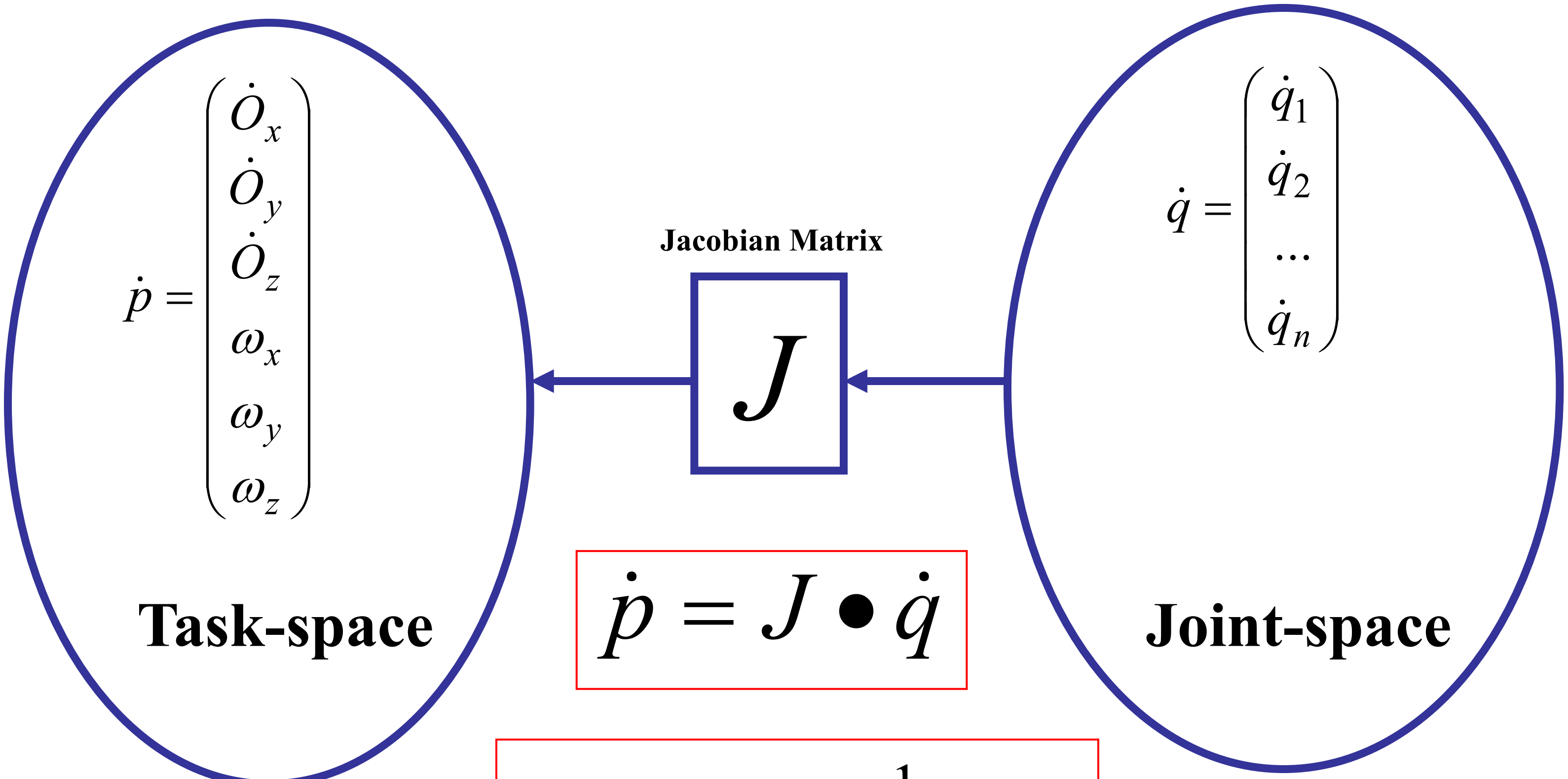
$$v_R = \frac{1}{2}(v_l + v_r)$$

c) Circular velocities at wheels, will create the same circular velocity of robot:

$$\omega_R = \frac{v_l}{R + L/2} = \frac{v_R}{R}$$

$$\omega_R = \frac{v_r}{R - L/2} = \frac{v_R}{R}$$

Use of Relationship between Task Space and Joint Space: Motion Kinematics

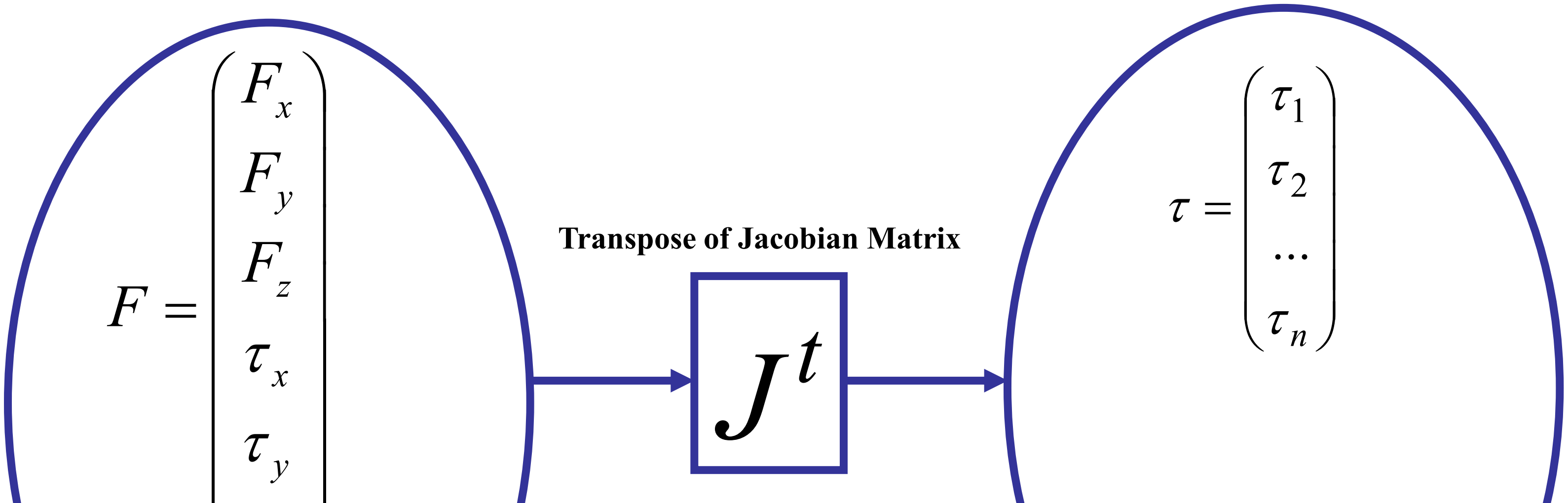


$$\dot{p} = \begin{pmatrix} \dot{O}_x \\ \dot{O}_y \\ \dot{O}_z \\ \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \qquad \dot{q} = \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \cdots \\ \dot{q}_n \end{pmatrix}$$

$$\dot{p} = J \bullet \dot{q}$$

$$\dot{p} = J \bullet K_r^{-1} \bullet \dot{q}_m$$

Use of Relationship between Task Space and Joint Space: Robot Statics

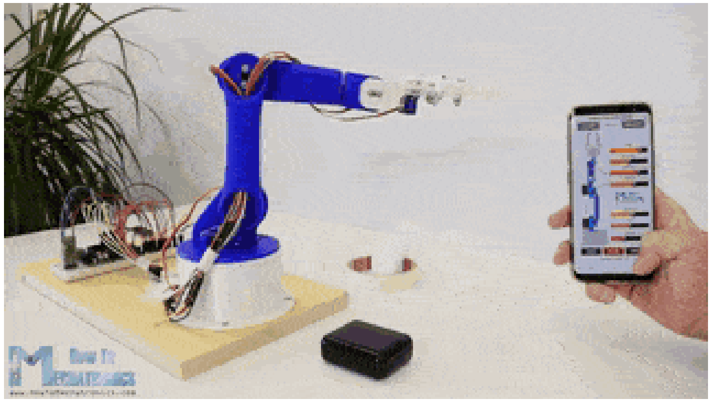


$$\tau = J^t \bullet F$$

$$\tau_m = K_r^{-1} \bullet J^t \bullet F$$

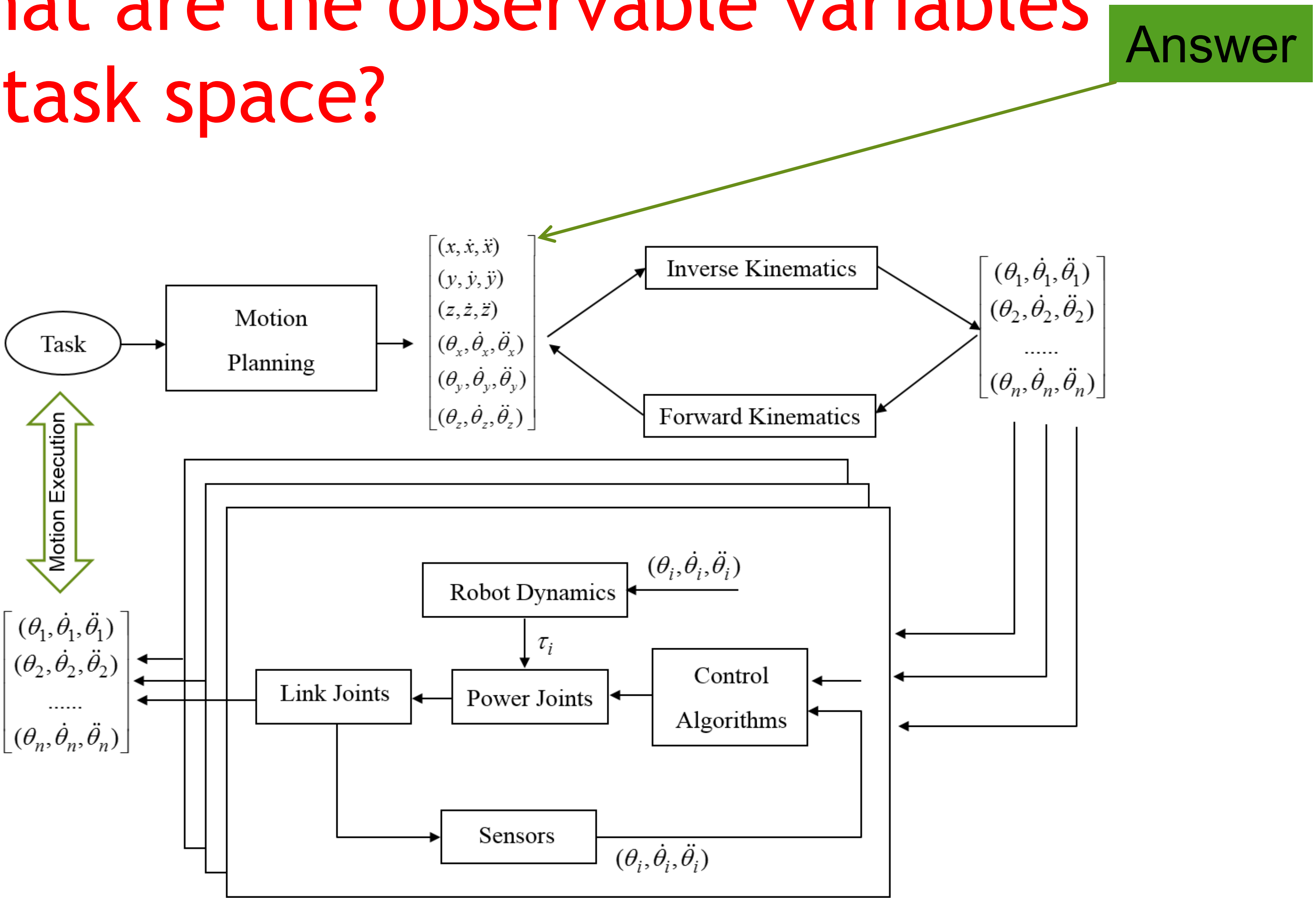
Outline of Lecture 5

- Task Space
- Control Input



- Control Feedback
- Control of Unconstrained Motion
- Control of Constrained Motion

What are the observable variables in task space?



Observable Variables in Task Space

- Positions of End-Effector
- Velocities of End-Effector
- Forces/Torques (related to accelerations)

How to achieve sensory feedback in Task Space?

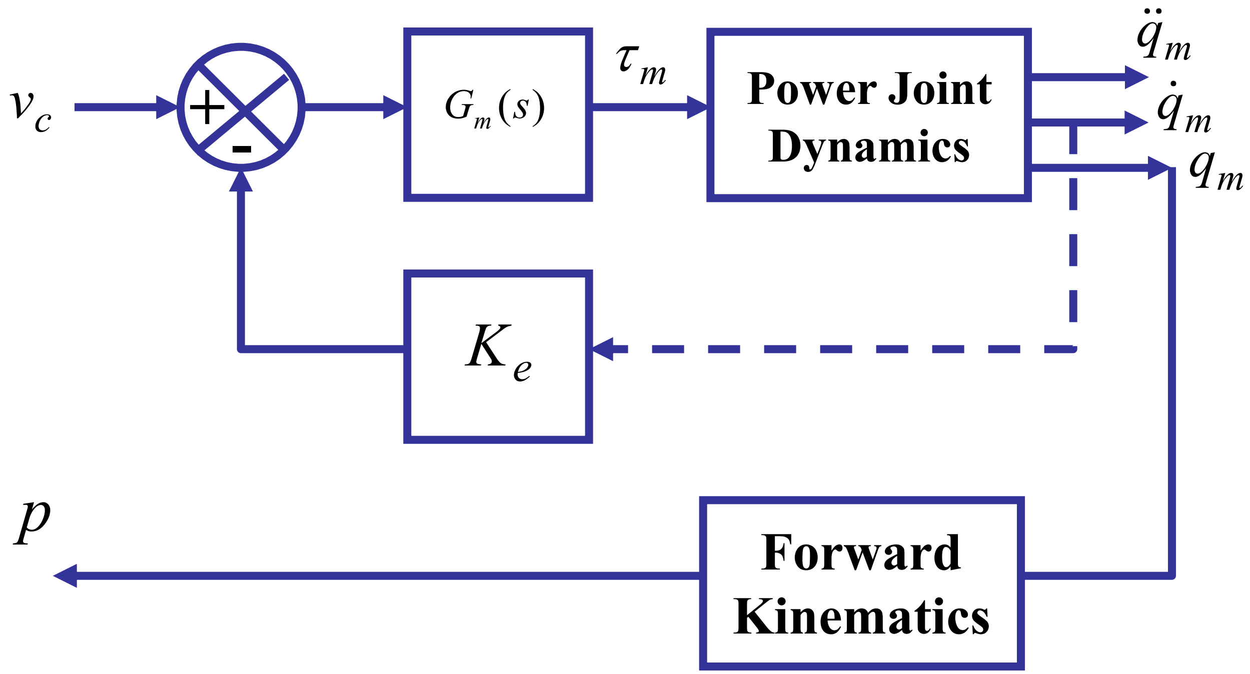
- Joint Position Sensors + Forward Kinematics
- Joint Velocity Sensors + Motion Kinematics

Indirect Measurement

- Vision System
- Force Sensors

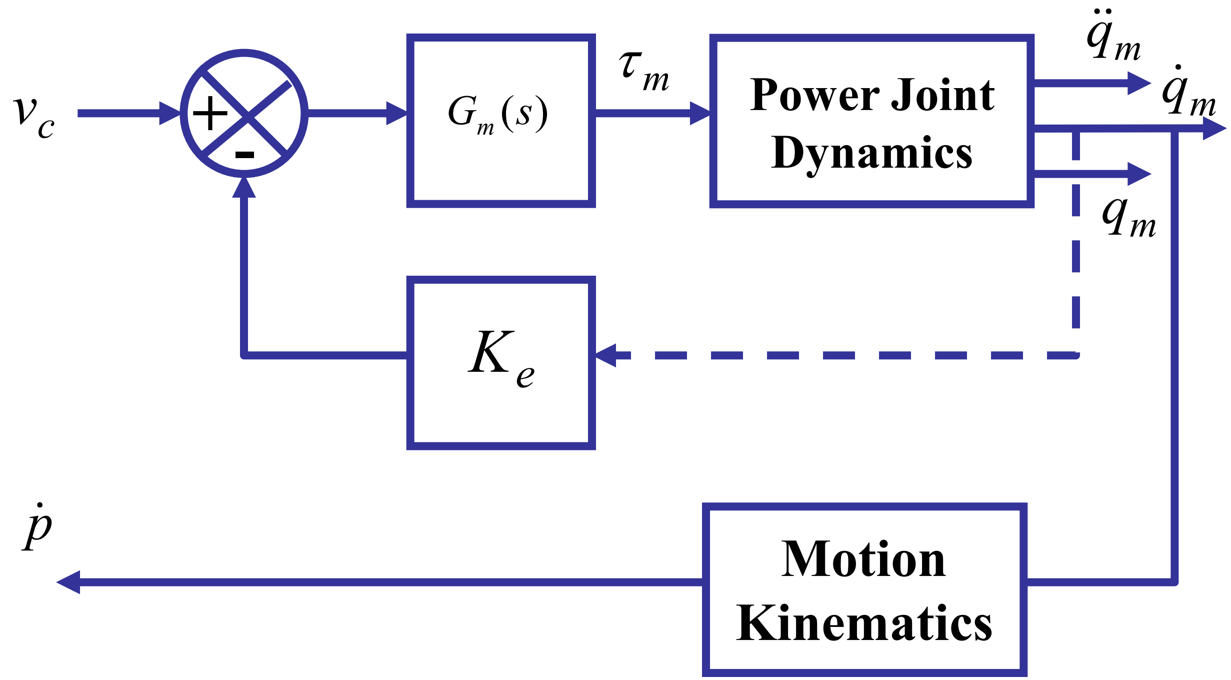
Direct Measurement

Solution of Using "Sensors + Forward Kinematics"

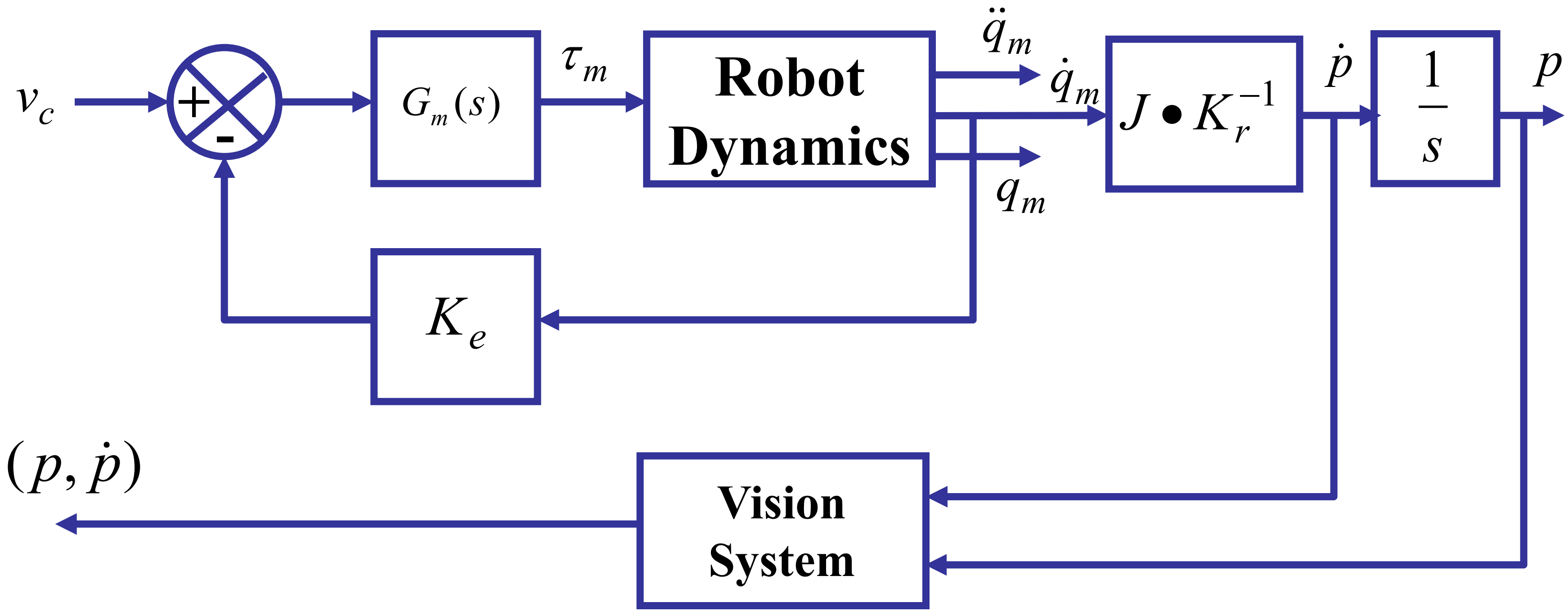


Example of Pick-and-Place Control in Task Space

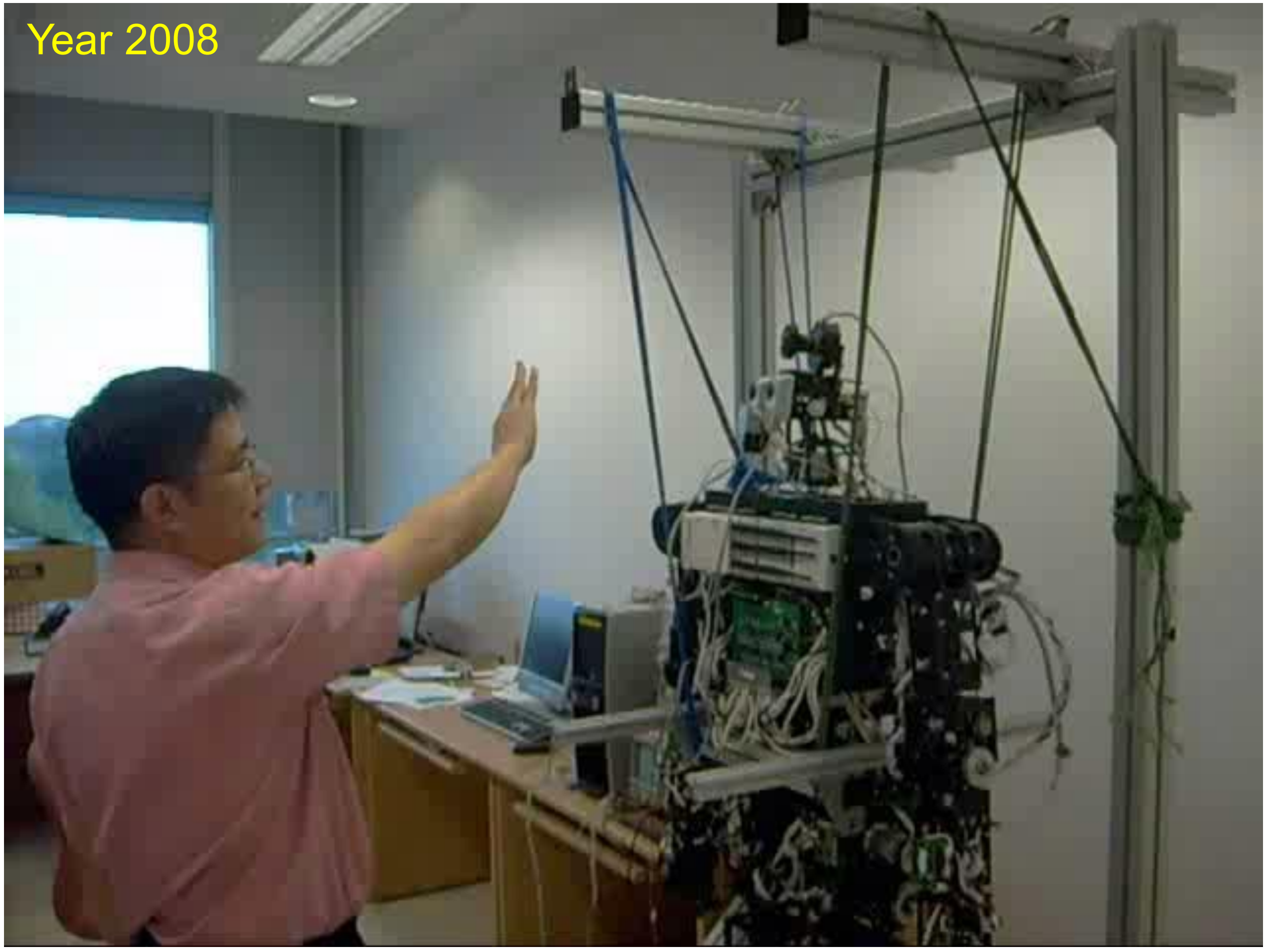
Solution of Using "Sensors + Motion Kinematics"



Solution of Using Vision Systems



Example 1:



Example 2:

Example 3:

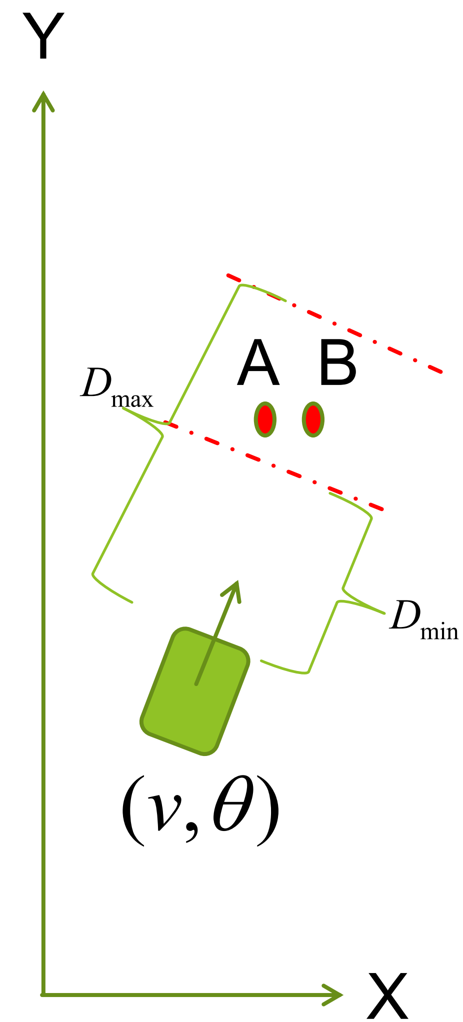
- A vehicle is following a moving target. The vehicle's vision system can measure the distances to points A and B of the moving target. What should be the trajectories of the vehicle?

- Answer:
  - Equation of velocity:

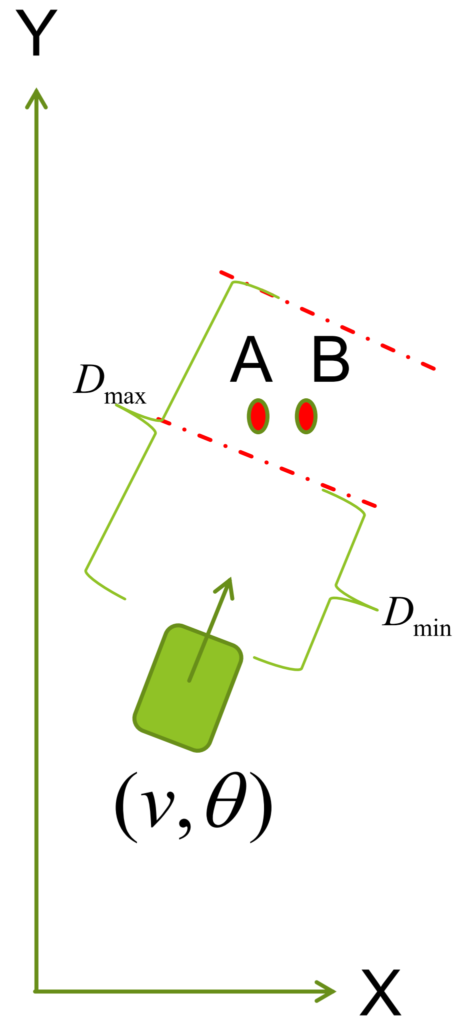
$$v(t_{k+1}) = \begin{cases} v(t_k) + \Delta v & \text{if } \mathrm{d} > \mathrm{D}_{\max} \\ v(t_k) & \text{if } \mathrm{D}_{\min} > \mathrm{d} > \mathrm{D}_{\max} \\ v(t_k) - \Delta v & \text{if } \mathrm{d} < \mathrm{D}_{\min} \end{cases}$$

  - Equation of steering:

$$\theta(t_{k+1}) = \begin{cases} \theta(t_k) + \Delta\theta & \text{if } d_A - d_B < -\varepsilon \\ \theta(t_k) & \text{if } |d_A - d_B| < \varepsilon \\ \theta(t_k) - \Delta\theta & \text{if } d_A - d_B > +\varepsilon \end{cases}$$



Experimental Result at NTU

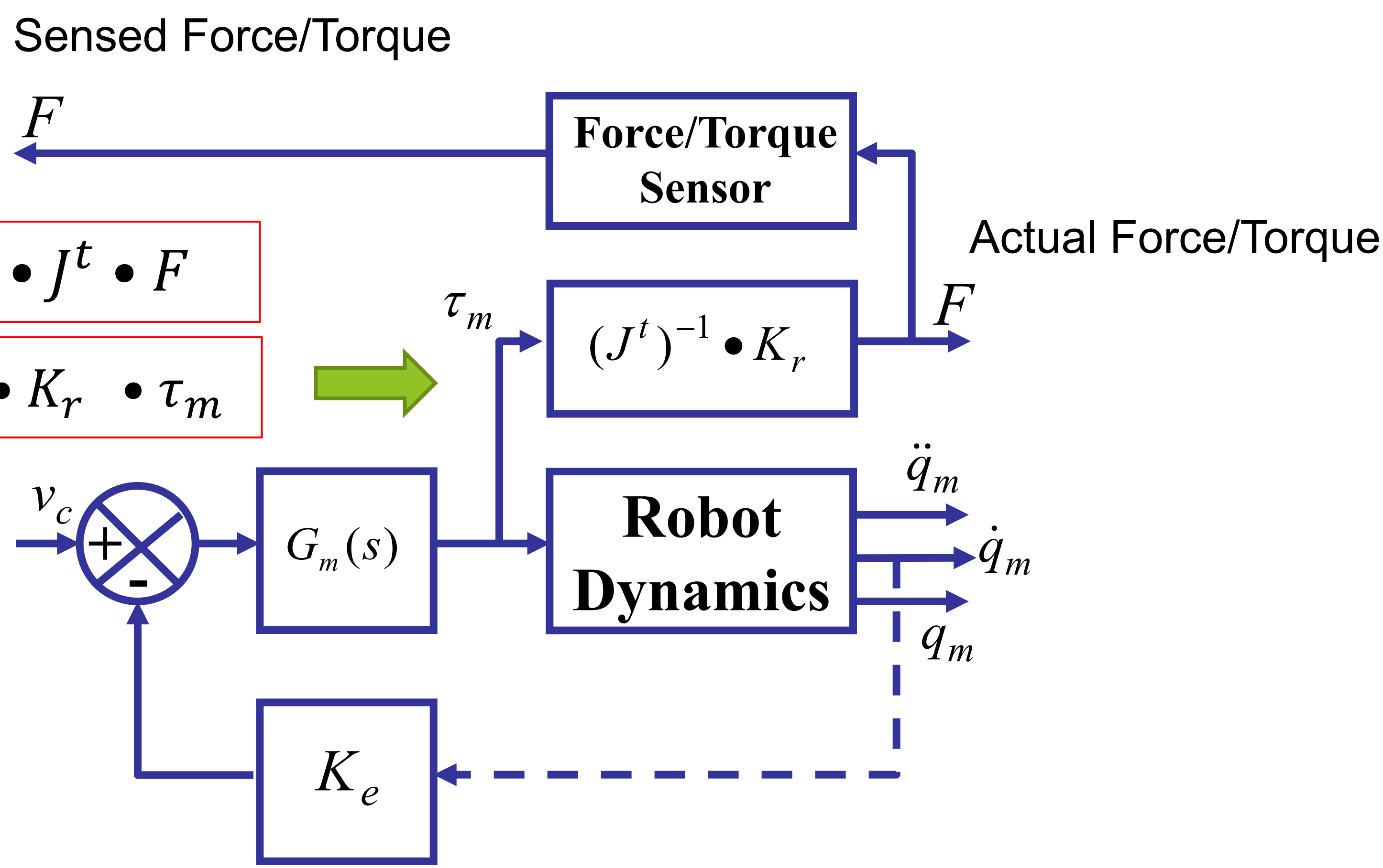


Solution of Using Force/Torque Sensors

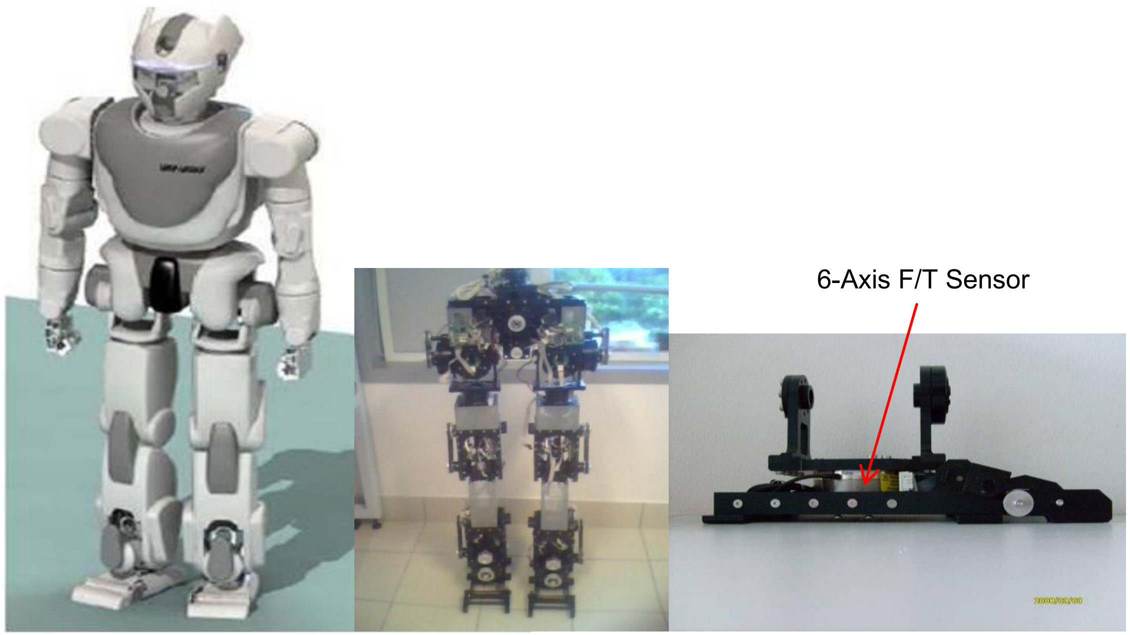
$$\tau = J^t \times F$$

$$\tau_m = K_r^{-1} \bullet J^t \bullet F$$

$$F = (J^t)^{-1} \bullet K_r \bullet \tau_m$$

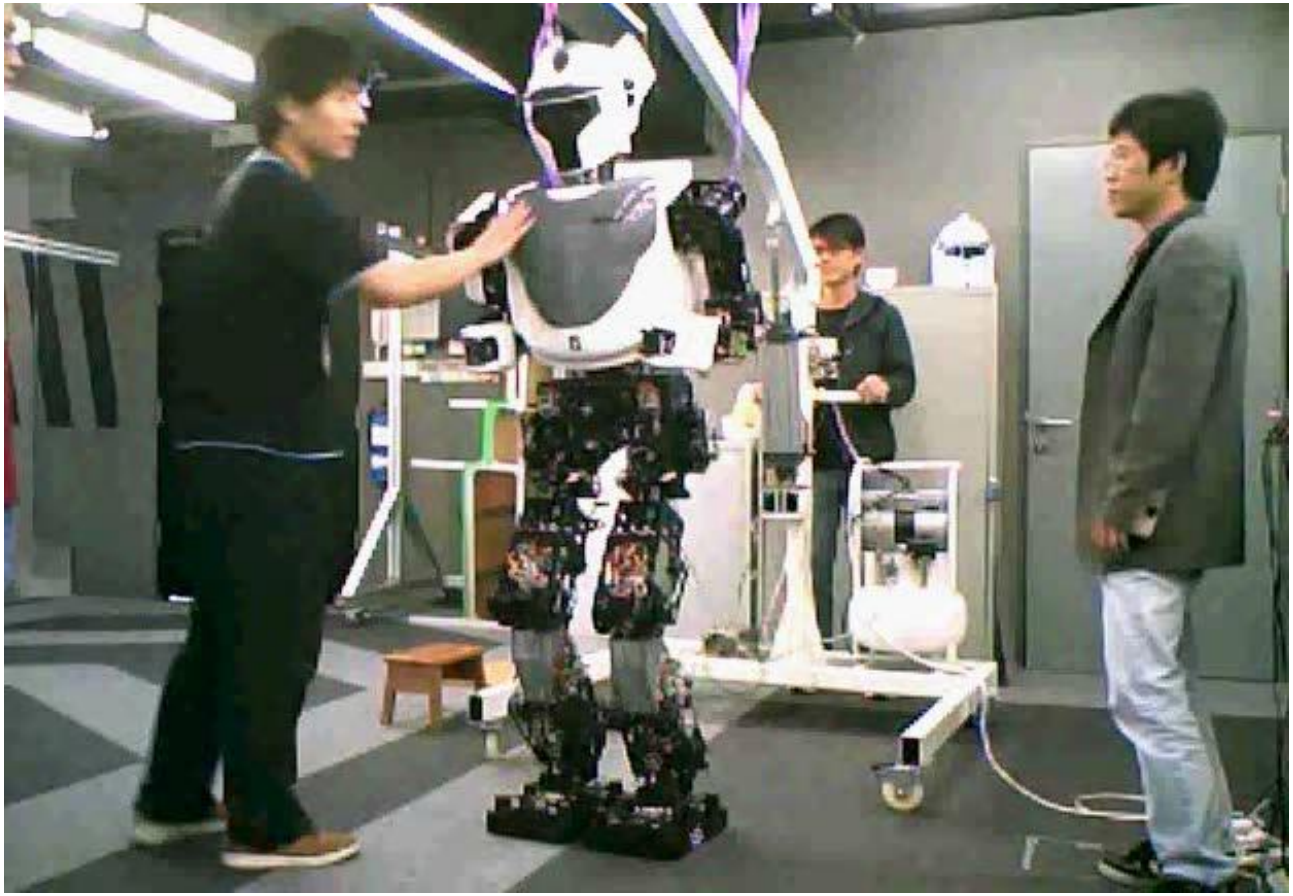


Robot Design with Incorporated Force/Torque Sensors



(a) Loch Humanoid Robot (b) Loch 's Biped (c) F/T Sensor in Foot

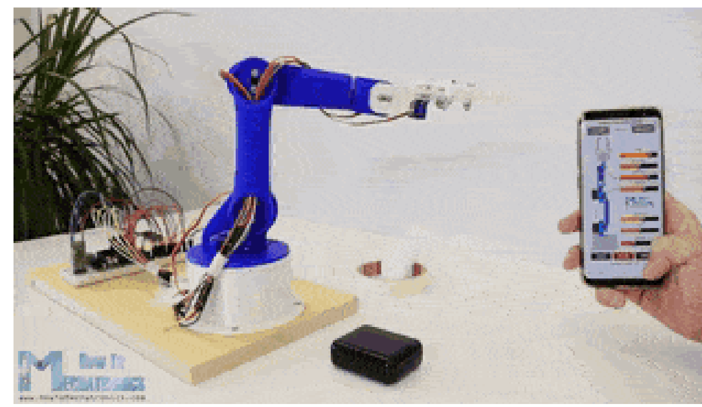
Example 1:



Example 2:

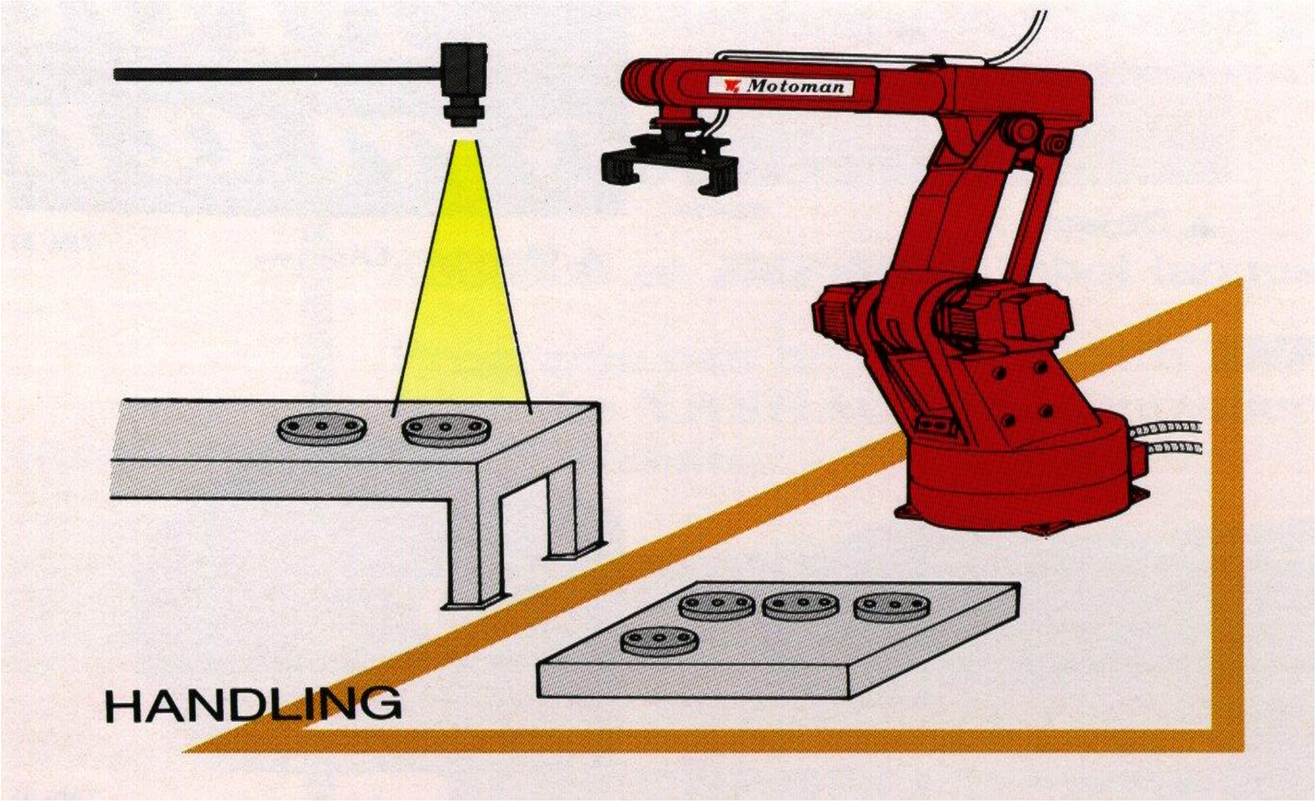
Outline of Lecture 5

- Task Space
- Control Input
- Control Feedback
- Control of Unconstrained Motion
- Control of Constrained Motion

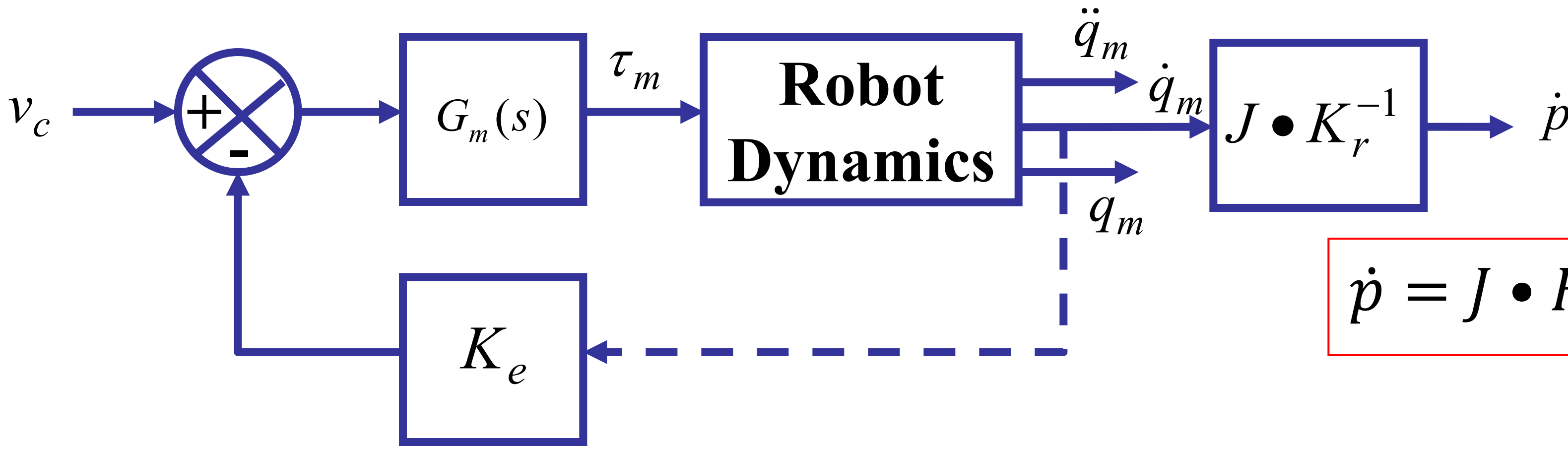


Definition of Unconstrained Motion

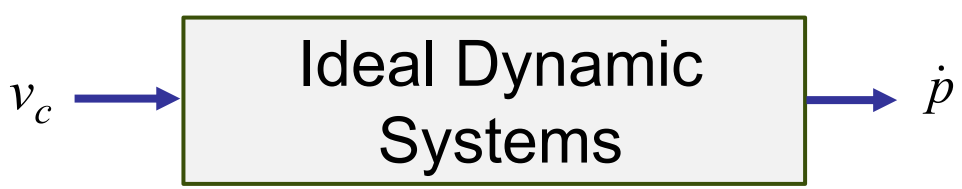
- When a robot's tool tip has no contact with its environment during motions, it performs unconstrained motions.



Robot's Dynamics under Control in Task Space for Unconstrained Motion Will Be:



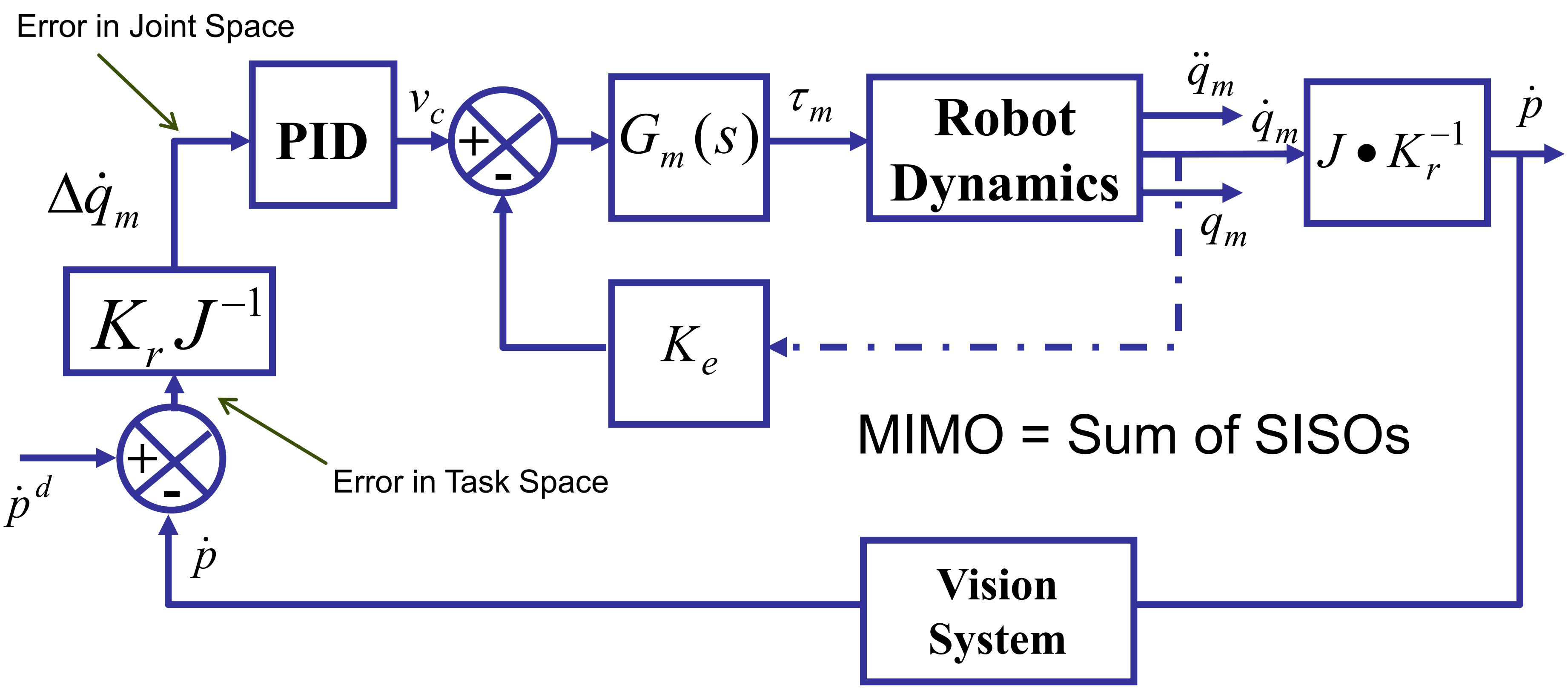
$$\dot{p} = J \bullet K_r^{-1} \bullet \dot{q}_m$$



MIMO = Sum of SISOs

Design of Error Control System …

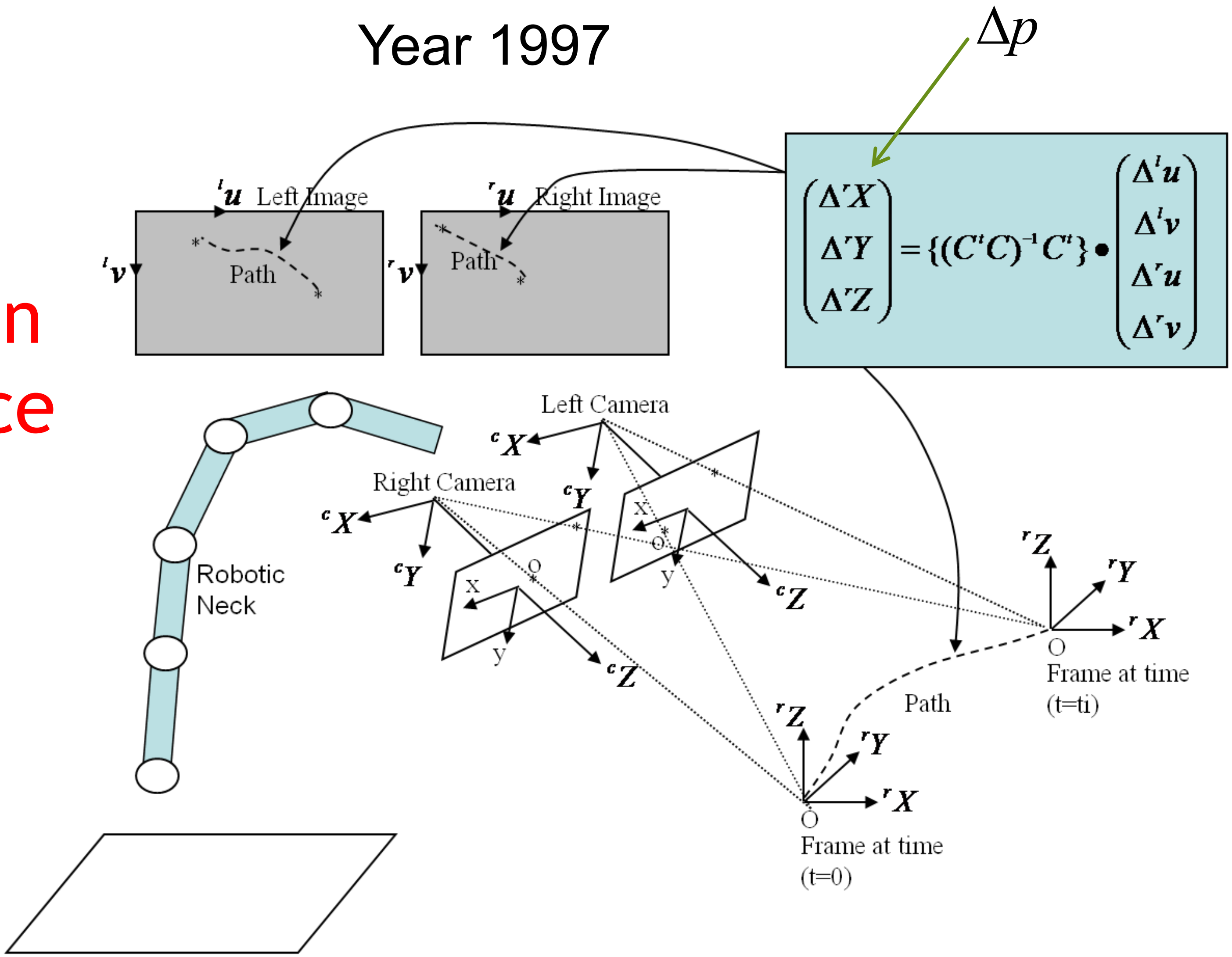
$$\dot{p} = J \bullet K_r^{-1} \bullet \dot{q}_m \quad \Rightarrow \quad \dot{q}_m = K_r \bullet J^{-1} \bullet \dot{p}$$



MIMO = Sum of SISOs

Principle of Human-like X-Eye Coordination in Task Space

Year 1997

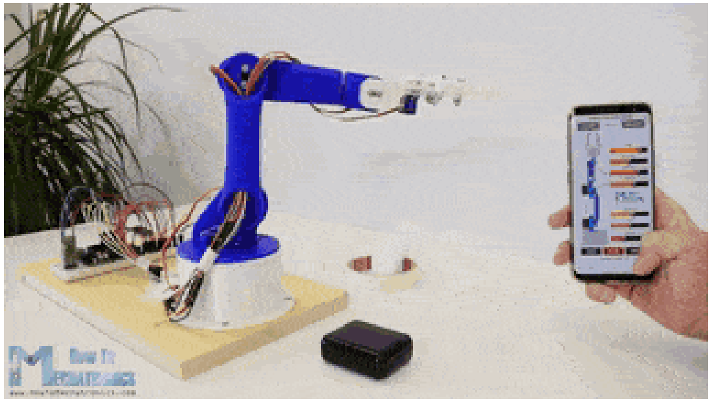


Xie M., Fang Yuhui and Lai Tingfeng, 2025, New Solution to 3D Projection in Human-like Binocular Vision, International Journal of Humanoid Robotics.

Experimental Verification Achieved in Year 2007 with the use of Monkey …

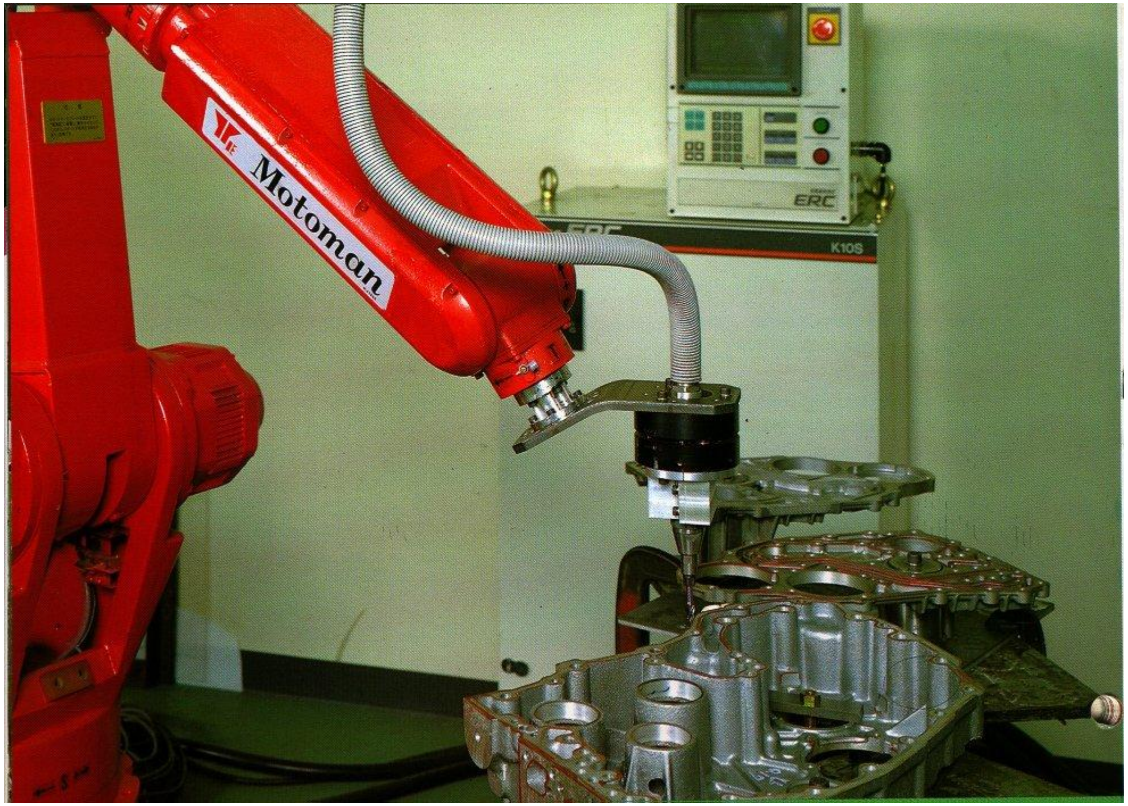
Outline of Lecture 5

- Task Space
- Control Input
- Control Feedback
- Control of Unconstrained Motion
- Control of Constrained Motion



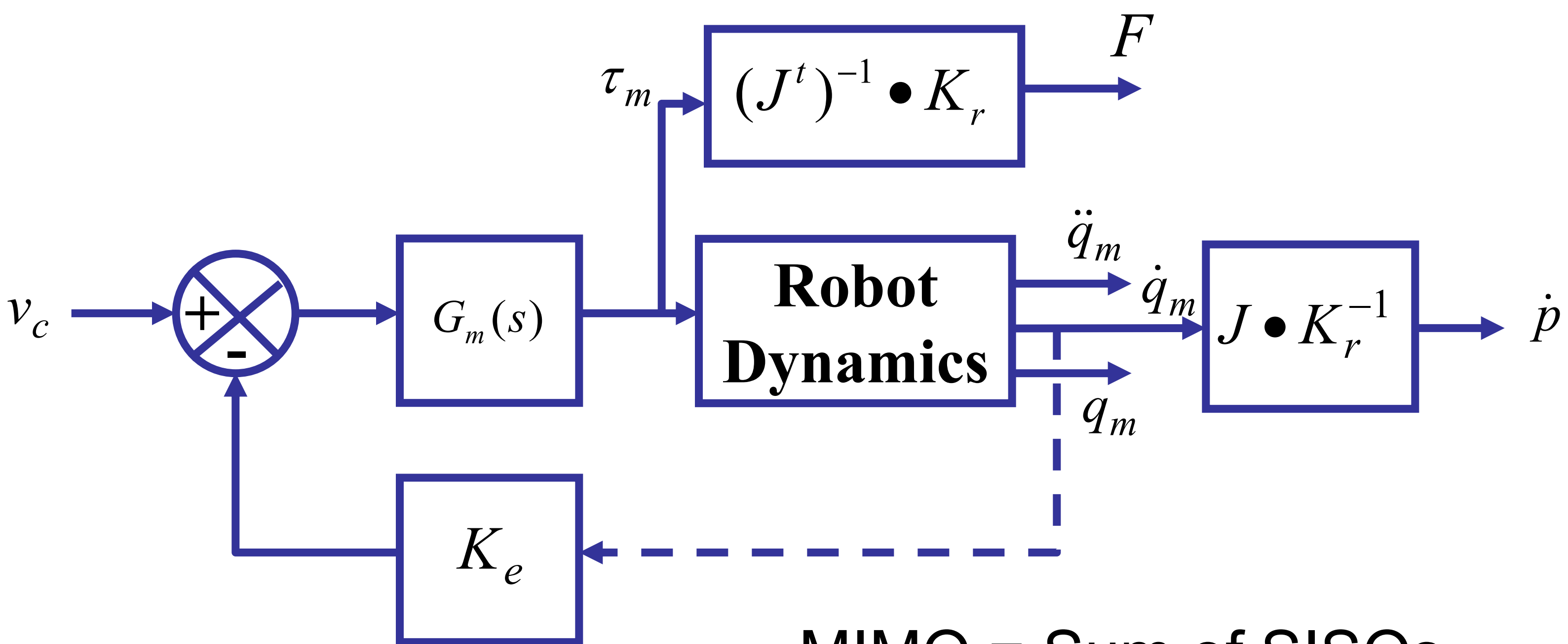
Definition of Constrained Motion

- When a robot's tool tip has contact with its environment during motions, it performs constrained motions.



Example of Robot Performing Constrained Motions

Robot's Dynamics under Control in Task Space for Constrained Motion Will Be:

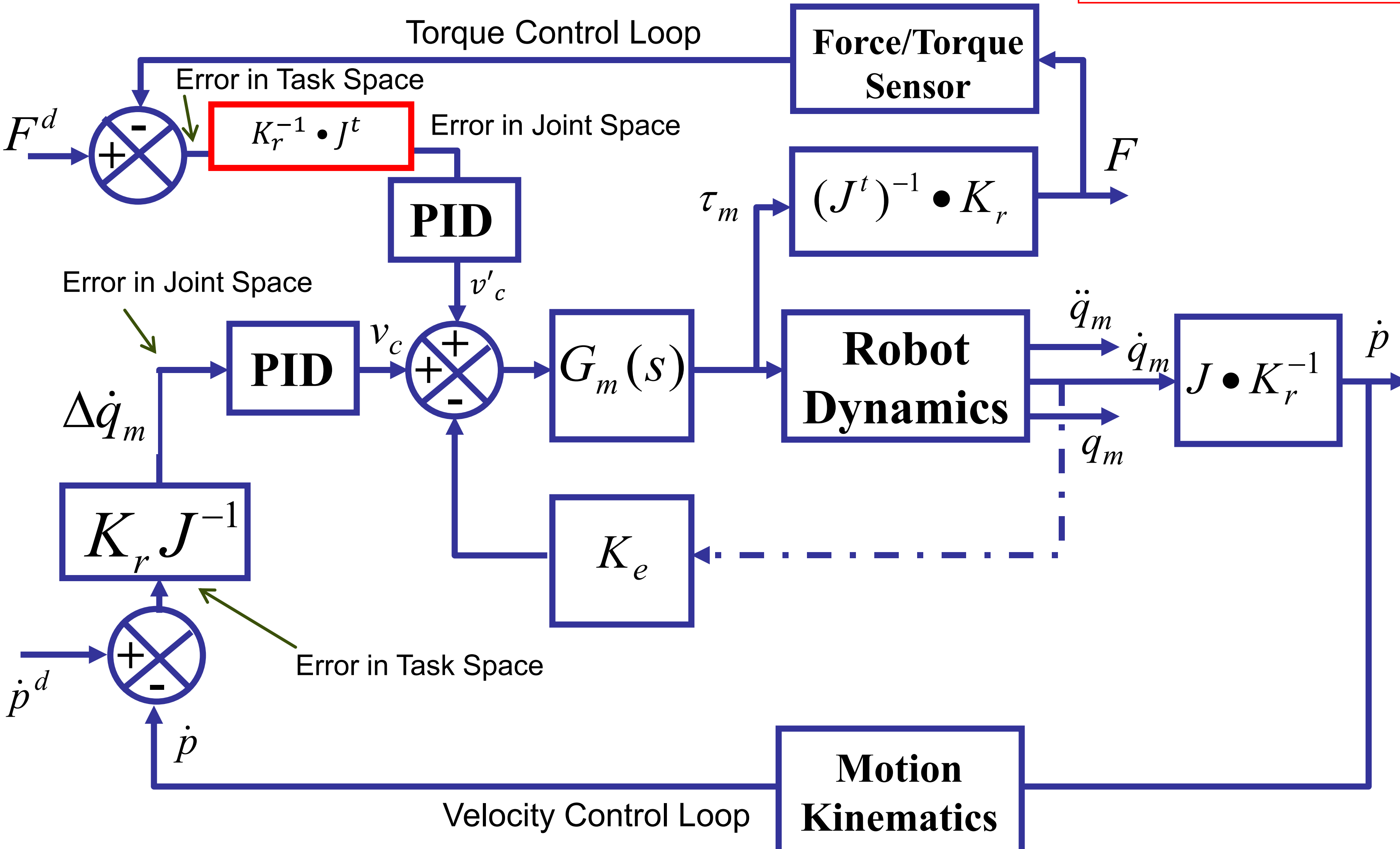


MIMO = Sum of SISOs

$$\tau_m = K_r^{-1} \bullet J^t \bullet F \quad \Rightarrow \quad F = (J^t)^{-1} \bullet K_r \bullet \tau_m$$

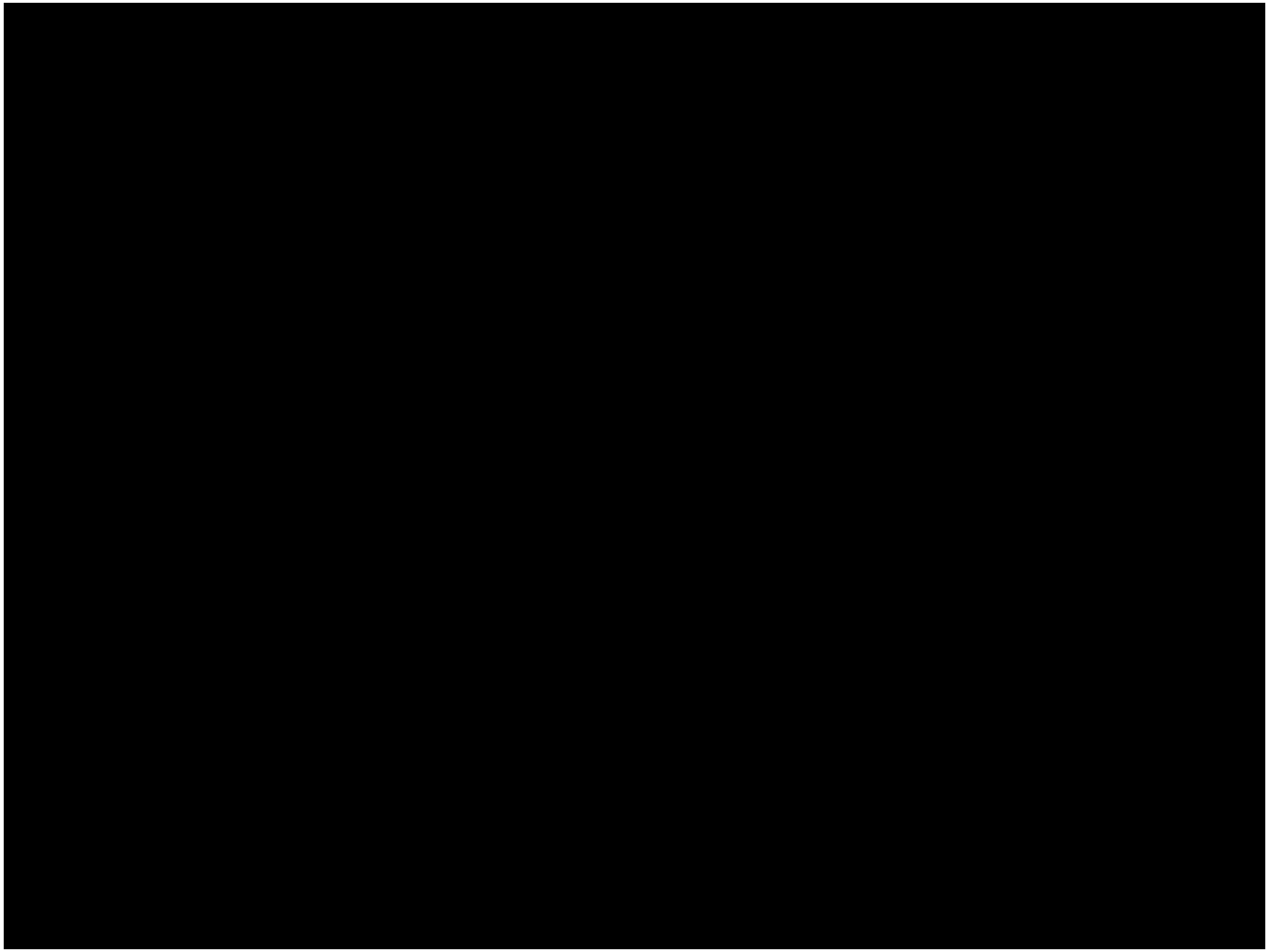
Design of Error Control System …

$$\tau_m = K_r^{-1} \bullet J^t \bullet F$$



Example 1 of Robot Performing Constrained Motions in Task Space

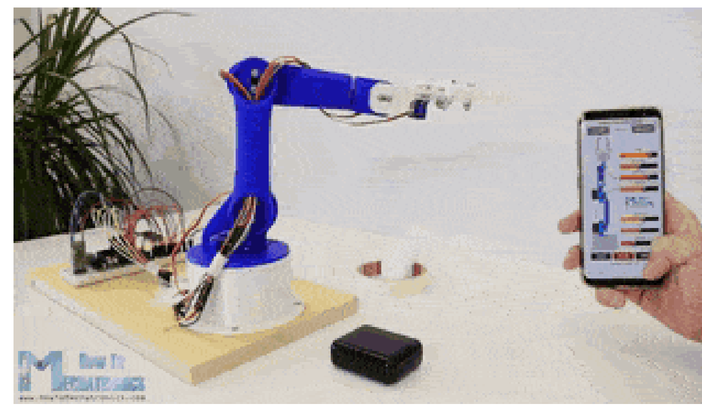
Year 2008



Example 2 of Robot Performing Constrained Motion in Task Space

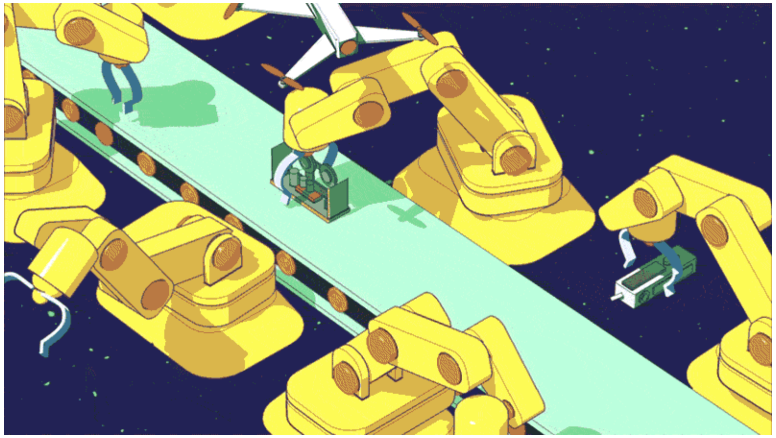
Summary of Lecture 5

- Task Space
- Control Input
- Control Feedback
- Control of Unconstrained Motion
- Control of Constrained Motion



Summary of Module 4

- Dynamics under Control
- Signal Flow Diagram
- Design of Control Systems
- Control in Joint-Space
- Control in Task-Space



"Ask not what your country can do for you – ask what you can do for your country," - John F. Kennedy

"Do not think that you are needy – think that you are needed in the world", - Manis Friedman

"Study will make you knowledgeable, resourceful, and hence more needed", - Xie Ming

Thank You for Listening!